Study Guide on ERM for Strategic Management for the Casualty Actuarial Society (CAS) Exam 7

(Based on Gary Venter's Paper, "ERM for Strategic Management—Status Report")

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Source: Venter, G.G., "ERM for Strategic Management—Status Report," Society of Actuaries, 2008.

This is an open-source study guide and may be revised pursuant to suggestions.

S7-ERMSM-1.
(a) According to Venter (p. 2), which entities initially advocated Enterprise Risk Management (ERM)?
(b) What two "key areas of application" in ERM does Venter's paper focus on in terms of the progress made and the needs still outstanding? (Venter, p. 2)
(c) What is ERM still only partially able to do? (Venter, p. 3)

Solution S7-ERMSM-1.
(a) Regulators and **rating agencies** initially advocated ERM.
(b) Venter's paper addresses **(i) the optimal capital level for an insurer** and **(ii) the risk-adjusted profitability of business units.**
(c) ERM is still only partially able to **characterize the capital need and profit adequacy of insurers.**

S7-ERMSM-2.
(a) What was the capital adequacy of an insurer measured by historically?
(b) What was another measure that began to be used as reserves began to grow?
(c) What is a weakness with these approaches, and what was the response?
(d) What are some remaining weaknesses with the response in part (c)? (Venter, pp. 3-4)

Solution S7-ERMSM-2.
(a) An insurer's capital adequacy was historically measured by its **premium-to-surplus ratio.**
(b) As reserves began to grow, the **reserve-to-surplus ratio** was also used.
(c) The weakness with the above ratios is that different types of premiums and reserves impose different degrees of risk by line of business and territory. The response was **risk-based capital (RBC),** where various classes of assets and liabilities get their own risk factors.
(d) Under RBC, it is still possible for inadequate premiums and reserves to generate lower capital charges than adequate levels. There is also an unmeasured difference in the risk of different companies that write the same business.

S7-ERMSM-3.
(a) Define *economic capital*.
(b) What are two problems with economic capital? (Venter, pp. 4-5)

Solution S7-ERMSM-3.
(a) Economic capital is the amount of capital needed to get the one-year probability of ruin below some target threshold.
(b) Problems with economic capital:
1. It may be difficult to determine what the target threshold for the probability of ruin should be.
2. Economic capital is beyond the capacity of current ERM models to quantify, because not much is known about loss levels at remote probabilities.

S7-ERMSM-4. How are "more sophisticated companies" responding to the modeling difficulties at remote probabilities? Give a simple quantitative example. (Venter, p. 5)

Solution S7-ERMSM-4. The response is to express capital as a multiple of loss levels. For instance, instead of saying that the capital allows for a single 1-in-1000-year loss event, it can be said that the capital allows for 3 1-in-200-year loss events or 4 1-in-100-year loss events. So, if one 1-in-100-year loss occurred, the company would lose one-fourth of its capital.

S7-ERMSM-5.
(a) According to Venter (p. 5), what is the real relationship between the capital need and the loss probability level?
(b) According to Venter (p. 5), to what can the ideal capital for an insurer be related?
(c) What frictional costs can push the ideal capital level up or down? (Venter, p. 5)

Solution S7-ERMSM-5.
(a) In reality, the loss probability level is calculated as a check *after* the capital level has been established.
(b) The ideal capital can be related to **the market value** of the insurer, or the **franchise value** (difference between the market value and the book value).
(c) Frictional costs that can push the ideal capital level up include the costs of raising new capital and the costs of financial distress (including adverse perception by regulatory agencies). Frictional costs that can push the ideal capital level down include taxation of investment income.

S7-ERMSM-6. If an ERM model cannot entirely derive the insurer's ideal capital level, then what can it do, according to Venter (p. 6)?

Solution S7-ERMSM-6. An ERM model can help quantify the risk inherent in the business in comparison to the capital decisions that have been made. They also enable what-if analysis to find how the capital need might change if the same relationship of risk and capital is maintained, but other strategic changes are made.

