# Study Guide on Financial Economics in Ratemaking for the Society of Actuaries (SOA) Exam GIADV: Advanced Topics in General Insurance

**(Based on Steven P. D'Arcy's and Michael A. Dyer's Paper, *"*[Ratemaking: A Financial Economics Approach]*")***

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**Source:** D'Arcy, Stephen P., and Dyer, Michael A., "Ratemaking: A Financial Economics Approach," PCAS LXXXIV, 1997.

**Problem FER-1.**
**(a)** In the Capital Asset Pricing Model (CAPM), for which type of risk are investors compensated, and for which are they not compensated?
**(b)** Given the following terms, provide the formula for $E(R_i)$, the expected return on a security, under the CAPM:
● $R_f$ = Risk-free rate;
● $E(R_m)$ = Expected return on the market portfolio;
● $\beta_i = Cov(R_i, R_m)/Var(R_m)$ = Beta of security i.
**(c)** What name is used for the expression $E(R_m) - R_f$? (D'Arcy and Dyer, p. 2)
**(d)** Based on what is the expression $E(R_m) - R_f$ frequently determined? (D'Arcy and Dyer, p. 3)

**Solution FER-1.**
**(a)** Investors **are compensated** for bearing **systematic risk**, which cannot be diversified away by adding more stocks to a portfolio. Investors **are not compensated** for bearing **unsystematic, diversifiable risk.** (D'Arcy and Dyer, p. 2)
**(b)** $E(R_i) = R_f + \beta_i[E(R_m)- R_f]$
**(c)** $E(R_m) - R_f$ is called the **market risk premium.**
**(d)** The market risk premium is frequently determined based on **historical experience.** (D'Arcy and Dyer, p. 3)

**Problem FER-2.** You are given the following for a particular security:
● The beta of the security is 0.9.
● The risk-free rate is assumed to be 3%.
● The expected return on the market portfolio is assumed to be 11%.
**(a)** Calculate the *market risk premium* using this situation.
**(b)** Use the Capital Asset Pricing Model (CAPM) to calculate the expected return for this security.

**Solution FER-2.**
**(a)** The market risk premium is $E(R_m)- R_f$ = 11% - 3% = **8%**.
**(b)** We use the formula $E(R_i) = R_f + \beta_i[E(R_m)- R_f]$ = 3% + 0.9*8% = $E(R_i)$ = **10.2%**.

**Problem FER-3.** You are given the following:
● The expected return on a security is 19%.
● The market risk premium is 10%.
● The risk-free rate is 2%.
Using the Capital Asset Pricing Model (CAPM), calculate the beta of this security.

**Solution FER-3.** We use the formula $E(R_i) = R_f + \beta_i[E(R_m)- R_f]$, rearranging it to $\beta_i[E(R_m)- R_f] = E(R_i) - R_f \rightarrow \beta_i = [E(R_i) - R_f ]/[E(R_m)- R_f]$.
We are given that $E(R_i)$ = 19%, $R_f$ = 2%, and $[E(R_m)- R_f]$ = 10%. Thus, $\beta_i$ = (19%-2%)/10% = $\boldsymbol{\beta_i = 1.7}$.

**Problem FER-4.** Give a reason why the Capital Asset Pricing Model (CAPM) cannot be fully tested. (D'Arcy and Dyer, p. 3)

**Solution FER-4.** The CAPM relies on the expected return of the market portfolio, which ought to include assets such as real estate, bonds, collectibles, and human capital. The total value of these assets is not practical to measure on a regular basis. In practice, empirical tests of the CAPM use a stock-market portfolio, which is an incomplete reflection of the true market portfolio. (D'Arcy and Dyer, p. 3)

**Problem FER-5.**
**(a)** What did Fama and French conclude about the relationship between beta and average returns over a long historical time period of 50 years?
**(b)** What two characteristics did Fama and French find to be more important than beta in explaining returns? (D'Arcy and Dyer, p. 3)

**Solution FER-5.**
**(a)** Fama and French concluded that the relationship between beta and average returns is insignificant.
**(b) Size** and **ratio of book value to market value** are more important than beta in explaining returns, according to Fama and French. (D'Arcy and Dyer, p. 3)

**Problem FER-6.** ***Fill in the blanks*** (D'Arcy and Dyer, p. 3): Merton Miller and Franco Modigliani posited in 1958 that the value of the firm is independent of the ________ and the _____________ chosen by the firm. This was a controversial conclusion because it used assumptions such as ____________.

**Solution FER-6.** Merton Miller and Franco Modigliani posited in 1958 that the value of the firm is independent of the **level of debt** and the **dividend payout level** chosen by the firm. This was a controversial conclusion because it used assumptions such as **no taxes**. (D'Arcy and Dyer, p. 3)

**Problem FER-7.** What option can stockholders of a corporation be thought to have merely by owning stock of that corporation? (D'Arcy and Dyer, p. 4)

**Solution FER-7.** Stockholders of a corporation can be thought to have an **option on the company's assets being greater than its liabilities**, since stockholders receive the difference between assets and liabilities when it is positive, but receive nothing when liabilities exceed assets. (D'Arcy and Dyer, p. 4)

**Problem FER-8.** You are given the following definitions for an insurer:
• IA = Investable assets;
• IR = Investment return;
• P = Premium;
• S = Owners' equity in the insurer;
• UPM = Underwriting profit margin.
Provide the formula for TRR, the target total rate of return for an insurer, using the Target Total Rate of Return Model. (D'Arcy and Dyer, pp. 5-6)

**Solution FER-8. TRR = (IA/S)(IR) + (P/S)(UPM)**

**Problem FER-9.** Using the Target Total Rate of Return Model, calculate the target total rate of return for an insurer with the following characteristics:
• Annual premium of 350,000
• Owners' equity of 900,000
• Investable assets of 1,200,000
• Investment return of 5%
• Underwriting profit margin of 3%

**Solution FER-9.** We use the formula TRR = (IA/S)(IR) + (P/S)(UPM) = (1,200,000/900,000)*(5%) + (350,000/900,000)*(3%) = **TRR = 7.833333333%**.

**Problem FER-10.** Using the Target Total Rate of Return Model, calculate the desired underwriting profit margin for an insurer with the following characteristics:
• Annual premium of 3,000,000
• Owners' equity of 8,600,000
• Investable assets of 5,000,000
• Investment return of 9%
• Target total rate of return of 7%

**Solution FER-10.** We use the formula TRR = (IA/S)(IR) + (P/S)(UPM) and rearrange it to calculate UPM: (P/S)(UPM) = TRR - (IA/S)(IR) → UPM = S[TRR - (IA/S)(IR)]/P = 8,600,000*[7% - (5,000,000/8,600,000)*9%]/3,000,000 = **UPM = 5.066666667%.**

**Problem FER-11.** You are given the following definitions for an insurer:
• IA = Investable assets;
• IR = Investment return;
• P = Premium;
• S = Owners' equity in the insurer;
• UPM = Underwriting profit margin.
● $R_f$ = Risk-free rate;
● $E(R_m)$ = Expected return on the market portfolio;
● $\beta_e$ = Beta of the insurer
Provide the formula for UPM, the insurer's underwriting profit margin, using the Target Total Rate of Return Model combined with a Capital Asset Pricing Model (CAPM). (D'Arcy and Dyer, p. 6)

**Solution FER-11.** We recall the formula UPM = S[TRR - (IA/S)(IR)]/P.
Under the CAPM, TRR = $R_f + \beta_e[E(R_m) - R_f]$, so **UPM = $S[R_f + \beta_e[E(R_m) - R_f] - (IA/S)(IR)]/P$**.

**Problem FER-12.** For a particular insurer, you know the following:
• Annual premium is 4,000,000.
• Owners' equity is 6,600,000.
• Investable assets are 11,000,000.
• The insurer's investment rate of return is 8%.
• The risk-free rate is 4%.
• The return on the market portfolio is 11%.
• The insurer's beta is 1.35.
Use the Target Total Rate of Return Model combined with a Capital Asset Pricing Model (CAPM) to calculate the insurer's underwriting profit margin.

**Solution FER-12.** We use the formula UPM = $S[R_f + \beta_e[E(R_m) - R_f], - (IA/S)(IR)]/P$ = 6,600,000*[4% + 1.35*(11% - 4%) – (11,000,000/6,600,000)*8%]/4,000,000 = **UPM = 0.1925%**.

**Problem FER-13.** You are performing an analysis of asset allocation among two investments – A and B, using the assumptions in a CAPM two-asset allocation approach.
● A is a risk-free investment with a rate of return of $R_f$.
● B is a risky investment with an expected rate of return of $E(R_k)$ and standard deviation of $\sigma_k$.
● Let W be the weight assigned to the risky investment B.

**(a)** Give the formula for $E(R_p)$, the expected return on the combination portfolio of A and B. (D'Arcy and Dyer, p. 11)

**(b)** Give the formula for $\sigma_p$, the standard deviation of the combination portfolio of A and B. (D'Arcy and Dyer, p. 12)
**(c)** On a plot of standard deviation (risk) on the horizontal axis and expected return on the vertical axis, what is the name of the line between the points denoting the risk/return combinations for A and B?

**Solution FER-13.**
**(a)** $E(R_p) = (1-W)*R_f + W*E(R_k)$.
**(b)** $\sigma_p = W*\sigma_k$.
**(c)** The line is called the **Capital Allocation Line (CAL)**

**Problem FER-14.** You are performing an analysis of asset allocation among two investments – A and B, using the assumptions in a CAPM two-asset allocation approach.
● A is a risk-free investment with a rate of return of 4%.
● B is a risky investment with an expected rate of return of 40% and a standard deviation of 33%.

**(a)** If a 70% of the portfolio is invested in asset B, what is the expected rate of return on the combination portfolio of A and B?
**(b)** If a 70% of the portfolio is invested in asset B, what is the standard deviation of the combination portfolio of A and B?
**(c)** What percentage of the portfolio should be invested in asset B in order to produce a portfolio expected rate of return of 18%?
**(d)** What percentage of the portfolio should be invested in asset B in order to produce a portfolio standard deviation of 12%?

**Solution FER-14.**
**(a)** We use the formula $E(R_p) = (1-W)*R_f + W*E(R_k)$, where W = 0.7.
Thus, $E(R_p) = (1-0.7)*4\% + 0.7*40\%$ = **$E(R_p) = 29.2\%$**.

