THE ACTUARY'S FREE STUDY GUIDE FOR THE OLD EXAM 6

Second Edition

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Section 1

Basic Concepts Regarding Unpaid Claim Estimation

This section of the study guide is intended to provide practice problems and solutions to accompany the pages of *Estimating Unpaid Claims Using Basic Techniques*, cited below. Students are encouraged to read these pages before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

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Formula for This Section

Formula 1.1: Reported claims = paid claims during period + case outstanding at end of period - case outstanding at beginning of period

The above formula applies to *incremental* reported claims -- i.e., reported claims applicable to a particular time period, irrespective of what happened during prior time periods.

Source:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapters 1 and 2, pp. 1-24.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-1-1. Friedland (p. 9) discusses two ways to categorize claim adjustment expenses: (1) allocated versus unallocated loss adjustment expenses (ALAE vs. ULAE) and (2) defense and cost containment versus adjusting and other expenses (DCC vs. A&O). For each of these classification systems, define the two categories of expenses.

Solution S6-1-1.

Allocated loss adjustment expenses (ALAE): Costs that can be assigned to the adjustment of a particular claim.

Unallocated loss adjustment expenses (ULAE): Costs that cannot be easily assigned to a particular claim - such as the insurer's rent, payroll, and computer expenses.

Defense and cost containment (DCC) expenses: All medical cost containment and defense litigation expenses.

Adjusting and other expenses: All claims adjusting expenses, whether or not they can be attributed to an individual claim.

Problem S6-1-2.

(a) What is the difference between an "unpaid claim estimate" and a "reserve" as officially defined in Actuarial Standard of Practice (ASOP) 43? (See Friedland, p. 13.)

(b) What is the difference between an "unpaid claim estimate" and a "carried reserve"? (See Friedland, p. 14.)

Solution S6-1-2.

(a) ASOP 43 confines the term "reserve" to "an amount booked in a financial statement." An "unpaid claim estimate" is an actuarial estimate of future payment obligations resulting from claims arising out of already occurred events.

(b) An "unpaid claim estimate" arises out of the use a particular method of estimation, and different methods can produce different estimates. A "carried reserve" for unpaid claims is the amount which is actually reported or published in the relevant financial statement.

Problem S6-1-3. List the five components of an unpaid claim estimate (discussed in Friedland, p. 14). Which of these constitute the broad term "incurred but not reported" (IBNR)?

Solution S6-1-3. According to Friedland, p. 14, the following are the five components of an unpaid claim estimate:

1. **Case outstanding on known claims = Unpaid case**

2. **Provision for future development on known claims**

3. **Estimate for reopened claims**

4. **Provision for claims incurred but not reported**

5. **Provision for claims in transit = claims reported but not recorded**

The term IBNR refers to items 2 through 5 above - i.e., every category *except* case outstanding on known claims.

Problem S6-1-4. Define the following dates related to insurance claims, described by Friedland (p. 22):

(a) Policy effective date

(b) Accident date

(c) Report date

(d) Transaction date

(e) Closing date

(f) Reopening date

Solution S6-1-4.

(a) Policy effective date: Date the insurance policy is issued.

(b) Accident date: Date of occurrence of covered loss.

(c) Report date: Date of insurer becoming notified of the claim.

(d) **Transaction date:** Date on which a payment is made or a case outstanding transaction occurs.

(e) Closing date: Date of either initial or final closure of the claim.

(f) Reopening date: Date on which an insurer begins to consider a previously closed claim open.

Problem S6-1-5. The case outstanding at the end of the year 2015 is $321,031. At the beginning of 2015, the case outstanding was $346,120. Paid claims during 2015 were $120,315. What is the dollar amount of reported claims during 2015? (See Friedland, p. 24.)

Solution S6-1-5. We use the formula
Reported claims = paid claims during period + case outstanding at end of period - case outstanding at beginning of period = 120315 + 321031 - 346120 = **$95,226.**

Section 2

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 1

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration – and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 14, pp. 329-330.

Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Slywotzky, A.J., and Drzik, J., "Countering the Biggest Risk of All," *Harvard Business Review*, April 2005, Harvard Business School Publishing.

Problems and Solutions from The Actuary's Free Study Guide

Problem S6-2-1. This problem is similar to Problem 1, Part (a), on the 2009 CAS Exam 6.

You are given the following information as of December 31, 2013:

(1) Paid Claims (including Salvage and Subrogation) by Accident Year (AY)

For AY 2011: 44,224
For AY 2012: 52,143
For AY 2013: 80,087

(2) Selected Ultimate Claims (including Salvage and Subrogation)

For AY 2011: 49,500
For AY 2012: 58,700
For AY 2013: 82,420

(3) Ratio of Received Salvage and Subrogation (S&S) to Paid Claims

For AY 2011: 0.151
For AY 2012: 0.176
For AY 2013: 0.210

(4) Development Factor to Ultimate for S&S Ratio

For AY 2011: 1.000
For AY 2012: 1.014
For AY 2013: 1.114

Using the ratio method, estimate the recoverables for Salvage and Subrogation (S&S) for accident years 2011-2013.

Solution S6-2-1. First, we estimate the ultimate S&S for each accident year. This is done by multiplying Ultimate Claims (2) by the S&S ratio (3) and the development factor to ultimate (4).

(5) Ultimate S&S by Accident Year: (5) = (2)*(3)*(4)

For AY 2011: 49,500*0.151*1.000 = 7474.5
For AY 2012: 58,700*0.176*1.014 = 10475.8368
For AY 2013: 82,420*0.210*1.114 = 19281.3348

Then we estimate paid S&S for each accident year. This is done by multiplying actual paid claims (1) by the S&S ratio (2). No development factors apply because we are only estimating what has already been paid.

(6) Paid S&S by Accident Year: (6) = (1)*(2)

For AY 2011: 44,224*0.151 = 6677.824
For AY 2012: 52,143*0.176 = 9177.168
For AY 2013: 80,087*0.210 = 16818.27

The S&S recoverables are the difference between ultimate S&S and paid S&S. They are *what remains to be recovered.*

(7) S&S Recoverables by Accident Year: (7) = (5) - (6)

For AY 2011: 7474.5 - 6677.824 = 796.676
For AY 2012: 10475.8368 - 9177.168 = 1298.6688

For AY 2013: 19281.3348 - 16818.27 = 2463.0648
Total: 796.676 + 1298.6688 + 2463.0648 = **4558.4096.**

Problem S6-2-2. How is the development for salvage recoveries typically different from the development for subrogation recoveries, and why? (See Friedland, p. 329).

Solution S6-2-2. Salvage is associated with property coverages, where the losses are often quickly reported and settled. Thus, the salvage can also be determined much faster.

Subrogation is associated with liability coverages, where the losses can take years to ascertain, and it may take years to determine who is liable and the ultimate claim payout. Also, subrogation recoveries may take years to materialize after the underlying claim is paid, because the insurer still has to pursue the responsible party.

Friedland (p. 329) notes that some subrogation age-to-age factors may be less than 1. This can happen for older claims where the prospect of recovering from the responsible party diminishes over time.

Problem S6-2-3. This problem is similar to Problem 8 on the 2008 CAS Exam 6.

You are analyzing a contract where the premium is paid in full at the start of the contract term. The upfront premium is $3650.

The expected incurred losses occur in the following percentages per year of the contract:

Year 1: 34%
Year 2: 15%
Year 3: 40%
Years 4-14: 1% per year

For the end of each the years 1, 2, 3, and 4, calculate the unearned premium reserve based on **(a)** the assumption that premium is earned in the same pattern as expected losses, **(b)** the assumption that premium is earned on a pro rata basis, and **(c)** the difference between the answers in (a) and (b).

(d) Based on your answer to part (c), explain the problem with applying the approach in part (b) to this situation.