S7-ERMSM-7. What are *silo risk measures*? Give examples for insurance. (Venter, p. 6)

Solution S7-ERMSM-7. Silo risk measures are measures that can be used for some risks but not all. Examples include premium volume, exposures in force, probable maximal loss (PML) for property lines, comparisons to similar market portfolios and option "Greeks" for silo risk measures.

S7-ERMSM-8.
(a) What is the shortcoming of standard deviation as an ERM risk measure?
(b) Conceptually define value at risk (VaR).
(c) Conceptually define tail value at risk (TVaR). (Venter, p. 7)

Solution S7-ERMSM-8.
(a) Standard deviation does not take into account the skewness of losses.
(b) VaR is a given percentile of the distribution of the account being quantified. This is also the same measure as economic capital.
(c) TVaR is the conditional mean of values that are greater than the VaR.

S7-ERMSM-9.
(a) Describe two problems with VaR.
(b) Describe a serious issue with TVarR and two measures that can address this issue. (Venter, p. 7)

Solution S7-ERMSM-9.
(a) Problems with VaR:
1. VaR is not subadditive: The VaR at a fixed probability for a number of risks combined can be greater than the sum of the VaRs for the individual risks, which makes it difficult to quantify the benefits of individual risks.
2. VaR is just a single point on the distribution and so does not give much information.
(b) A serious issue with TVaR is that it is linear for losses in the excess region, whereas usual risk attitudes are that the adverse effect of losses increases more than linearly with the size of the losses. Measures to address this problem include weighted-TVaR (**WTVaR**), which uses an adjusted probability distribution that increases the weight given to larger losses, and risk-adjusted TVaR (**RTVaR**), which is defined as the conditional tail mean plus some fraction of the conditional tail standard deviation

S7-ERMSM-10.
(a) For a risk measure $\rho(Y)$, define what it means for m to be *homogeneous*.
(b) Under what circumstances are VaR and TVaR homogeneous and non-homogeneous?
(c) Let ρ be a homogeneous risk measure on a random variable $Y = X_1 + \ldots + X_n$. Give the mathematical definition of $r(X_k)$, the marginal decomposition of $\rho(Y)$ for X_k. (Venter, pp. 8-9)

Solution S7-ERMSM-10.
(a) ρ is homogeneous if for any constant a > 0, **ρ(a*Y) = a*ρ(Y)**.
(b) VaR and TVaR are homogeneous if defined for a given probability level, but are not homogeneous if defined in excess of a given monetary amount.
(c) $r(X_k) = \lim_{\varepsilon \to 0}[(\rho(Y) - \rho(Y-\varepsilon*X_k))/\varepsilon]$

S7-ERMSM-11.
(a) Given that $\rho(Y) = E[\sum_i(h_i(Y)*L_i(Y)) \mid$ condition on Y], and for each h_i and some V and W, $h_i(V) + h_i(W) = h_i(V + W)$, and the L_i are any functions for which this conditional expected value exists, provide the formula for the co-measure for component k: $r(X_k)$.
(b) Now provide the mathematical relationship between $\rho(Y)$ and $r(X_k)$. (Venter, p. 8)

Solution S7-ERMSM-11.
(a) $r(X_k) = E[\sum_i(h_i(X_k)*L_i(Y)) \mid$ condition on Y].
(b) $\rho(Y) = \sum_k(r(X_k))$.