**(b)** We use the formula $\sigma_p = W*\sigma_k$. Thus, $\sigma_p = 0.7*33\%$ = **$\sigma_p = 23.1\%$**.

**(c)** We use the formula $E(R_p) = (1-W)*R_f + W*E(R_k)$, where we are given $E(R_p) = 18\%$. We rearrange the formula to solve for W: $E(R_p) = R_f - W*R_f + W*E(R_k) \rightarrow$
$E(R_p) - R_f = W*(E(R_k) - R_f) \rightarrow W = [E(R_p) - R_f]/[E(R_k) - R_f] = (18\% - 4\%)/(40\% - 4\%) = W =$ 0.388888889, so **approximately 38.89% of the portfolio should be invested in asset B**.

**(d)** We use the formula $\sigma_p = W*\sigma_k$. We are given that $\sigma_p = 12\%$, and thus $W = \sigma_p / \sigma_k = 12\%/33\% = W = 0.363636363636$, so **approximately 36.36% of the portfolio should be invested in asset B**.

**Problem FER-15.** Given a portfolio P comprised of asset 1 and asset 2, with weight W assigned to asset 2, what is the general formula for $\sigma_p$, the standard deviation of the portfolio? Use the following terms:
- $\sigma_1$ = Standard deviation of asset 1
- $\sigma_2$ = Standard deviation of asset 2
- $R_1$ = Rate of return on asset 1
- $R_2$ = Rate of return on asset 2

(D'Arcy and Dyer, p. 11)

**Solution FER-15.** $\boldsymbol{\sigma_p = \sqrt{[(1-W)^2*\sigma_1^2 + 2*(1-W)*W*Cov(R_1, R_2) + W^2*\sigma_2^2]}}$

**Problem FER-16.** You are given the following information about a portfolio comprised of two assets, Q and H:
- 55% of the portfolio consists of Q, and 45% consists of H.
- The covariance of the rates of return of Q and H is 0.36.
- The standard deviation of Q is 35%.
- The standard deviation of H is 19%.

What is the portfolio standard deviation?

**Solution FER-16.** We let W be the weight assigned to H, i.e., 45%. Then we use the formula $\sigma_p = \sqrt{[(1-W)^2*\sigma_Q^2 + 2*(1-W)*W*Cov(R_Q, R_H) + W^2*\sigma_H^2]} = \sqrt{[(0.55)^2*0.35^2 + 2*0.55*0.45*0.36 + (0.45)^2*0.19^2]} = \sqrt{(0.2225665)} = \sigma_p = 0.4717695412 =$ **47.17695412%**.

**Problem FER-17.** You are performing an analysis of asset allocation among two investments – A and B, using the assumptions in a CAPM two-asset allocation approach.
- A is a risk-free investment with a rate of return of 4%.
- B is a risky investment with an expected rate of return of 40% and a standard deviation of 33%.

**(a)** If an investor who has $100,000 desires a return of 50% and can only invest in assets A or B, what could the investor do under these assumptions?

**(b)** What would be the standard deviation of the portfolio selected by this investor? (D'Arcy and Dyer, p. 14)

**Solution FER-17.**

**(a)** We use the formula $W = [E(R_p) - R_f]/[E(R_k) - R_f]$ to calculate the weight in the risky asset B.
Here, $E(R_p) = 50\%$, $R_f = 4\%$, and $E(R_k) = 40\%$.
Thus, W = (50% - 4%)/(40% - 4%) = 1.277777777778.

This means that the investor would need to invest $127,777.78 in Asset B in order to get the desired return of 50%. Since the investor only has $100,000, the investor would need to borrow the remaining $27,777.78 at the risk-free rate and invest this money in Asset B.

**(b)** We use the formula $\sigma_p = W*\sigma_k$., where W was calculated as 1.277777777778 and $\sigma_k$ = 33%. Thus, $\sigma_p$ = 1.277777777778*33% = **42.1666666667%.**

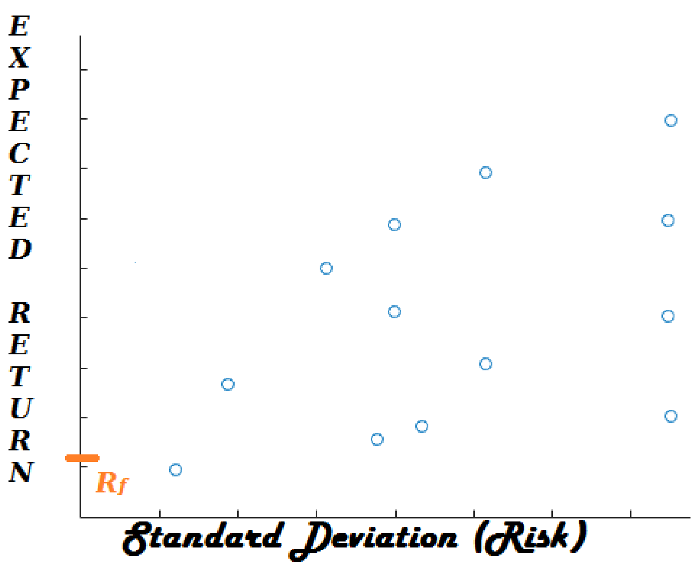
**Problem FER-18.** Given any number of assets i with returns $R_i$, with each asset being given a weight $W_i$ for the portfolio p, provide formulas for the following.

**(a)** $E(R_p)$, the expected return on portfolio p.
**(b)** $\sigma_p^2$, the variance of portfolio p.

You are given that $\sigma_{i,j}$ is the covariance between stocks i and j, where $i \neq j$.

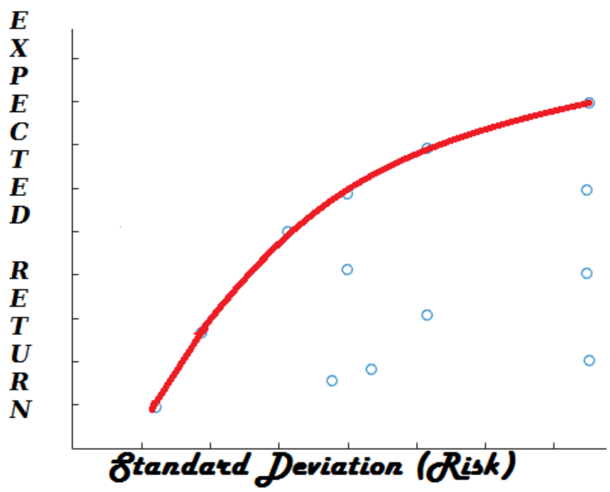
**Solution FER-18.**
**(a)** $E(R_p) = \Sigma_i[W_i*E(R_i)]$
**(b)** $\sigma_p^2 = \Sigma_i[W_i*\sigma_i^2] + \Sigma_j\Sigma_i[W_i*W_j*\sigma_{i,j}]$

**Problem FER-19.** On the plot below representing possible portfolios (where $R_f$, the risk-free rate, is identified on the vertical axis), draw the following:
**(a)** The efficient frontier (D'Arcy and Dyer, p. 17);
**(b)** The Capital Allocation Line (D'Arcy and Dyer, p. 18);
**(c)** The efficient risky portfolio M (D'Arcy and Dyer, p. 18).



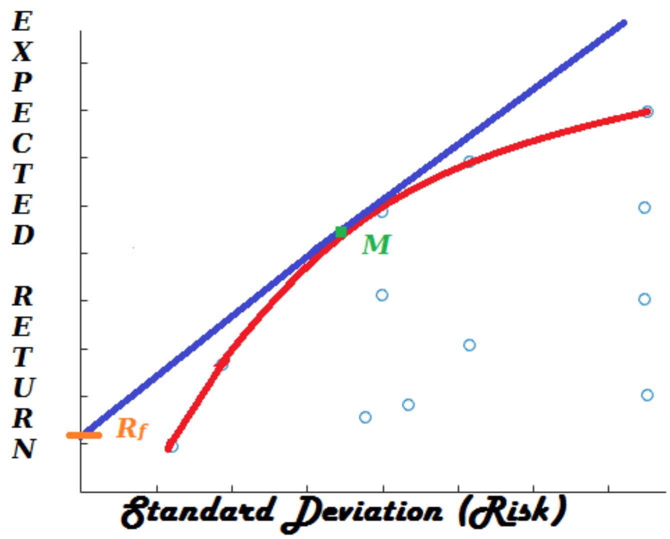
**Solution FER-19.**

**(a)** The efficient frontier is the curve representing the portfolios which provide the highest return for a given level of risk and the lowest risk for a given level of return. (D'Arcy and Dyer, p. 17). This is represented by the red curve below.




**(b)** The Capital Allocation Line is tangent to the efficient frontier and intersects the vertical axis at $R_f$, the risk-free rate. This is the blue line in the diagram below.

**(c)** The efficient risky portfolio M is the point at which the Capital Allocation Line touches the efficient frontier.



**Problem FER-20.** If the Capital Asset Pricing Model (CAPM) is correct and an investor wished to maximize returns for a given level of risk, why would the investor *not* invest in a portfolio Q along the efficient frontier, which is *not* the efficient risky-asset portfolio M at which the Capital Allocation Line (CAL) is tangent to the efficient frontier? (D'Arcy and Dyer, p. 18)

**Solution FER-20.** If the CAPM is correct and Q is on the efficient frontier but is not M, then there will be a point on the CAL that is above Q for the same value of standard deviation (risk) – meaning that this point provides a higher expected return for the same amount of risk. The way for an investor to realize this risk/return combination would be to invest a portion of assets in the efficient risky-asset portfolio M and to invest the remaining portion in the risk-free asset (earning the risk-free rate of return). The weighted-average return and standard deviation of this combination would correspond to the point on the CAL that is above Q and would provide the desired higher expected return, so investing in Q would be sub-optimal.

**Problem FER-21.** ***Fill in the blanks*** (D'Arcy and Dyer, p. 19): Generally, by choosing assets at random and adding them to the portfolio, the investor can __________ [increase or reduce?] the overall risk of a portfolio. However, eventually, the investor reaches a ________________ where more assets added to the portfolio do not significantly __________ [increase or reduce?] the total risk of this portfolio. Three names for the remaining risk at this stage are ____________, ____________, or ____________ risk.

**Solution FER-21.** Generally, by choosing assets at random and adding them to the portfolio, the investor can **reduce** the overall risk of a portfolio. However, eventually, the investor reaches a **saturation point** where more assets added to the portfolio do not significantly **reduce** the total risk of this portfolio. Three names for the remaining risk at this stage are **nondiversifiable**, **systematic**, or **market** risk. (D'Arcy and Dyer, p. 19)

**Problem FER-22.** For this problem, assume that the Capital Asset Pricing Model (CAPM) is correct.
**(a)** What are three names for risk associated with individual assets that can be diversified away? (D'Arcy and Dyer, p. 19)
**(b)** If an investor holds a portfolio that consists of a combination of the risk-free asset and the efficient market portfolio M, explain whether the investor would need to be concerned with the risk in type (a). (D'Arcy and Dyer, pp. 19-21)

**Solution FER-22.**
**(a)** Risk that can be diversified away is **unsystematic, company-specific, diversifiable risk.**
**(b)** Diversifiable risk is not a matter of concern if an investor holds a combination of the risk-free asset and the efficient market portfolio M, since this portfolio is well-diversified and is only subject to market risk that cannot be diversified away.