(e) Name three kinds of insurance-related products for which assuming that premium is earned on a pro rata basis would not be appropriate.

Solution S6-2-3. (a) The unearned premium reserve (here, UPR) is equal to

(Total premium)*(1 - Fraction of premium that is earned).

For Year 1, UPR = 3650*(1 - 0.34) = **2409**.
For Year 2, UPR = 3650*(1 - 0.34 - 0.15) = **1861.5**.
For Year 3, UPR = 3650*(1 - 0.34 - 0.15 - 0.40) = **401.5**.
For Year 4, UPR = 3650*(1 - 0.34 - 0.15 - 0.40 - 0.01) = **365**.

(b) There are 14 years over which the policy is expected to have losses. Thus, the pro rata method assumes that each year, $1/14^{th}$ of the premium is earned, leaving the unearned premium reserve to be (Total premium)*(1 - (1/14)*Number of years elapsed).

For Year 1, UPR = 3650*(1 - 1/14) = **3389.286714**
For Year 2, UPR = 3650*(1 - 2/14) = **3128.571429**
For Year 3, UPR = 3650*(1 - 3/14) = **2867.857143**
For Year 4, UPR = 3650*(1 - 4/14) = **2607.142857**

(c) These answers are simply the difference between the corresponding values in (a) and (b):

For Year 1: 2409 - 3389.286714 = **-980.286714**
For Year 2: 1861.5 - 3128.571429 =**-1267.071429**
For Year 3: 401.5 - 2867.857143 =**-2466.357143**
For Year 4: 365 - 2607.142857 =**-2242.142857**

(d) The answers in part (c) can be thought of as the degree to which the pro rata method of estimating earned premium *underestimates* the true profitability of this product. Most of the losses for this product occur early on - during the first four years. But the pro rata method assumes that the losses occur evenly throughout the 14 years. This might, for instance, lead the company to assume that this product is not profitable and withdraw from offering it, when the product might in fact be a decent revenue source.

(e) The pro rata assumption for earned premium is not appropriate for the following kinds of products:

1. **Warranties**, where losses typically occur later during the contract term;

2. Policies covering **seasonal exposures**, such as hurricane risk. More premium should be earned during the season(s) of peak exposure.

3. **Aggregate excess insurance policies**, which cover losses above a certain attachment point. The attachment point is likely to be reached only later in the policy term, so that is when premium should start to be earned.

Problem S6-2-4. This problem is similar to Problem 42 on the 2008 CAS Exam 6.

What are the four considerations for identifying and assessing a risk, as mentioned by Slywotzky and Drzik (2005)? Briefly explain each consideration in your own words - not in the words of the authors or the CAS solutions.

Solution S6-2-4. The following is a sample answer. Students are encouraged to develop their own phrasings to help internalize the ideas.

1. **Severity** - What portion of the company's overall value (measured in shareholders' equity, market value, earnings, etc.) could be jeopardized by the risk?

2. **Probability** - How likely is the risk to happen?

3. **Timing** - Will the risk occur sooner or later? Can the time of occurrence be pinpointed, or can the risk occur at any of a broad range of times?

4. **Changing probability over time** - Will the passage of time raise, lower, or not affect the chances of the risk occurring?

Problem S6-2-5. This problem is similar to Problem 34 on the 2009 CAS Exam 6.

Explain each of the following strategic risks identified by Slywotzky and Drzik (2005) and state an approach for how a company might overcome such a risk.

(a) Brand erosion

(b) Customer priority shift

(c) New-project failure

(d) Market stagnation

Solution S6-2-5.

(a) Brand erosion: Company experiences a significant drop in market share either because a sudden event (e.g., a well-publicized product defect or a large accident) has rendered its brand less attractive to consumers or a gradual erosion of the company's brand occurred because the company failed to innovate and satisfy consumer expectations.