S7-ERMSM-12.
(a) Give the formula for ρ(Y), the excess tail value at risk (XTVaR) in excess of level α, such that the corresponding co-measure $r(X_k)$ is marginal and ρ is homogeneous. Also give the formula for such an $r(X_k)$.
(b) What is the marginal co-measure of standard deviation (give the formula), and what is it called?
(c) Describe Tasche's requirement for suitable allocation and the situations in which marginal decomposition meets that requirement.
(Venter, pp. 8-9)

Solution S7-ERMSM-12.
(a) $\rho(Y) = E[(Y – E(Y)) \mid F(Y) > \alpha]$. (Note that using E(Y) is more accurate in terms of notation.) The marginal co-measure is $r(X_k) = E[(X_k – E(X_k)) \mid F(Y) > \alpha]$.
(b) The marginal co-measure of standard deviation is **$r(X_k) = Cov(X_k, Y)/Std(Y)$**, called the **co-standard deviation.**
(c) Tasche's requirement for suitable allocation is that, if capital is allocated by a risk measure in order to calculate risk-adjusted return by line, it is desirable to conclude that growing a line with higher-than-average premium will increase the return for the whole company. Marginal decomposition meets this requirement in situations of proportional growth (taking higher percentages of business already written) and usually in situations where similar business units are added.

S7-ERMSM-13. According to Venter (p. 9), what are the most reasonable risk measures for calculating risk-adjusted profitability? Why?

Solution S7-ERMSM-13. WTVar and RTVaR are the most reasonable risk measures, as they avoid VaR's single-point nature and TVaR's linearity. They also relate to the value of the risk transfer.

S7-ERMSM-14. Fill in the blanks (Venter, p. 9):
RTVaR is based on the historical risk-pricing mechanism of _____. This mechanism has the shortcoming that it does not capture the _____ of most insurance business.

Solution S7-ERMSM-14. RTVaR is based on the historical risk-pricing mechanism of **standard deviation loading**. This mechanism has the shortcoming that it does not capture the **heavy tail** of most insurance business.

S7-ERMSM-15. What stock-market behavior would seem to require the use of much lower threshold probability levels than those often used? (Venter, pp. 9-10)

Solution S7-ERMSM-15. The stock market sometimes punishes a company's failure to meet a plan more than it punishes the financial shortfall itself. If this is considered, then the random variable measured might be the shortfall from the plan, rather than the absolute financial loss.

S7-ERMSM-16. Give three reasons discussed by Venter (p. 10) for why capital allocation can be considered artificial.

Solution S7-ERMSM-16.
1. The capital allocated is not actually assigned to the business units.
2. Capital allocation relies on sometimes arbitrary risk measures and selections of probability levels.
3. It is doubtful that each business unit will have the same target return on allocated capital.

S7-ERMSM-17.
(a) Describe the basic premise of *capital consumption* as an alternative to capital allocation.
(b) With regard to calculating capital consumption, what would it be difficult to use option-pricing formulas in insurance?
(c) How is the methodology of capital consumption similar to the pricing of stop-loss reinsurance?
(Venter, pp. 10-11)

Solution S7-ERMSM-17.
(a) Capital consumption calculates the cost to the firm of bearing the risk of each business unit and subtracts this cost from unit profits. This gives the value added for each unit.
(b) It would be difficult to use option-pricing formulas in insurance because (i) there is no fixed date when capital will be attached (the unit can use the firm's capital at any time and stop only when losses are paid), and (ii) the loss distributions are heavy-tailed, which renders pricing at the mean (as is done with options) ill-advised.
(c) It can be thought that the firm provides each business unit with stop-loss reinsurance, with a retention at break-even. The value of the stop-loss is the cost of carrying the unit.

S7-ERMSM-18.
(a) What definition does Venter provide for on p. 13 for *optimal capital*?
(b) What is the general recommendation of actuarial literature with regard to the firm achieving the optimal level of capital?
(c) Venter contrasts the assumptions of the traditional actuarial approach and the Modigliani-Miller approach of 1950s financial economists. What is the difference with regard to such issues as reinsurance and capital structure? What is the middle ground between these approaches?
(Venter, pp.13-14)

Solution S7-ERMSM-18.
(a) Venter defines optimal capital as the capital strategy that over time would maximize the expected present value of cash flows ("dividends") to shareholders.
(b) The firm should let its profits accumulate until reaching the ideal level of capital. Then, if there is any excess, the firm could distribute it as dividends. This approach reduces the probability of early ruin.
(c) The actuarial approach assumed that the costs of refinancing with regard to reinsurance and the capital structure were "inifinite", while the Modigliani-Miller framework assumed that such costs were zero and that capital structure, reinsurance, and risk management (!) are irrelevant to firm value. These approaches are at the opposite ends of the spectrum in terms of assumed impacts on existing shareholders of raising new capital. The middle ground is to combine the actuarial approach with the possibility of external finance at a finite cost.