**Problem FER-23.** If the Capital Asset Pricing Model (CAPM) is correct, $R_f$ is the risk-free rate, $E(R_m)$ is the expected return of the market portfolio, and $\beta_i$ is an individual asset i's sensitivity to movements in the market portfolio, then address the following:

**(a)** What is the expression for the *asset risk premium* – the excess return demanded on an individual asset added to a well-diversified portfolio?
**(b)** What is the formula for the expected return $E(R_i)$ of asset i?
(D'Arcy and Dyer, p. 22)

**Solution FER-23.**
**(a)** The asset risk premium is **$\beta_i*[E(R_m) - R_f]$**.
**(b) $E(R_i) = R_f + \beta_i*[E(R_m) - R_f]$**.

**Problem FER-24.** For this problem, assume that the Capital Asset Pricing Model (CAPM) is correct. You are considering adding an asset Z to a well-diversified portfolio. Asset Z's beta is 1.44, the risk-free rate is 5%, and the expected return of the market portfolio is 16%.
**(a)** What is the asset risk premium for Z?
**(b)** What is the expected return on asset Z?

**Solution FER-24.**
**(a)** The asset risk premium is $\beta_Z[E(R_m) - R_f]$ = 1.44*[16% - 5%] = **15.84%**.
**(b)** The expected return on asset Z is $R_f + \beta_Z[E(R_m) - R_f]$ = 5% + 15.84% = **20.84%**.

**Problem FER-25.** Identify the six key assumptions of the Capital Asset Pricing Model (CAPM). (D'Arcy and Dyer, pp. 22-23)

**Solution FER-25.** The six key assumptions of the Capital Asset Pricing Model (CAPM) are as follows:
**1.** Investors are risk-averse diversifiers who try to maximize expected return and minimize risk.
**2.** Investors are price takers, in that they act as if their trades have no effect on asset prices.
**3.** Investors have homogeneous or identical expectations about asset expected returns and standard deviations.
**4.** Investors have no transaction costs or taxes.
**5.** Investors can borrow or invest at the risk-free rate without any limit.
**6.** Assets are infinitely divisible.

**Problem FER-26.** Assume that the Capital Asset Pricing Model (CAPM) is correct, $R_f$ is the risk-free rate, and $E(R_m)$ is the expected return of the market portfolio. What is the name of the line with slope $E(R_m) – R_f$ and vertical-axis intercept of $R_f$, where the horizontal axis consists of values of beta, and the vertical axis consists of values for the expected return? (D'Arcy and Dyer, p. 23)

**Solution FER-26.** This line is the **Security Market Line (SML).**

**Problem FER-27.**
**(a)** What does it mean for an asset to have negative beta?
**(b)** Give an example of a type of asset that might have a negative beta. (D'Arcy and Dyer, p. 23)

**Solution FER-27.**
**(a)** An asset with negative beta would have returns that move in the opposite direction of the return of the market portfolio.
**(b) Gold** and **gold-mining stocks** are examples of assets with negative beta, since they tend to have positive returns when the market falls. (D'Arcy and Dyer, p. 23)

**Problem FER-28.**
**(a)** What is the implication of the Capital Asset Pricing Model (CAPM) for prices of assets whose returns are *above* the Security Market Line (SML)?
**(b)** What is the implication of the Capital Asset Pricing Model (CAPM) for prices of assets whose returns are *below* the Security Market Line (SML)?
(D'Arcy and Dyer, pp. 23-25)

**Solution FER-28.**
**(a)** An asset whose market return is above the SML will have its price be bid up until the rate of return decreases to a point on the SML.
**(b)** An asset whose market return is below the SML will have its price be bid down until the rate of return increases to a point on the SML.

**Problem FER-29.** D'Arcy and Dyer mention (p. 25) that the Capital Asset Pricing Model (CAPM) requires the use of the market risk premium and past market portfolio returns and individual asset returns to arrive at beta estimates for individual assets. What likely unrealistic assumptions does such use make?

**Solution FER-29.** Reliance on these relationships assumes that they are stable, whereas they are likely to change over time. (D'Arcy and Dyer, p. 25)

**Problem FER-30.** The following questions pertain to Fairley's insurance version of the Capital Asset Pricing Model (CAPM). (D'Arcy and Dyer, p. 26)
**(a)** What does the funds-generating coefficient k represent?
**(b)** What does the insurer's underwriting beta $\beta_u$ signify?
**(c)** If $R_f$ is the risk-free rate and $E(R_m)$ is the expected return of the market portfolio, what is the formula for the underwriting profit margin, UPM?

**Solution FER-30.**
**(a)** The funds-generating coefficient k represents the average time the insurer holds premiums.
**(b)** The insurer's underwriting beta $\beta_u$ signifies the relation of the insurer's underwriting returns to the market portfolio returns.
**(c)** $\mathbf{UPM = -k*R_f + \beta_u *[E(R_m) - R_f]}$. (D'Arcy and Dyer, p. 26)

**Problem FER-31.** The following questions pertain to Fairley's insurance version of the Capital Asset Pricing Model (CAPM). (D'Arcy and Dyer, p. 26)

**(a)** What does the Fairley CAPM ignore about the insurer's situation?
**(b)** What does the Fairley CAPM assume about insurers' investment earnings?
**(c)** What is the name for the term $-k*R_f$ in the formula for underwriting profit margin, where k is the funds-generating coefficient and $R_f$ is the risk-free rate?

**Solution FER-31.**
**(a)** The Fairley CAPM ignores the **actual insurance company investment performance.**
**(b)** The Fairley CAPM assumes that **insurers will earn the risk-free rate of return** and will **incur the gain and loss on any risky investment.**
**(c)** The term $-k*R_f$ is called the **investment inflow rate of return.** (D'Arcy and Dyer, p. 26)

**Problem FER-32.**
**(a)** For Fairley's insurance version of the Capital Asset Pricing Model (CAPM), how is the underwriting beta $\beta_u$ frequently estimated?
**(b)** Let $R_m$ be the return on the market portfolio and $R_u$ be the insurer's underwriting return. What is the formula for the estimate for $\beta_u$ in the approach discussed in part (a)? (D'Arcy and Dyer, p. 26)

**Solution FER-32.**
**(a)** The underwriting beta $\beta_u$ is frequently estimated by running a simple linear regression of underwriting returns against the returns of the market portfolio.
**(b)** The estimate of $\beta_u$ from the linear regression is $\mathbf{Cov(R_u,R_m)/Var(R_m)}$.

**Problem FER-33.** ***Fill in the blanks*** (D'Arcy and Dyer, p. 27): For Fairley's insurance version of the Capital Asset Pricing Model (CAPM), the funds-generating coefficient k can be estimated using the insurer's projection of the ____________ and ____________ _____________ pattern expected from the insurer's ___________. The estimate of k would be the weighted average of the length of time expected between the ______________ and the _______________ among these different ____________.

**Solution FER-33.** For Fairley's insurance version of the Capital Asset Pricing Model (CAPM), the funds-generating coefficient k can be estimated using the insurer's projection of the **loss** and **expense payment** pattern expected from the insurer's **current exposures**. The estimate of k would be the weighted average of the length of time expected between the **receipt of premium** and the **payment of losses and expenses** among these different **exposures**. (D'Arcy and Dyer, p. 27)

**Problem FER-34.** You are given the following about an insurer's book of business:
• The correlation coefficient of the insurer's returns with those of the market portfolio is 0.44.
• 65% of the insurer's book of business consists of home insurance policies where the expected loss payment occurs 0.8 years after receipt of premium.
• 20% of the insurer's book of business consists of workers' compensation policies where the expected loss payment occurs 2.1 years after receipt of premium.
• 15% of the insurer's book of business consists of medical malpractice policies where the expected loss payment occurs 5.5 years after receipt of premium.
• The risk-free rate is 3%.
• The rate of return on the market portfolio is 16%.

Use Fairley's insurance version of the Capital Asset Pricing Model (CAPM) to calculate the following:
**(a)** An estimate of the funds-generating coefficient k;
**(b)** The underwriting profit margin of the insurer.

**Solution FER-34.**
**(a)** The funds-generating coefficient k would be a weighted average of the average loss-payment times within the insurer's book of business: 0.65*0.8 + 0.2*2.1 + 0.15*5.5 = **k = 1.765**.

**(b)** We use the formula UPM = -k*$R_f$ + $\beta_u$*[E($R_m$) - $R_f$]. We are given $R_f$ = 3%, E($R_m$) = 16%, and $\beta_u$ = 0.44. Thus, UPM = -1.765*3% + 0.44*(16% - 3%) = **UPM = 0.425%**.

**Problem FER-35.** For the Hill/Modigliani tax version of the Capital Asset Pricing Model (CAPM), you are given the following:
• $R_f$ is the risk-free rate.
• E($R_m$) is the expected return of the market portfolio.
• k is the funds-generating coefficient.
• $\beta_u$ is the insurer's underwriting beta.
• S is the insurer's equity.
• P are the insurer's annual premiums.
• T is the tax rate on underwriting income.
• $T_A$ is the tax rate on investment income.

**(a)** What is the formula for UPM, the underwriting profit margin? (D'Arcy and Dyer, pp. 28-29)
**(b)** If an insurer's investment portfolio contains investments that are taxable at different rates, what would $T_A$ represent? (D'Arcy and Dyer, p. 29)

**Solution FER-35.**
**(a) UPM = -k*$R_f$*(1- $T_A$)/(1-T) + $\beta_u$*[E($R_m$) - $R_f$] + (S/P)*$R_f$*($T_A$)/(1-T).**
**(b)** $T_A$ would represent the **weighted average** of the different tax rates on the insurer's investment portfolio. (D'Arcy and Dyer, p. 29)

**Problem FER-36.** The following questions apply to the Hill/Modigliani tax version of the Capital Asset Pricing Model (CAPM).
**(a)** Describe conceptually the first term $-k*R_f*(1-T_A)/(1-T)$ in the formula for the underwriting profit margin.
**(b)** Describe conceptually the second term $\beta_u*[E(R_m) - R_f]$ in the formula for the underwriting profit margin. (D'Arcy and Dyer, p. 29)

**Solution FER-36.**
**(a)** The first term $-k*R_f*(1-T_A)/(1-T)$ is the after-tax adjusted risk-free return on the insurer's investment portfolio during the time lag between receipt of premiums and payment of losses.
**(b)** The second term $\beta_u*[E(R_m) - R_f]$ is the underwriting risk premium.
(D'Arcy and Dyer, p. 29)

**Problem FER-37.**
You are given the following about an insurer's book of business:
• The correlation coefficient of the insurer's returns with those of the market portfolio is 0.44.
• The risk-free rate is 3%.
• The rate of return on the market portfolio is 16%.
• The funds-generating coefficient k for this insurer is 1.765.
• The corporate tax rate is 30%.
• 12% of the insurer's investment holdings are in tax-exempt bonds.
• 35% of the insurer's investment holdings are in stocks that are taxed at 22%.
• 53% of the insurer's investment holdings are taxed as ordinary taxable income.
• The insurer's equity-to-premium ratio is 1.4.