A way to overcome brand erosion is to "redefine the scope of brand investment" to focus on other aspects than mere marketing, such as quality of the product or service being offered.

(b) Customer priority shift: Shifts in the customers' preferences or a shift in the balance of power toward consumers may reduce the company's value and profitability.

A way to overcome this problem is through "fast and cheap experimentation", where the company engages in relatively flexible and inexpensive ways to assess changing consumer tastes and receive feedback from consumers in various segments of the market.

(c) New-project failure: A company suffers because its new venture failed to attract customers or to function as intended, or competitors seized on the idea so quickly that the company could not earn sufficient profits.

The stepping-stone method, which uses a series of projects instead of a single lump-sum project, is a way of countering this risk. Each step in this method can be self-contained but can also lead to further undertakings if successful. The farther along on the stepping-stone approach the company is, the more likely success is, since earlier steps would have addressed and adapted to some of the challenges involved.

(d) Market stagnation: Growth in the market is diminished or halted, and new sources of growth cannot be easily found.

Demand innovation, the focus on the value the company brings to the customer, can help overcome market stagnation. The company can focus on more than just how the product functions but also on how consumers use the product. The company can then provide its consumers with services that would help them employ its product more effectively.

Section 3

Basic Concepts Regarding Data Used in Estimating Unpaid Claims

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Source:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 3, pp. 28-33.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-3-1. Give three ways in which external data may differ from an insurance company's internal data so as to render the two data sets non-comparable. (See Friedland, p. 29.)

Solution S6-3-1. The following are ways in which external data may substantively differ from an insurance company's internal data (Friedland, p. 29):

1. Differences in insurance products being monitored

2. Differences in case outstanding and settlement practices

3. Differences in insurers' operations

4. Differences in coding

5. Different geographical areas

6. Differences in mix of business

Any three of the above suffice as an answer. Other valid answers may also be possible.

Problem S6-3-2. To achieve homogeneous data sets, actuaries often separate insurance data into groups based on certain characteristics. Give four examples of desirable attributes for a single group of data. (See Friedland, p. 29.)

Solution S6-3-2. The following are examples of desirable attributes for a single group of data (Friedland, p. 29):

1. Large volume of observations/claim counts

2. Consistent coverage triggers

3. Similar length of time for reporting the claims after they occur

4. Ability to quickly develop appropriate case outstanding estimates

5. Similar lengths of time to settle the claims

6. Similar likelihoods of claims reopening

7. Similar average settlement values

Any four of the above suffice as an answer. Other valid answers may also be possible.

Problem S6-3-3. Insurers often use both internal (staff) claim adjusters and external (independent) adjusters. For which of these two types of adjusters is it typically easier to allocate adjustment expenses to a specific claim? Why? (See Friedland, p. 31.)

Solution S6-3-3. It is easier to allocate claim adjustment expenses to **external adjusters**, because external adjusters are often hired to adjust specific claims, as when the insurer's claim volume is higher than its staff adjusters can handle. For internal adjusters, it is often not clear how the relevant salaries and overhead expenses are to be distributed among individual claims, so the claim expenses are more easily categorized as unallocated.

Problem S6-3-4. Friedland, p. 32, lists five criteria that actuaries consider in establishing thresholds for what is deemed a "large claim" and might have different unpaid claim estimation techniques applied to it. List four of these criteria.

Solution S6-3-4. The following are the five criteria for determining the large claim threshold, as provided in Friedland, p. 32:

1. Number of claims exceeding the threshold per year

2. Claim size compared to policy limits

3. Claim size compared to reinsurance limits

4. Credibility of internal large claim data

5. Degree to which relevant external data are available

Any four of the above suffice as an answer. Other valid answers may also be possible.

Problem S6-3-5. Briefly describe four kinds of *recoveries* that might be relevant in determining unpaid claim estimates. (See Friedland, pp. 33-34.)

Solution S6-3-5. The following are four kinds of recoveries that might be relevant in determining unpaid claim estimates:

1. **Deductible recoveries:** Insurer pays the entire claim and seeks reimbursement from the insured for the deductible.