S7-ERMSM-19. What characteristics of insurance policyholders increase the insurer's capital need and the value of risk management? (Venter, p. 14)

Solution S7-ERMSM-19. Insurance policyholders are non-diversified in their insurance companies, and they are more risk-averse toward insurance-company failure, since they would lose their insurance protection in the event of failure.

S7-ERMSM-20. On page 14, Venter suggests the possibility of incorporating the complex modeling of *what* into models of optimal capital?

Solution S7-ERMSM-20. Venter suggests the possibility of incorporating the complex modeling of such attributes as the variable growth of premiums by time and line, cyclical profitability, and complex reinsurance deals.

S7-ERMSM-21.
(a) Besides not taking tail risk into account, in what other situation could a company's capital not be considered sufficient?
(b) According to Venter, what sorts of risk are relevant to consider in order to prevent a company with sufficient capital from having its capital become insufficient? (Venter, p. 15)

Solution S7-ERMSM-21.
(a) Capital could be considered insufficient if it is too low to take advantage of profitable business opportunities.
(b) Any risk for which a price is charged is relevant in terms of addressing the prospect of insufficient capital.

S7-ERMSM-22. Identify the three types of risk measures discussed by Venter and define/describe each. (Venter, pp. 15-16)

Solution S7-ERMSM-22. The three types of risk measures are as follows:
1. **Moment-based measures:** Use moments to measure risk. Examples include standard deviation, skewness, and semi-standard deviation (using only unfavorable deviations). They incorporate either the whole distribution of results or only adverse results.
2. **Tail-based measures:** Look only at the tail of the distribution. These include VaR, TVaR, XTVaR, RTVaR, and EPD (expected policyholder deficit).
3. **Transformed distribution measures:** Change the probabilities to give more weight to adverse results and then take the mean or use some other risk measure with the transformed probabilities. These measures use the entire distribution of events but put more weight on the tails. The two subsets of this type of measure are (i) transforms of the probabilities of financial results and (ii) transforms of the probabilities of the underlying events that lead to the results.

S7-ERMSM-23.
(a) Give the formula for EPD (expected policyholder deficit) at probability level α.
(b) What does EPD represent conceptually if α is the probability of default? (Venter, p. 16)

Solution S7-ERMSM-23.
(a) $\mathbf{EPD_\alpha = (1-\alpha)*(TVaR_\alpha - VaR_\alpha)}$.
(b) EPD represents the average uncovered loss that would occur if capital is carried at VaR_α and a default occurs.

S7-ERMSM-24.
(a) Give the general formula for a *spectral risk measure*.
(b) Express TVaR as a spectral risk measure. (Venter, pp. 16-17)

Solution S7-ERMSM-24.
(a) $\mathbf{\rho(Y) = E[Y*\eta(F(Y))]}$, where η is a nonnegative scalar function.
(b) $\mathbf{TVaR_q = \int_{y>[F^{-1}](q)} [y*f(y)/(1-q)]*dy}$. In this case, $\mathbf{\eta = 1/(1-q)}$ **for** $\mathbf{p > q}$ **and 0 otherwise.**

S7-ERMSM-25.
(a) Give the formula for a *distortion risk measure* $\rho(X)$ if $S(x)$ is the survival function of X and $g(x)$ is a non-decreasing function to and from the unit interval such that $g(0) = 0$ and $g(1) = 1$.
(b) How could a distortion risk measure be seen as a mean? (Venter, p. 18)

Solution S7-ERMSM-25.
(a) $\rho(X) = \int_0^\infty g[S(x)]*dx$.
(b) The distorted survival function, g[S(x)] is itself a survival function that could be called S*(x). Its corresponding distorted cumulative distribution function could be called F*(x). Since the integral of the survival function is the mean of the corresponding distribution, ρ(X) is the mean of the distribution F*.