Use the Hill/Modigliani tax version of the Capital Asset Pricing Model (CAPM) to calculate the following:
**(a)** The tax rate on the insurer's investment income, $T_A$;
**(b)** The insurer's underwriting profit margin.

**Solution FER-37.**
**(a)** The tax rate on the insurer's investment income is the weighted average of tax rates on the investment portfolio: 0.12*0% + 0.35*22% + 0.53*30% = $\mathbf{T_A = 23.6\%}$.
**(b)** We use the formula $UPM = -k*R_f*(1-T_A)/(1-T) + \beta_u*[E(R_m) - R_f] + (S/P)*R_f*(T_A)/(1-T)$ = -1.765*3%*(1-0.236)/(1-0.3) + 0.44*(16% - 3%) + 1.4*3%*0.236/(1-0.3) = -5.779114286% + 5.72% + 1.416% = **UPM = 1.356885714%**.

**Problem FER-38.**
**(a)** What risk have models that apply the Capital Asset Pricing Model (CAPM) to insurance been criticized for ignoring? (D'Arcy and Dyer, p. 30)
**(b)** What other cost would the CAPM ignore? (D'Arcy and Dyer, p. 31)
**(c)** If this criticism is correct, what is the implication with regard to the results of the use of CAPM in insurance pricing? (D'Arcy and Dyer, p. 30)

**Solution FER-38.**
**(a)** Models that apply the CAPM to insurance have been criticized for ignoring **risk unique to insurance that is not systematic with investment risk**. Such risk would be unrelated to movements of the stock market. (D'Arcy and Dyer, p. 30)
**(b)** The CAPM would also ignore **bankruptcy costs**. (D'Arcy and Dyer, p. 31)
**(c)** If this criticism is correct, then models that apply CAPM to insurance pricing would **underprice** insurance (since they are ignoring additional sources of risk and cost). (D'Arcy and Dyer, p. 30)

**Problem FER-39.** ***Fill in the blanks*** (D'Arcy and Dyer, p. 31): Discounted cash-flow (DCF) analysis converts cash flows from different times to a common ____________ based on the ____________ so that cash inflows and outflows can be more easily compared. It can be useful in insurance where differences in timing between ____________ and ____________ are common.

**Solution FER-39.** Discounted cash-flow (DCF) analysis converts cash flows from different times to a common **point** based on the **time value of money** so that cash inflows and outflows can be more easily compared. It can be useful in insurance where differences in timing between **receipt of premiums** and **payment of losses** are common. (D'Arcy and Dyer, p. 31)

**Problem FER-40.** An insurer collects a premium of $5555 on a policy, where a fixed expense of $222 is paid outright. One year later, there will be a loss on the policy of $5033. The annual interest rate, at which the excess of premium over expenses will be invested, is 8%. What is the present value of the insurer's profit in this situation? (See D'Arcy and Dyer, p. 32.)

**Solution FER-40.** The present value of the insurer's profit is (Premium – Expense) – PV(Loss), where the present value of the loss, PV(Loss) = 5033/1.08 = 4660.185185.

Thus, the present value of the insurer's profit is (5555 – 222) – 4660.185185 = 672.8148148 = **$672.81.**

**Problem FER-41.** Assume the annual interest rate is 10%.
**(a)** What is the future value at time t = 3 years of an amount whose present value is 5353?
**(b)** What is the present value of an amount whose future value at time t = 5 years will be 6666? (See D'Arcy and Dyer, p. 33.)

**Solution FER-41.**
**(a)** We use the formula $FV_t = PV*(1+r)^t$, where we are given t = 3, PV = 5353, and r = 0.1. Thus, $FV_3 = 5353*1.1^3 =$ **7124.843.**
**(b)** We use the formula $PV = FV_t /(1+r)^t = 6666/1.1^5 =$ **4139.06154.**

**Problem FER-42.** Assume that a bond has a maturity value of 2000, payable in 3 years. The bond also pays annual interest of 300 per year, with the first payment occurring in 1 year and the last payment occurring in 3 years. The annual rate of return on the bond is 5%.
**(a)** What is the value of the bond today? (See D'Arcy and Dyer, p. 34.)
**(b)** If an investor were offered an opportunity to purchase this bond for 2400, should the investor accept the offer?

**Solution FER-42.**
**(a)** The value of the bond today is the sum of the present values of each of its cash flows. Each cash flow is subject to the formula $PV = FV_t /(1+r)^t$. Thus, the present value of the bond is $300/1.05 + 300/1.05^2 + (2000 + 300)/1.05^3 =$ **2544.649606.**
**(b)** Because the purchase price of 2400 is less than the bond's value of 2544.649606, the investor would realize a larger rate of return than the required 5% from the purchase and so should accept the offer.

**Problem FER-43.** Let $CF_t$ be the cash flow at each time t. Let r be the annual rate of return. What is the formula for net present value (NPV) that can be used to determine whether to invest in a given project with cash flows $CF_t$? (D'Arcy and Dyer, p. 34)

**Solution FER-43.** $\mathbf{NPV = CF_0 + \Sigma_t[CF_t/(1+r)^t].}$

**Problem FER-44.** Assume you have a project with the following cash flows:
• Year 0 (now): -25,000
• Year 1: -12,000
• Year 2: 20,000
• Year 3: 25,000
• Year 4: 30,000
The annual interest rate used to discount cash flows is 6%.

**(a)** Find the net present value (NPV) of this project.
**(b)** On the basis of NPV, decide whether or not the project should be invested in. (See D'Arcy and Dyer, p. 35.)

**Solution FER-44.**
**(a)** We use the formula $NPV = CF_0 + \Sigma_t[CF_t/(1+r)^t] = -25000 + (-12000)/1.06 + 20000/1.06^2 + 25000/1.06^3 + 30000/1.06^4 =$ **NPV = 26,232.46606**.
**(b)** Because the NPV > 0, the project should indeed be invested in.

**Problem FER-45.**
**(a)** Define the *internal rate of return (IRR)* of a project.
**(b)** By what method is the IRR calculated?
**(c)** What is the decision rule that compares the IRR to the required rate of return for a project? (D'Arcy and Dyer, p. 36)

**Solution FER-45.**
**(a)** The IRR is the discount rate that gives the project a net present value (NPV) of zero.
**(b)** The IRR is calculated by trial and error or by iteration using computer programs.
**(c)** If the IRR is greater than the project's required rate of return, then accept the project. If the IRR is less than the project's required rate of return, then reject the project.
(D'Arcy and Dyer, p. 36)

**Problem FER-46.**
**(a)** What is a problem with the internal rate of return (IRR) calculation if cash flows for a project change signs (alternate between positive and negative) more than once?
**(b)** How can this problem be overcome in many cases? (D'Arcy and Dyer, pp. 36-37)

**Solution FER-46.**
**(a)** In a situation where cash flows alternate signs more than once, multiple IRRs can be generated.
**(b)** Even though multiple IRRs are generated, typically, only one of those will be reasonable. For instance, it a positive IRR and a negative IRR are generated, and a project has a positive NPV, then the positive IRR can be accepted as the reasonable value. (D'Arcy and Dyer, pp. 36-37)

**Problem FER-47.**
**(a)** For the Gordon growth model of stock valuation, if $r_s$ is the required return on a stock, and $D_t$ is the dividend expected at time t, what is the formula for the value of the stock V?
**(b)** If it is assumed that, after the current dividend $D_0$, dividends will grow at a constant annual growth rate g, what is the formula for V? (D'Arcy and Dyer, p. 37)
**(c)** In what circumstances could the formula in part **(b)** *not* be used? (D'Arcy and Dyer, p. 38)

**Solution FER-47.**
**(a)** $\mathbf{V = \Sigma_t[D_t/(1+r_s)^t]}$.
**(b)** $\mathbf{V = D_0*(1 + g)/(r_s - g)}$.
**(c)** The formula in part **(b)** cannot be used **if the growth rate is greater than or equal to the required stock return rate.** (D'Arcy and Dyer, p. 38)

**Problem FER-47.** Stock Q will be paying a dividend of 46 one second from now. Thereafter, the annual dividend will always grow by 5%. The required annual rate of return on Stock Q is 13%. What is the value of Stock Q using the Gordon growth model?

**Solution FER-47.** We use the formula $V = D_0*(1 + g)/(r_s - g) = 46*(1.05)/(0.13 – 0.05)$ = **V = 603.75.**

**Problem FER-48.**
**(a)** What is the basic premise of the Risk-Adjusted Discount Technique?
**(b)** What does the term "risk-adjusted" mean in the context of this technique?
(D'Arcy and Dyer, p. 39)

**Solution FER-48.**
**(a)** On a risk-adjusted basis, the present value of the premium equals the present value of all the cash flows resulting from writing an insurance policy – losses, expenses, and taxes on both underwriting and investment income.
**(b)** The term "risk-adjusted" means that the interest rate selected to discount cash flows varies to account for the degree of risk inherent in the cash flow.
(D'Arcy and Dyer, p. 39)

**Problem FER-49.** The following notation is given:
• PV(x) = Present value of any x;
• L = Losses and loss-adjustment expenses;
• P = Premiums;
• E = Underwriting expenses;
• TUW = Taxes on underwriting profit or loss;
• TII = Taxes on investment income;
• UPM = Underwriting profit margin.

Using the Risk-Adjusted Discount Technique, provide formulas for the following:
**(a)** PV(P), the present value of premiums;
**(b)** UPM, the underwriting profit margin.
(D'Arcy and Dyer, p. 40)

**Solution FER-49.**
**(a) PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII)**
**(b) UPM = 1 – (L+E)/P.**

**Problem FER-50.** An insurer writes a policy with the following characteristics:
• All cash flows are discounted at the same interest rate of 5%, which is also the rate of return on the insurer's investments.
• Premium is collected immediately.
• Losses on the policy will be 300 and will be paid in exactly one year.
• Expenses on the policy are 43 and are paid immediately upon issuance.
• The policy is supported by equity of 200.
• All taxes on both underwriting and investment income are imposed at a rate of 20%.