2. **Salvage recoveries:** Insurer takes possession of damaged property and sells it to recoup some of the claim costs.

3. **Subrogation recoveries:** Insurer seeks money from the third party responsible for a loss so as to recoup some of its claim costs.

4. **Reinsurance recoveries:** Insurer's liability for losses is reduced by having had certain claims ceded to a reinsurer.

Section 4

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 2

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Source: Past Casualty Actuarial Society exams: 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-4-1. This problem is similar to Problem 17 on the 2009 CAS Exam 6.

An insurance policy was sold on August 1, 2016, and takes effect on February 1, 2017. The full premium for the policy is $1800. Premium is earned evenly over the course of every month, and the policy has a term of 6 months.

Use the *deferral-matching premium recognition approach* to calculate (i) earned premium and (ii) unearned premium for each of the following time periods and evaluation dates.

(a) Fourth quarter of 2016, evaluated as of December 31, 2016.
(b) First quarter of 2017, evaluated as of March 31, 2017.
(c) Second quarter of 2017, evaluated as of June 30, 2017.
(d) Third quarter of 2017, evaluated as of September 30, 2017.

Solution S6-4-1.

(a) In 2016, the policy is still not in effect, so no premium is earned. Thus, (i) **earned premium = $0** and (ii) **unearned premium = $1800**.

(b) During the first quarter of 2017, the policy is in effect during February and March, so two months of premium, or 1800*2/6 = $600 are earned. The rest of the full-term premium remains unearned. Thus, (i) **earned premium = $600** and (ii) **unearned premium = $1200**.

(c) During the second quarter of 2017, the policy is in effect the entire time, so three months of premium, or 1800*3/6 = $900 are earned. This, added to the $600 earned in the first quarter of 2017, implies that, as of the end of the second quarter of 2007, (i) **earned premium = $1500** and (ii) **unearned premium = $300**.

(d) Since the policy has a six-month term, it expires on August 1, 2017. Thus, only one month of premium, or 1800*1/6 = $300 is earned during the third quarter. At the end of the third quarter, the policy has already expired, so the full-term premium is earned. Thus, (i) **earned premium = $1800** and (ii) **unearned premium = $0.**

The following conditions apply to Problems S6-4-2 through S6-4-3:

A reinsurance contract stipulates that the primary insurer shall retain the first $100,000 of every loss for every risk, whereafter the reinsurer will assume liability for the remaining portion of the loss. However, the reinsurer's liability per loss per risk cannot exceed $500,000, and the reinsurer's liability per loss occurrence cannot exceed $700,000.

The following losses occurred:

Occurrence 1:
Risk 1 suffered $431,000 in bodily injury losses and $316,000 in property damage losses.
Risk 2 suffered $50,000 in bodily injury losses and $150,000 in property damage losses.

Occurrence 2:
Risk 3 suffered $51,000 in bodily injury losses and $20,000 in property damage losses.
Risk 4 suffered $360,000 in bodily injury losses and $10,000 in property damage losses.
Risk 5 suffered $100,000 in bodily injury losses and $100,000 losses.

Problem S6-4-2. This problem is similar to Problem 21(a) on the 2009 CAS Exam 6.

For these two occurrences, what will be total amount for which the *reinsurer* will be liable?

Solution S6-4-2.

We analyze Occurrence 1:
For Risk 1, the total loss amount is 431000 + 316000 = $747,000.
For Risk 2, the total loss amount is 50000 + 150000 = $200,000.
For each risk, the primary insurer retains $100,000, leaving at most
$647,000 for Risk 1 and $100,000 for Risk 2 to be paid by the reinsurer. However, the reinsurer's liability is limited to $500,000 per risk, meaning that the reinsurer will pay $500,000 for Risk 1 and $100,000 for Risk 2, for a total of $600,000 for Occurrence 1.