S7-ERMSM-26. Give the formula for the *Wang transform* g(p) with parameters a, b, where Φ is the standard normal cumulative distribution function, and T_a is the t-distribution function with a degrees of freedom. (Venter, p. 18)

Solution S7-ERMSM-26. $g(p) = 1 - T_a(\Phi^{-1}(1-p) - b)$.

S7-ERMSM-27. According to Venter (p. 18), which of the following is able to produce arbitrage-free pricing: (i) transforming probabilities of financial results or (ii) transforming probabilities of underlying events? What is the technical requirement that allows this to occur with these transforms?

Solution S7-ERMSM-27. According to Venter, **(ii) transforming probabilities of underlying events** is able to produce arbitrage-free pricing. The technical requirement for these transforms is that the **zero-probability events have to be the same before and after the transform**.

S7-ERMSM-28.
(a) What is a "complete market"?
(b) In an incomplete market, what is a *minimum martingale transform*? (Venter, p. 19)

Solution S7-ERMSM-28.
(a) A "complete market" is one where there is a perfect hedging strategy available, associated with a single specific martingale transform.
(b) In an incomplete market, a minimum martingale transform is the transform associated with the hedging strategy which will minimize the variance of the payoff risk, given that it is impossible to altogether eliminate that variance.

S7-ERMSM-29.
(a) Give the formula for the *relative entropy* between two measures P and Q.
(b) In an insurance-pricing context, what could P and Q stand for? (Venter, p. 19)

Solution S7-ERMSM-29.
(a) The relative entropy is $\mathbf{E_p*[(dQ/dP)*\log(dQ/dP)]}$.
(b) In an insurance-pricing context, P could be the real-world measure, and Q could be the martingale. A Q is sought so as to minimize the relative entropy.

S7-ERMSM-30. According to Venter (p. 20), how can one use risk measures to take account of the risk below the tail which companies are not willing to take for free?

Solution S7-ERMSM-30. One can use RTVaR and WTVaR at low probability levels, or transformed probability measures that emphasize the tail but use the entire distribution.

S7-ERMSM-31.
(a) According to Venter (p. 21), risk pricing is often viewed as having two chief components. What are they?
(b) According to Venter (p. 21), what are the two primary pricing paradigms in the financial literature?

Solution S7-ERMSM-31.
(a) The two chief components of risk pricing are (1) return for taking on risk and (2) compensation for the frictional costs of holding capital.
(b) The two primary pricing paradigms in the financial literature are (1) the capital asset pricing model (CAPM), and (2) no-arbitrage pricing.

S7-ERMSM-32.
(a) Give the CAPM formula for the return on an asset, R_Y, in terms of the risk-free return R_F and the market rate of return R_M.
(b) On what kind of distribution of returns is CAPM based? (Venter, p. 22)

Solution S7-ERMSM-32.
(a) $E(R_Y) = R_F + \gamma * \mathbf{Cov}[R_Y, R_M]$
(b) CAPM is based on a **normal** distribution of returns.

S7-ERMSM-33.
(a) Define the *frictional costs of holding capital*. (Venter, p. 24)
(b) Venter, on p. 25, discusses the recommendation of Grundl and Schmeiser to treat each frictional cost individually and to include it in a contingent-claims approach to pricing. How might this be done with respect to corporate taxation?

Solution S7-ERMSM-33.
(a) The frictional costs of holding capital are costs that accrue whether or not risk is taken, such as taxes on investment income or agency costs resulting from managers using the capital in their own interests rather than those of the company.
(b) With respect to corporate taxation, the government could be seen as holding an option on the profits of the firm, and the value of the option could be calculated using the appropriate martingale transform.