**(a)** Calculate the premium that this insurer would charge using a discounted cash-flow technique.
**(b)** Calculate the underwriting profit margin for this policy.

**Solution FER-50.**
**(a)** We use the formula PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII), where PV(P) = P, since premium is collected immediately.
PV(L) = 300/1.05 = 285.7142857.
PV(E) = E = 43.

The underwriting income in one year will be P – L – E = P – (300 + 43) = P – 343. The tax on this amount will be 0.2*(P - 343), and the present value of this tax will be
PV(TUW) = 0.2*(P - 343)/1.05 = 0.1904761905P – 65.33333333333.
The investment income in one year will be (Equity + P – E)*0.05 = (P + 200 – 43)*0.05 = (P + 157)*0.05. The tax on this amount will be 0.2*0.05(P + 157) = 0.01*(P + 157). The present value of this tax will be P(TII) = 0.01*(P + 157)/1.05 = 0.0095238095P + 1.495238095.

Thus, P = 285.7142857 + 43 + 0.1904761905P – 65.33333333333 + 0.0095238095P + 1.495238095 → P = 0.2P + 264.8761905 → 0.8P = 264.8761905 → **P = 331.0952381**.
**(b)** We use the formula UPM = 1 – (L+E)/P = 1 – (300 + 43)/331.0952381 = -0.0359557026 = **UPM = -3.59557026%.**

**Problem FER-51.** Answer the following questions based on the Risk-Adjusted Discount Technique. (D'Arcy and Dyer, p. 41)
**(a)** Which cash flows would it *not* be reasonable to discount at a risk-free rate and why?
**(b)** For which cash flows would a risk-free discount rate be appropriate?

**Solution FER-51.**
**(a) Losses** should not be discounted at a risk-free rate, because they are not known with certainty but will rather vary around an expected value.
**(b)** It is appropriate to use a risk-free rate to discount **premium income, underwriting expenses,** and **taxes** that emanate from these certain cash flows – since these cash flows are known once the policy has been written. (D'Arcy and Dyer, p. 41)

**Problem FER-52.**
**(a)** Why could an insurance policy be seen as an asset with a negative beta within a Capital Asset Pricing Model (CAPM) framework?
**(b)** What does the view of an insurance policy as an asset with a negative beta imply about the relationship of the risk-adjusted discount rate for the insurance policy with respect to the risk-free rate? (D'Arcy and Dyer, p. 42)

**Solution FER-52.**
**(a)** An asset with a negative beta has value when the policyholder's tangible assets are reduced in value – which is the function of an insurance policy.
**(b)** The required return on an asset with negative beta is below the risk-free rate, and therefore the risk-adjusted discount rate for the insurance policy would be ***less than*** the risk-free rate.

**Problem FER-53.** An insurer writes a policy with the following characteristics:
• The risk-free rate is 5%, which is also the rate of return on the insurer's investments.
• The risk-adjusted discount rate is 2%.
• Premium is collected immediately.
• Losses on the policy will be 300 and will be paid in exactly one year.
• Expenses on the policy are 43 and are paid immediately upon issuance.
• The policy is supported by equity of 200.
• All taxes on both underwriting and investment income are imposed at a rate of 20%.

**(a)** Calculate the premium that this insurer would charge using a discounted cash-flow technique.
**(b)** Calculate the underwriting profit margin for this policy.

**Solution FER-53.**
**(a)** We use the formula PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII), where PV(P) = P, since premium is collected immediately.
PV(L) = 300/1.02 = 294.1176471.
PV(E) = E = 43.
The underwriting income in one year will be P – E – L. However, the discount rate applied to (P – E) (the risk-free rate) will be different from the discount rate applied to (-L) (the risk-adjusted discount rate. The present value of the underwriting income will be (P – 43)/1.05 – 300/1.02, and the present value of the tax on the underwriting income will be PV(TUW) = 0.2*(P – 43)/1.05 – 0.2*300/1.02 = 0.1904761905P – 67.0140056.
The investment income in one year will be (Equity + P – E)*0.05 = (P + 200 – 43)*0.05 = (P + 157)*0.05. The tax on this amount will be 0.2*0.05(P + 157) = 0.01*(P + 157). The present value of this tax will be P(TII) = 0.01*(P + 157)/1.05 = 0.0095238095P + 1.495238095.

Thus, P = 294.1176471 + 43 + 0.1904761905P – 67.0140056 + 0.0095238095P + 1.495238095 → P = 0.2P + 270.1036415 → 0.8P = 270.1036415 → **P = 337.6295518**.

**(b)** We use the formula UPM = 1 – (L+E)/P = 1 – (300 + 43)/337.6295518 = -0.0159063333 = **UPM = -1.59063333%**.

**Problem FER-54.** ***Fill in the blanks*** (D'Arcy and Dyer, p. 43): The effect of discounting loss payments at a risk-adjusted rate is to _____________ [increase or decrease?] the appropriate premium level and _____________ [increase or decrease?] the underwriting loss. The higher the tax rate, the _______________ [more or less?] the overall effect of a lower risk-adjusted discount rate would be.

**Solution FER-54.** The effect of discounting loss payments at a risk-adjusted rate is to **increase** the appropriate premium level and **decrease** the underwriting loss. The higher the tax rate, the **less** the overall effect of a lower risk-adjusted discount rate would be. (D'Arcy and Dyer, p. 43)

**Problem FER-55.** ***Fill in the blanks*** (D'Arcy and Dyer, p. 43): Surplus, or equity, is required to support not the writing of policies, but the ______________. Surplus is required in the event that _______ exceed the expected values so that the insurer can absorb the excess without ___________.

**Solution FER-55.** Surplus, or equity, is required to support not the writing of policies, but the **assumption of the obligation to pay claims**. Surplus is required in the event that **claims** exceed the expected values so that the insurer can absorb the excess without **defaulting on the commitment to pay claims**. (D'Arcy and Dyer, p. 43)

**Problem FER-56.**
**(a)** For a realistic allocation of equity to support a policy, how long should equity continue to be allocated?
**(b)** What approach can provide for such a realistic allocation of equity to the policy if the payment pattern of losses is known? (D'Arcy and Dyer, p. 43)

**Solution FER-56.**
**(a)** Equity should continue to be allocated to a given policy **until the obligation to pay claims is extinguished, i.e., all losses are settled**.
**(b)** A **proportional release of equity** using the same pattern as the payment pattern of losses could provide for such a realistic allocation. (D'Arcy and Dyer, p. 43)

**Problem FER-57.** What complication to insurers' selection of discount rates was brought about by the Tax Reform Act of 1986? (D'Arcy and Dyer, pp. 43-44)

**Solution FER-57.** The Tax Reform Act of 1986 requires loss reserves to be discounted based on a five-year moving average of mid-maturity US government obligations. These required discount rates may differ from the risk-free rate and may have no relationship to rates actually earned by the insurer or even available to the insurer. (D'Arcy and Dyer, pp. 43-44)

**Problem FER-58.**
**(a)** A common assumption in ratemaking is that expenses are paid when the premium is received. Why is this sometimes an unrealistic assumption? (D'Arcy and Dyer, p. 45)
**(b)** What is a consequence of the failure to reflect a more realistic treatment of expenses in a risk-adjusted discounted cash-flow model? (D'Arcy and Dyer, p. 46)

**Solution FER-58.**
**(a)** Many expenses are actually incurred long before a premium is collected – including setting up computer systems, underwriting guidelines, contract language, advertising, and training of personnel. (D'Arcy and Dyer, p. 45)
**(b)** If the fact that some expenses are incurred before the writing of the policy is not taken into account, then the premium would be understated. (D'Arcy and Dyer, p. 46)

**Problem FER-59.** What is a common adjustment within risk-adjusted discounted cash flow models with respect to *premiums*? (D'Arcy and Dyer, p. 46)

**Solution FER-59.** A common adjustment with respect to premiums reflects the fact that they are not received immediately at the inception of the policy term. Rather, there may be delays of several months, particularly if the agent is given a certain amount of time to remit premiums to the insurer. The delay reflects a form of agent compensation and should be reflected as an expense rather than a discount to the premium. (D'Arcy and Dyer, p. 46)

**Problem FER-60.** An insurer writes a policy with the following characteristics:
• The risk-free rate is 5%, which is also the rate of return on the insurer's investments.
• The risk-adjusted discount rate is 2%.
• Taxes are calculated at the end of each year. The tax authorities require that loss reserves for losses that are unpaid at the end of a given year be discounted at an interest rate of 7%.
• Premium is collected one month after policy issuance.
• Losses on the policy will be 300, half of which will be paid in exactly one year, and the other half of which will be paid in exactly two years.
• Total expenses on the policy are 43. Of these, 23 are paid immediately upon issuance, but 20 are paid 3 years prior to policy issuance.
• The policy is supported by equity of 200 which is released proportionally to the payment of losses.
• All taxes on both underwriting and investment income are imposed at a rate of 20%.

**(a)** Calculate the premium that this insurer would charge using a discounted cash-flow technique.
**(b)** Calculate the underwriting profit margin for this policy.

**Solution FER-60.**
**(a)** We use the formula PV(P) = PV(L) + PV(E) + PV(TUW) + PV(TII), where PV(P) = $P/1.05^{(1/12)}$ = 0.9959424074P.
PV(L) = 150/1.02 + $150/1.02^2$ = 291.2341407.
PV(E) = 23 + $20*1.05^3$ = 46.1525,since 20 of the expenses were paid 3 years in the past and must be discounted by -3 years.

Taxes on underwriting income can be separated into two components, taxes on the certain cash flows (premiums and expenses) and tax deductions due to the variable cash flows (losses, as well as loss reserves).
The certain cash flows are P – PV(E) = P - 46.1525, on which the tax at the end of year 1 will be 0.2*(P - 46.1525). The present value of this amount is 0.2*(P - 46.1525)/1.05 = 0.1904761905P – 8.790952381.
For the variable cash flows, at the end of year 1, the tax-deductible amount will equal to paid losses at the end of year 1, plus remaining loss reserves (discounted). The amount subject to the deduction will be 150 + 150/1.07 = 290.1869159. The tax savings on this amount will be

0.2*290.1869159 = 58.03738318. Now we discount this amount for one year by the risk-adjusted discount rate: 58.03738318/1.02 = 56.89939527.
At the end of year 2, there is additional tax savings due to the paid loss of 150, but it is offset by the elimination of the previous discounted loss reserve of 150/1.07. The net amount subject to the deduction will be 150 - 150/1.07 = 9.813084112. The tax savings on this amount will be 0.2*9.813084112 = 1.962616822. Now we discount this amount for two years by the risk-adjusted discount rate: $1.962616822/1.02^2 = 1.886406019$.
Thus, the total PV(TUW) = 0.1904761905P – 8.790952381 - 56.89939527 - 1.886406019 = PV(TUW) = 0.1904761905P – 67.57675367.