We analyze Occurrence 2:
For Risk 3, the total loss amount is 51000 + 20000 = $71,000. This entire risk is retained by the primary insurer.
For Risk 4, the total loss amount is 360000 + 10000 = $370,000. Of this, the primary insurer retains $100,000, leaving $270,000 for the reinsurer.
For Risk 5, the total loss amount is 100000 + 100000 = $200,000. Of this, the primary insurer retains $100,000, leaving $100,000 for the reinsurer. The reinsurer thus pays a total of $370,000 for Occurrence 2.
The reinsurer's total payout will thus be 600000 + 370000 = **$970,000.**

Problem S6-4-3. This problem is similar to Problem 21(b) on the 2009 CAS Exam 6.

Assume that there is now a co-participation provision of 10%, where the primary insurer must contribute this percentage to the losses where the reinsurer is otherwise liable. This provision does not reduce either the reinsurer's per-risk limit or its per-occurrence limit. How much would the *primary insurer* need to pay in this situation?

Solution S6-4-3.

We analyze Occurrence 1:

For Risk 1, the total loss amount is 431000 + 316000 = $747,000.
Primary insurer retains $100,000, while the reinsurer still has to pay $500,000, since, with the 10% participation requirement, the primary insurer would pay only an additional $50,000, leaving enough (747000 - 150000 = $597,000) for the reinsurer to exhaust its per-risk limit. The primary insurer would be responsible for the remaining $97,000, making the primary insurer's liability equal to 100000 + 50000 + 97000 = $247,000.

For Risk 2, the total loss amount is 50000 + 150000 = $200,000.
For each risk, the primary insurer retains $100,000, Of this, the primary insurer retains $100,000, and pays 10% of the rest, or an additional $10,000, for a total of $110,000.

Thus, the primary insurer's total liability for Occurrence 1 is 247000 + 110000 = $357,000.

We analyze Occurrence 2:

For Risk 3, the total loss amount is 51000 + 20000 = $71,000. This entire risk is retained by the primary insurer.

For Risk 4, the total loss amount is 360000 + 10000 = $370,000. Of this, the primary insurer retains $100,000, and pays 10% of the rest, or an additional $27,000, for a total of $127,000.

For Risk 5, the total loss amount is 100000 + 100000 = $200,000. Of this, the primary insurer retains $100,000, and pays 10% of the rest, or an additional $10,000, for a total of $110,000.

Thus, the primary insurer's total liability for Occurrence 2 is 71000 + 127000 + 110000 = $308,000.

The primary insurer's total payout will be 357000 + 308000 = **$665,000.**

The following conditions apply to Problems S6-4-4 through S6-4-5:

When there are multiple reinsurance treaties, one treaty can *inure to the benefit of* another treaty by being applied to the loss amount first, potentially reducing what the other treaty would need to pay.

Suppose there is a 60% quota share treaty (the reinsurer gets ceded 60% of a loss) and a $5 million excess of $15 million catastrophe treaty (the reinsurer pays at most $5 million in losses above $15 million).

Problem S6-4-4. This problem is similar to Problem 25(a) on the 2009 CAS Exam 6.

If a loss of $30 million occurs and the catastrophe treaty inures to the benefit of the quota share treaty, what is the liability of (i) the catastrophe treaty, (ii) the quota share treaty, and (iii) the primary insurer?

Solution S6-4-4. The catastrophe treaty is applied first, so it pays the $5 million over $15 million (which is less than the total loss amount). The quota share treaty applies to the rest, and the reinsurer gets 60% of the remaining loss amount, or 60% of ($30 million - $5 million = $25 million). The quota share treaty thus pays 25*0.6 million = $15 million. The primary insurer will pay the rest -- $10 million.

(i) **Catastrophe treaty pays $5 million.**
(ii) **Quota share treaty pays $15 million.**
(iii) **Primary insurer pays $10 million.**

Problem S6-4-5. This problem is similar to Problem 25(b) on the 2009 CAS Exam 6.