Taxes on investment income will be assessed at the end of years 1 and 2.
At the end of year 1, the insurer will have been able to earn investment income on the amount of (Equity + P – E), and the investment income in one year will be (Equity + P – E)*0.05 = (P + 200 – 43)*0.05 = (P + 157)*0.05. The tax on this amount will be 0.2*0.05(P + 157) = 0.01*(P + 157). The present value of this tax will be 0.01*(P + 157)/1.05 = 0.0095238095P + 1.495238095.

At the end of year 1, half of the losses will be paid, and half of the equity (200/2 = 100) will be released, leaving only the remaining half (100) to earn investment income in year 2. Also, 150 in losses will have been paid, so that amount will no longer be available to earn a return. The remaining amount available to earn investment income in year 2 will be (P + 100 – 43 – 150) = (P – 93). The investment income will be (P – 93)*0.05. The tax on this amount will be 0.2*0.05(P – 93) = 0.01*(P – 93). The present value of this tax will be $0.01*(P – 93)/1.05^2$ = 0.0090702948P – 0.843537415.

Thus, PV(TII) = 0.0095238095P + 1.495238095 + 0.0090702948P – 0.843537415 = PV(TII) = 0.0185941043P + 0.65170068.

Hence, our complete equation becomes
0.9959424074P = 291.2341407 + 46.1525 + 0.1904761905P – 67.57675367 + 0.0185941043P + 0.65170068 → 0.9959424074P = 0.209070295P + 270.4615877 → 0.786872112P = 270.4615877 → **P = 343.7173379**.

**(b)** We use the formula UPM = 1 – (L+E)/P = 1 – (300 + 43)/343.7173379 = 0.002086999
= **UPM = 0.2086999%**.

**Problem FER-61.** What additional adjustment to the treatment of *expenses* within the Risk-Adjusted Discount Technique could be included to enable a more realistic treatment? (D’Arcy and Dyer, pp. 49-50)

**Solution FER-61.** Some expenses are not fixed absolute quantities, but are rather dependent on the amount of premium. Examples include commissions and premium taxes. Using the Risk-Adjusted Discount Technique, one could set only some expenses as fixed and others as varying with the amount of premium P.

**Problem FER-62.** Why is in not always realistic to determine the present value of premiums based on the risk-free rate? (D'Arcy and Dyer, p. 50)

**Solution FER-62.** The lag in premium collection is different from investment in a risk-free security. Some risk is involved. Some premiums are never paid, resulting in policy cancellation. Other premiums are paid, but only after losses have occurred, and the insured might not have paid the premium if there had been no losses. (D'Arcy and Dyer, p. 50)

**Problem FER-63.** According to D'Arcy and Dyer (pp. 50-51), what are three serious drawbacks to the Risk-Adjusted Discount Technique, which cannot be fixed by adjusting the formula used?

**Solution FER-63.**
**Drawback 1.** There is no widely accepted approach for setting the risk-adjusted discount rate.
**Drawback 2.** If the CAPM is used, then it may not be a valid model. Research in finance has raised serious questions about the validity of the CAPM's application to investment returns in general.
**Drawback 3.** It is difficult to appropriately allocate equity to a policy, and the Risk-Adjusted Discount Technique relies, in the calculation of taxes on investment income, on an accurate allocation of equity both with regard to amount and to time. (D'Arcy and Dyer, p. 50)
**Drawback 4.** The Risk-Adjusted Discount Technique considers only one policy term, whereas the profitability of insurance policies depends on how many renewal cycles the policy has been through, with long-term business becoming more profitable. (D'Arcy and Dyer, p. 51)
Any three of the above would suffice.

**Problem FER-64.** What are three considerations that may be involved in determining the allocation of equity to an insurance policy? (D'Arcy and Dyer, p. 51)

**Solution FER-64.** Considerations would include the following:
**1.** Degree of variability in losses;
**2.** Length of time loss payments will be made;
**3.** Covariability among different lines of insurance;
**4.** Type of insurance product and its susceptibility to catastrophe losses;
**5.** Presence of reinsurance agreements that would limit losses.

Any three of the above would suffice.

**Problem FER-65.** Describe the *aging phenomenon* with regard to insurance policies. (D'Arcy and Dyer, p. 51)

**Solution FER-65.** The aging phenomenon is the observation that, while new insurance business tends to be unprofitable, long-term business becomes increasingly profitable over the course of renewals. This appears to occur for all insurers and all lines of business. (D'Arcy and Dyer, p. 51)

**Problem FER-66.** Identify and briefly describe two basic types of options. (D'Arcy and Dyer, p. 52)

**Solution FER-66.**
**1.** A **call option** gives the owner the right to *buy* the underlying asset at the specified price, called the strike or exercise price.
**2.** A **put option** gives the owner the right to *sell* the underlying asset at the specified price, called the strike or exercise price.
(D'Arcy and Dyer, p. 52)

**Problem FER-67.** What is the difference between a European option and an American option? (D'Arcy and Dyer, p. 52)

**Solution FER-67.** A European option may only be exercised on the expiration date, whereas an American option can be exercised at any time until expiration. (D'Arcy and Dyer, p. 52)

**Problem FER-68.** Let S be the price of a stock and X be the strike price of an option.
**(a)** Give the formula for C, the payoff of a call option. (D'Arcy and Dyer, p. 53)
**(b)** Give the formula for P, the payoff of a put option. (D'Arcy and Dyer, p. 55)

**Solution FER-68.**
**(a) C = max[S – X, 0].**
**(b) P = max[X – S, 0].**

**Problem FER-69.**
**(a)** What is the payoff for the buyer of a *call* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 60?
**(b)** What is the payoff for the buyer of a *put* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 60?
**(c)** What is the payoff for the buyer of a *call* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 15?
**(d)** What is the payoff for the buyer of a *put* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 15?

**Solution FER-69.**
**(a)** We use the formula C = max[S – X, 0] = max[35-60, 0] = **C = 0**.
**(b)** We use the formula P = max[X – S, 0] = max[60-35, 0] = **P = 25**.
**(c)** We use the formula C = max[S – X, 0] = max[35-15, 0] = **C = 20**.
**(d)** We use the formula P = max[X – S, 0] = max[15-35, 0] = **P = 0**.

**Problem FER-70.** Let S be the price of a stock and X be the strike price of an option based on that stock.
**(a)** For a call option, what is the formula for $W_C$, the payoff at expiration for the call *writer (seller)*? (D'Arcy and Dyer, p. 57)
**(b)** For a put option, what is the formula for $W_P$, the payoff at expiration for the put *writer (seller)*? (D'Arcy and Dyer, p. 60)

**Solution FER-70.**
**(a)** $\mathbf{W_C = min[X - S, 0]}$
**(b)** $\mathbf{W_P = min[S - X, 0]}$

**Problem FER-71.**
**(a)** What is the payoff for the *seller (writer)* of a *call* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 60?
**(b)** What is the payoff for the *seller (writer)* of a *put* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 60?
**(c)** What is the payoff for the *seller (writer)* of a *call* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 15?
**(d)** What is the payoff for the *seller (writer)* of a *put* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 15?

**Solution FER-71.**
**(a)** We use the formula $W_C = \min[X - S, 0] = \min[60\text{-}35, 0] = \mathbf{W_C = 0}$.
**(b)** We use the formula $W_P = \min[S - X, 0] = \min[35\text{-}60, 0] = \mathbf{W_P = -25}$.
**(c)** We use the formula $W_C = \min[X - S, 0] = \min[15\text{-}35, 0] = \mathbf{W_C = -20}$.
**(d)** We use the formula $W_P = \min[S - X, 0] = \min[35\text{-}15, 0] = \mathbf{W_P = 0}$.

**Problem FER-72.** Suppose that each of the options below can be purchased for a price of 10.
**(a)** What is the total payoff for the buyer and the total payoff for the seller of a *call* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 60?
**(b)** What is total payoff for the buyer and the total payoff for the seller of a *put* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 60?
**(c)** What is total payoff for the buyer and the total payoff for the seller of a *call* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 15?
**(d)** What is total payoff for the buyer and the total payoff for the seller of a *put* option on a stock, where the stock price at expiration is 35, and the strike price of the option is 15?

**Solution FER-72.** Let $C_0$ be the initial call purchase price and $P_0$ be the initial put purchase price. The formulas for the respective payoffs become the following (D'Arcy and Dyer, p. 60):
$C = \max[S - X - C_0, -C_0]$;
$P = \max[X - S - P_0, -P_0]$;
$W_C = \min[X - S + C_0, C_0]$;
$W_P = \min[S - X + P_0, P_0]$.

**(a)** C = max[S – X – $C_0$, -$C_0$] = max[35-60-10, -10] = **C = -10** = Payoff for the buyer.
The payoff for the seller is the inverse of the payoff for the buyer, so **$W_C$ = 10** = Payoff for the seller.
**(b)** P = max[X – S – $P_0$, -$P_0$] = max[60-35-10, -10] = **P = 15** = Payoff for the buyer.
The payoff for the seller is the inverse of the payoff for the buyer, so **$W_P$ = -15** = Payoff for the seller.
**(c)** C = max[S – X – $C_0$, -$C_0$] = max[35-15-10, -10] = **C = 10** = Payoff for the buyer.
The payoff for the seller is the inverse of the payoff for the buyer, so **$W_C$ = -10** = Payoff for the seller.
**(d)** P = max[X – S – $P_0$, -$P_0$] = max[15-35-10, -10] = **P = -10** = Payoff for the buyer.
The payoff for the seller is the inverse of the payoff for the buyer, so **$W_P$ = 10** = Payoff for the seller.

**Problem FER-73.** What are two primary reasons for which investors trade options? (D'Arcy and Dyer, pp. 69-70)

**Solution FER-73.**
**Reason 1.** Speculative motive of the potential to profit from the price movements of the underlying asset at a fraction of the cost of buying the asset itself. (D'Arcy and Dyer, p. 69)
**Reason 2.** Hedging a position taken in the underlying asset. (D'Arcy and Dyer, p. 70)

**Problem FER-74.** An investor purchases a stock at a price of $55 per share. To hedge the transaction, the investor also purchases a put option, giving him the right to sell the stock at $55 per share during the next three months. The cost of the put option is $6.

**(a)** If the stock and put option are both held to expiration, and the price of the stock at expiration is $66, what the investor's total payoff?
**(b)** If the stock and put option are both held to expiration, and the price of the stock at expiration is $45, what the investor's total payoff?

**Solution FER-74.**
**(a)** A stock price at expiration of $66 would mean that the put option would expire worthless. The investor's payoff would be (Expiration Stock Price – Purchase Stock Price) – (Cost of Put Option) = (66-55) – 6 = **$5.**
**(b)** Since the stock price at expiration is less than the strike price for the put option, it would be to the investor's advantage to exercise the put option, selling the stock at the same price the investor bought it: $55. The net cost to the investor is the cost of the put option, and thus the investor's payoff would be **-$6.**

**Problem FER-75.**
**(a)** What hedged position has the same payoff characteristics as owning a call option?
**(b)** What hedged position has the same payoff characteristics as owning a put option?
(D'Arcy and Dyer, p. 71)

**Solution FER-75.**
**(a) Buying a stock and buying a put option** on the stock has the same payoff characteristics as owning a call option.
**(b) Short-selling a stock and buying a call option** on the stock has the same payoff characteristics as owning a put option.

**Problem FER-76.** What five variables that affect option prices does the Black-Scholes option-pricing model take into account? (D'Arcy and Dyer, p. 74)

**Solution FER-76.** The Black-Scholes option-pricing model takes into account the following variables (D'Arcy and Dyer, p. 74):
**1.** The underlying stock price;
**2.** The exercise price;
**3.** The time to expiration of the option;
**4.** The volatility of price movements in the underlying stock;
**5.** The risk-free rate of interest.

**Problem FER-77.** What is the central idea behind the derivation of the Black-Scholes option-pricing model? (D'Arcy and Dyer, p. 74)

**Solution FER-77.** If an investor is able to continuously maintain a perfect hedge using an option on the underlying stock or asset, and borrow or lend at the risk-free rate, then this hedging portfolio must yield the risk-free rate of return to the investor. (D'Arcy and Dyer, p. 74)

**Problem FER-78.**
**(a)** What manner of compounding of interest is used in the Black-Scholes option-pricing model?
**(b)** What distribution of asset prices is used in the Black-Scholes option-pricing model?
**(c)** Why is the distribution in part (b) used? (D'Arcy and Dyer, p. 74)

**Solution FER-78.**
**(a) Continuous-time compounding** of interest is used.
**(b)** A **lognormal** distribution of asset prices is used.
**(c)** A lognormal distribution is used because an asset cannot sell for a price of less than zero, and a lognormal distribution will always produce non-negative asset prices. (D'Arcy and Dyer, p. 74)

**Problem FER-79.** You are given the following variables:
● C = Black-Scholes model price for a European call option
● S = Price of the underlying stock or asset
● X = Exercise price of the option
● N(*) = Normal distribution function evaluated at *
● $R_f$ = Risk-free rate of return
● t = Time to option expiration
● σ = Standard deviation of the continuously compounded returns of the underlying asset

**(a)** Give the formula for the $d_1$ component of the Black-Scholes formula for C.
**(b)** Give the formula for the $d_2$ component of the Black-Scholes formula for C. You may use $d_1$ as a term.
**(c)** Give the Black-Scholes formula for C, using $d_1$ and $d_2$ in the formula.
(D'Arcy and Dyer, pp. 74-75)

**Solution FER-79.**
**(a)** $d_1 = [\ln(S/X) + (R_f + 0.5\sigma^2)*t]/(\sigma*t^{1/2})$
**(b)** $d_2 = d_1 - \sigma*t^{1/2}$
**(c)** $C = S*N(d_1) - X*\exp(-R_f*t)*N(d_2)$

**Problem FER-80.** Given a stock with current price 100, a standard deviation of stock-price returns of 0.44, and an annual continuously compounded risk-free rate of interest of 4%, and a call option on the stock with an exercise price of 90 and time to expiration of 3 years, find the following:

**(a)** The $d_1$ component of the Black-Scholes formula for the price of the call option;
**(b)** The $d_2$ component of the Black-Scholes formula for the price of the call option;
**(c)** The price of the call option using the Black-Scholes formula.

You may use a spreadsheet program to perform the calculation.

**Solution FER-80.**

**(a)** We use the formula $d_1 = [\ln(S/X) + (R_f + 0.5\sigma^2)*t]/(\sigma*t^{1/2}) = [\ln(100/80) + (0.04 + 0.5*0.44^2)*3]/(0.44*3^{1/2})$ = $\mathbf{d_1 = 0.831310318}$.

**(b)** We use the formula $d_2 = d_1 - \sigma*t^{1/2} = 0.831310318 - 0.44*3^{1/2}$ = $\mathbf{d_2 = 0.069207963}$.

**(c)** We use the formula $C = S*N(d_1) - X*\exp(-R_f*t)*N(d_2) =$
$100*N(0.831310318) - 80*\exp(-0.04*3)*N(0.069207963)$.

In Microsoft Excel, the input for this calculation is "=
100*NORMSDIST(0.831310318) - 80*exp(-0.04*3)*NORMSDIST(0.069207963)".

The result is **C = 42.2757993**.

**Problem FER-81.** Answer the following questions based on the Black-Scholes option-pricing model (D'Arcy and Dyer, p. 76).
**(a)** What relationship – positive, negative, or none – exists between the stock price and the call price?
**(b)** What relationship – positive, negative, or none – exists between the exercise price and the call price?

**(c)** What relationship – positive, negative, or none – exists between the time to expiration and the call price?
**(d)** What relationship – positive, negative, or none – exists between the risk-free rate and the call price?
**(e)** What relationship – positive, negative, or none – exists between the standard deviation and the call price?

**Solution FER-81.**
**(a)** A **positive** relationship exists between the stock price and the call price.
**(b)** A **negative** relationship exists between the exercise price and the call price.
**(c)** A **positive** relationship exists between the time to expiration and the call price.
**(d)** A **positive** relationship exists between the risk-free rate and the call price.
**(e)** A **positive** relationship exists between the standard deviation and the call price.

**Problem FER-82.** What kind of option can describe the value of a corporation that will liquidate at the end of a given time period? Provide an expression for the payoff of this option. (D'Arcy and Dyer, p. 76)

**Solution FER-82.** A **European call option** can describe the value of a corporation that will liquidate at the end of a given time period. The payoff of the option is the corporation's equity E = max[A – D, 0], where A = the value of the corporation's assets and D = the value of the corporation's debts/liabilities at expiration. If A exceeds D, the shareholders will receive the difference. If D exceeds A, the shareholders will receive nothing.

**Problem FER-83.** If a corporation will liquidate at the end of a set period of time and the assets of the corporation will be A, while the liabilities will be D, what is the formula for the value $V_D$ of the debtholders' claims at liquidation? What option transaction does this formula resemble? (D'Arcy and Dyer, p. 77)

**Solution FER-83. $V_D$ = min[D, A].** This resembles a **written put option** (where the debtholders have "sold" the option) with a maximum value of D – the face value of the debtholders' claims.

**Problem FER-84.** For an insurance contract, suppose P is the premium, B is the deductible amount, L is an unknown loss amount, and time value of money is ignored.

**(a)** What is the formula for the policy value $V_p$ at the end of the policy period?
**(b)** What option transaction does the formula in part (a) resemble?
**(c)** What is the formula for the value of the policyholder's claim, $V_h$?

**Solution FER-84.**
**(a) $V_p$ = min[P, P – L + B]**
**(b)** This resembles the payoff at expiration to a **written European call option** (where the insurer has "sold" the option) with an exercise price at the deductible amount B. The policyholder would be analogous to the buyer/owner of the call option.
**(c) $V_h$ = max[L – B – P, -P].**

**Problem FER-85.**
**(a)** For the Doherty-Garven insurance-pricing model that applies option pricing, what type of insurer is assumed, and with what two initial attributes?
**(b)** What is the aim of the model?
**(c)** By what means is this aim achieved?
**(d)** What are the two types of claimholders within this model?
(D'Arcy and Dyer, p. 78)

**Solution FER-85.**
**(a)** The model assumes a **single-period insurer** with **(i)** initial equity of $S_0$ and **(ii)** premiums collected (net of expenses) of $P_0$.
**(b)** The aim of the model is to find the premium that gives the insurer a "fair" or adequate return on equity.
**(c)** This aim is achieved by setting the present value of the expected end-of-period market value of equity equal to the beginning-of-period amount of equity.
**(d)** The two types of claimholders within this model are **(i)** policyholders who expect their losses to be paid and **(ii)** the government that expects taxes to be paid. (D'Arcy and Dyer, p. 78)

**Problem FER-86.** You are given the following terms in the Doherty-Garven insurance-pricing model that applies option pricing:
- $S_0$ = Initial equity
- $P_0$ = Initial premiums collected (net of expenses)
- k = Funds-generating coefficient
- R = Rate of investment return on the insurer's initial asset portfolio

What is the formula for $Y_1$, the end-of-period asset portfolio available to the insurer to pay claimholders? (D'Arcy and Dyer, p. 78)

**Solution FER-86. $Y_1 = S_0 + P_0 + (S_0 + k*P_0)R$**

**Problem FER-87.** Suppose an insurer intends to exist for one year and begins with equity of 350,000 and premiums collected, net of all expenses, of 390,000. The average time between collection of premiums and payment of losses for this insurer is 0.35 years, and the insurer can earn a return on its investment portfolio at an annual rate of 10%.

Using the Doherty-Garven insurance-pricing model, at the end of the insurer's existence, what will be the total assets available to pay claimholders? (Note: Some of these assets may have been expended to make claim payments during the course of the year.)

**Solution FER-87.** We use the formula $Y_1 = S_0 + P_0 + (S_0 + k*P_0)R$. We are given that $S_0$ = 350000, $P_0$ = 390000, k = 0.35, and R = 0.1. Thus, $Y_1$ = 350000 + 390000 + (350000 + 0.35*390000)*0.1 = $\mathbf{Y_1}$ **= $788,650**.

**Problem FER-88.** You are given the following terms in the Doherty-Garven insurance-pricing model that applies option pricing:
• $Y_1$ = End-of-period asset portfolio available to the insurer to pay claimholders
• L = Total losses of policyholders.

**(a)** What is the formula for $H_1$, the end-of-period total claim by the policyholders?
**(b)** To what option payoff is this formula equivalent?
(D'Arcy and Dyer, p. 79)

**Solution FER-88.**
**(a)** $\mathbf{H_1 = max(min[L, Y_1], 0)}$
**(b)** This formula is equivalent to the expiration payoff to the owner of a **European call option with exercise price L.**

**Problem FER-89.** You are given the following terms in the Doherty-Garven model of insurance pricing that incorporates option pricing.
● $Y_0$ = Insurer's initial asset portfolio;
● $Y_1$ = Insurer's end-of-period asset portfolio;
● t = Insurer's corporate tax rate;
● i = Taxable portion of insurer's investment income;
● L = Loss payments to policyholders;
● $P_0$ = Premiums collected at inception (net of expenses).

**(a)** Give the formula for $T_1$, the value of the government's end-of-period tax claim on the insurer.
**(b)** What type of option transaction does this resemble?
**(c)** What does the term $(Y_1 - Y_0)$ represent?
**(d)** Let $H_1$ be the end-of-period total policyholder claims. What is the expression for $V_e$, the end-of-period value of the insurer's equity, using variables among those given?
(D'Arcy and Dyer, p. 79)

**Solution FER-89.**
**(a)** $\mathbf{T_1 = max[t(i[Y_1 - Y_0] + P_0 – L), 0]}$
**(b)** This resembles a **European call option** transaction.
**(c)** The term $(Y_1 - Y_0)$ represents the insurer's **investment income**.

**(d) $V_e = Y_1 - H_1 - T_1$.** (D'Arcy and Dyer, p. 79)

**Problem FER-90.** You are given the following terms in the Doherty-Garven model of insurance pricing that incorporates option pricing.
● $Y_0$ = Insurer's initial asset portfolio;
● $Y_1$ = Insurer's end-of-period asset portfolio;
● $V(Y_1)$ = Market value of the insurer's asset portfolio;
● t = Insurer's corporate tax rate;
● i = Taxable portion of insurer's investment income;
● L = Loss payments to policyholders;
● E(L) = Expected value of losses and loss-adjustment expenses during the period;
● $P_0$ = Premiums collected at inception (net of expenses);
● C[A; B] = Current value of a European call option with exercise price B written on an asset with a value of A.

**(a)** Give the formula for $H_0$, the present value of the policyholders' total claim, which includes a call option in one of the terms.
**(b)** Give the formula for $T_0$, the present value of the government's tax claim, which includes a call option in one of the terms.
**(c)** Give the formula for $V_m$, the market value of the insurer's equity, using terms including $H_0$ and $T_0$.
**(d)** Give the formula for $V_m$, the market value of the insurer's equity, in terms of the options in parts (a) and (b). Let the option in the formula for $H_0$ be called $C_1$, and let the option in the formula for $T_0$ be called $C_2$.
(D'Arcy and Dyer, p. 80)

**Solution FER-90.**
**(a) $H_0 = V(Y_1) - C[Y_0; E(L)]$**
**(b) $T_0 = t*C[i(Y_1 - Y_0) + P_0; E(L)]]$**
**(c) $V_m = V(Y_1) - H_0 - T_0$**
**(d) $V_m = C_1 – t*C_2$**

**Problem FER-91.** You are given the following facts for an insurer that intends to exist for one year and then liquidate.

**Initial Equity:** 600,000
**Premiums Written:** 1,540,000
**Expenses:** 255,000
**Expected Losses:** 1,000,000
**Standard Deviation of Investment Returns:** 0.33
**Risk-Free Interest Rate** (annual, continuously compounded): 2%
**Funds-Generating Coefficient:** 0.7
**Proportion of Investment Income That is Taxable:** 0.6

**Tax Rate:** 40%
This insurer's investment portfolio earns a return equivalent to the risk-free rate.

You are using the Doherty-Garven insurance-pricing model that involves option prices, in order to determine the value of the insurer.

**(a)** Identify and describe mathematically the option in this situation, pertaining to the *policyholders'* claim on the insurer. What is the exercise price of this option?
**(b)** Identify and describe mathematically the option in this situation, pertaining to the *government's* claim on the insurer. (This is the option whose value is multiplied by the tax rate.) What is the exercise price of this option?
**(c)** Use the Black-Scholes formula to calculate $C_1$, the value of the option pertaining to the *policyholders'* claim on the insurer.
**(d)** Use the Black-Scholes formula to calculate $C_2$, the value of the option pertaining to the *government's* claim on the insurer.
**(e)** Use the Doherty-Garven model to calculate the value of the insurer.

You may use a spreadsheet program where necessary.

**Solution FER-91.**
**(a)** The option pertaining to the *policyholders'* claim on the insurer is the European call option whose value is described as $C[Y_0; E(L)]$, where **E(L) = exercise price = 1,000,000**. $Y_0$ = (Equity + Premiums – Expenses) = (600,000 + 1,540,000 - 255,000) = 1,885,000. Thus, this is the European call option with value **C[1,885,000; 1,000,000]**.

**(b)** The option pertaining to the *government's* claim on the insurer (prior to the application of the tax rate T) is the European call option whose value is described as $C[i(Y_1 - Y_0) + P_0; E(L)]]$, where **E(L) = exercise price = 1,000,000**. We calculate $Y_1$ as $S_0 + P_0 + (S_0 + k*P_0)i$ = 1,885,000 + (600,000 + 0.7*(1,540,000-255,000))*0.02 = 1,914,990. Thus, $i(Y_1 - Y_0) + P_0$ = 0.02(1,914,990 - 1,885,000) + (1,540,000 - 255,000) = 1,285,599.8. Thus, this is the European call option with value **C[1,285,599.8; 1,000,000]**.

**(c)** We seek to find $C_1$ = C[1,885,000; 1,000,000].
Within the Black-Scholes formula, we first find $d_1 = [\ln(S/X) + (R_f + 0.5\sigma^2)*t]/(\sigma*t^{1/2})$ = $[\ln(1{,}885{,}000/1{,}000{,}000) + (0.02 + 0.5*0.33^2)*1]/(0.33*1^{1/2}) = d_1 = 2.146599457$.
We can also find $N(d_1)$ using the Excel input "=NORMSDIST(2.146599457)", which gives $N(d_1) = 0.984087408$.
Next, we find $d_2 = d_1 - \sigma*t^{1/2} = 2.146599457 - 0.33*1^{1/2} = d_2 = 1.816599457$.
We can also find $N(d_2)$ using the Excel input "=NORMSDIST(1.816599457)", which gives $N(d_2) = 0.965360767$.
Next, we calculate $C_1 = S*N(d_1) – X*\exp(-R_f*t)*N(d_2)$ = 1885000*0.984087408 – 1000000*exp(-0.02*1)*0.965360767 = $\mathbf{C_1}$ **= 908,759.421**.

**(d)** We seek to find $C_2$ = C[1,285,599.8; 1,000,000].
Within the Black-Scholes formula, we first find $d_1 = [\ln(S/X) + (R_f + 0.5\sigma^2)*t]/(\sigma*t^{1/2})$ = $[\ln(1{,}285{,}599.8/1{,}000{,}000) + (0.02 + 0.5*0.33^2)*1]/(0.33*1^{1/2})$ = $d_1$ = 0.9868950905.
We can also find $N(d_1)$ using the Excel input "=NORMSDIST(0.9868950905)", which gives $N(d_1)$ = 0.838152964.
Next, we find $d_2 = d_1 - \sigma*t^{1/2} = 0.9868950905 - 0.33*1^{1/2} = d_2$ = 0.6568950905.
We can also find $N(d_2)$ using the Excel input "=NORMSDIST(0.6568950905)", which gives $N(d_2)$ = 0.744375812.
Next, we calculate $C_2 = S*N(d_1) – X*\exp(-R_f*t)*N(d_2)$ = 1285599.8*0.838152964 - 1000000*exp(-0.02*1)*0.744375812 = **$C_2$ = 347,893.0995**.

**(e)** We use the formula $V_m = C_1 – t*C_2$ = 908759.421 - 0.4*347893.0995 = Value of the insurer = **769,602.1812**.

**Problem FER-92.** ***Fill in the blanks*** (D'Arcy and Dyer, p. 83): In the Doherty-Garven insurance-pricing model, when losses are allowed to vary, the exercise prices of the options for the stockholders and government are ____________.

**Solution FER-92.** In the Doherty-Garven insurance-pricing model, when losses are allowed to vary, the exercise prices of the options for the stockholders and government are **random variables**. (D'Arcy and Dyer, p. 83)

**Problem FER-93.** The Doherty-Garven insurance-pricing model is used to arrive at the appropriate premium P* that the insurer should charge in order to achieve a "fair" rate of return to shareholders.
**(a)** What is the mathematical condition by which the Doherty-Garven model considers the rate of return to be fair?
**(b)** Once the "fair" amount of premium P* is computed, give the formula for the insurer's fair underwriting profit margin UPM.
(D'Arcy and Dyer, p. 83)

**Solution FER-93.**
**(a)** The rate of return is considered fair if the market value of the insurer's equity $V_m = C_1 – t*C_2$ is equal to the initial equity amount $S_0$.
**(b) UPM = [P* – E(L)]/P*.** (D'Arcy and Dyer, p. 83)

**Problem FER-94. (a)** Identify and briefly describe the two different option-pricing models used by Doherty and Garven.
**(b)** Generally describe how are the solutions within these two option-pricing models are found, and the reason for taking this approach.
**(c)** Does the Doherty-Garven approach tend to result in higher or lower underwriting profit margins than calculated under the Capital Asset Pricing Model (CAPM)? (D'Arcy and Dyer, p. 83)

**Solution FER-94.**
**(a)** The two different option-pricing models used by Doherty and Garven are the following:
**1.** Model based on **Constant Absolute Risk Aversion (CARA)** and **normal distribution** of asset prices.
**2.** Model based on **Constant Relative Risk Aversion (CRRA)** and **lognormal distribution** of asset prices.

**(b)** The solutions within these two option-pricing models are found **by trial and error from properly parametrized versions of the models**. Necessary parameter estimates are the initial equity level, standard deviation of claim costs and investment returns, and correlation between claim costs and investment returns. This approach is taken because **the models do not have closed-form solutions**.

**(c)** The Doherty-Garven approach tends to result in **higher** underwriting profit margins than the CAPM.
(D'Arcy and Dyer, p. 83)

**Problem FER-95.** What three problems of the Capital Asset Pricing Model (CAPM) and the Discounted Cash Flow approaches does the Doherty-Garven insurance-pricing model avoid? (D'Arcy and Dyer, p. 84)

**Solution FER-95.** The following problems are avoided by the Doherty-Garven insurance-pricing model:
**1.** Estimation of betas
**2.** Estimation of market risk premiums
**3.** Consideration solely of systematic risk. (The Doherty-Garven model considers the *total* risk of the insurer's investment portfolio and underwriting operations.) (D'Arcy and Dyer, p. 84)

**Problem FER-96.** What is one documented tendency of the Black-Scholes option-pricing model that poses a problem in applying it to insurance cases? Why does it pose a problem for insurance applications? (D'Arcy and Dyer, p. 84)

**Solution FER-96.** The Black-Scholes option-pricing model tends to underprice options in which the stock price is well above the exercise price (in-the-money options). However, these are exactly the types of options used in the application of the option-pricing model to insurance, since the terminal value of the insurer's assets is expected to be much higher than losses.
(D'Arcy and Dyer, p. 84)