If a loss of $22 million occurs and the quota share treaty inures to the benefit of the catastrophe treaty, what is the liability of (i) the quota share treaty, (ii) the catastrophe treaty, and (iii) the primary insurer?

Solution S6-4-5. The quota share treaty is applied first, meaning that the quota share treaty pays 0.6*22 million = $13.2 million. The remaining loss amount is (22-13.2) million = $8.8 million. This is not enough to meet the threshold where the catastrophe treaty is triggered, so the catastrophe treaty pays nothing. The primary insurer pays the remaining $8.8 million.

(i) **Quota share treaty pays $13.2 million.**
(ii) **Catastrophe treaty pays nothing.**
(iii) **Primary insurer pays $8.8 million.**

Section 5

Basic Concepts Regarding Data Aggregation and Other Data Treatments in Unpaid Claim Estimation

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Source:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapters 3-5, pp. 34-52.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-5-1. What are the three possible treatments of allocated loss adjustment expenses (ALAE) in excess-of-loss reinsurance contracts? (See Friedland, p. 34.) Which of these is the most common treatment?

Solution S6-5-1. The following are the three possible treatments of allocated loss adjustment expenses (ALAE) in excess-of-loss reinsurance contracts (Friedland, p. 34):
1. All ALAE are included in the claim amount for the purposes of determining whether reinsurance coverage applies. This is the most common treatment.
2. All ALAE are excluded from coverage.
3. ALAE are included on a pro rata basis. The amount of ALAE covered is the total ALAE multiplied by the ratio of the excess amount of the claim to the total claim amount.

Problem S6-5-2.

(a) What is the most common type of exposure base for insurers?
(b) For self-insurers, what are three possible alternative exposure bases if the exposure base in part (a) is not available? For each alternative, identify a type of insurance to which it would apply. Why would such alternatives be necessary?

Solution S6-5-2. This question is based on the discussion in Friedland, p. 35.

(a) The most common exposure base for insurers is **earned premium.**

(b) Self-insurers most often do not pay premiums, so alternative exposure bases are needed. The following exposure bases are described by Friedland, p. 35:
1. Number of employees - for crime insurance
2. Property values - for property insurance
3. Number of vehicles - for automobile liability insurance
4. Miles driven - for automobile liability insurance
5. Payroll - for workers' compensation insurance in the U. S.
6. Sales - for general liability for corporations
7. Square footage - for general liability for corporations
8. Population - for general liability for public agencies
9. Operating expenditures - for general liability for public agencies
10. Outpatient visits - for hospital professional liability
11. Average occupied beds - for hospital professional liability

Any three of the above suffice as an answer. Other valid answers may also be possible.

Problem S6-5-3. According to Friedland, p. 37, what are the four elements that a data review may examine to "verify that the data utilized are reliable and sufficient for the intended purpose"?

Solution S6-5-3. The following four elements are mentioned by Friedland, p. 37:
1. Consistency with prior data
2. Data definitions
3. Consistency with financial statement data
4. Data reasonableness

Problem S6-5-4. For each of the following methods of aggregation, give (i) an advantage and (ii) a disadvantage of the method. (See Friedland, pp. 40-43.)
(a) Calendar year aggregation
(b) Accident year aggregation
(c) Policy year aggregation
(d) Report year aggregation

Solution S6-5-4.

(a) Calendar year aggregation

(i) ***Advantages*** (any one is acceptable):
1. No future development exists; data remain fixed at the end of the calendar year.
2. Data are easily available from insurers' financial reports.
(ii) ***Disadvantages*** (any one is acceptable):
1. The issue of development cannot be addressed.

2. Mismatch between premiums and losses (the premiums collected during a year do not necessarily pertain to the policies on which losses have occurred, and vice versa).

(b) Accident year aggregation
(i) ***Advantages*** (any one is acceptable):
1. Easy to understand and produce
2. Shorter typical development timeframes than for the policy year method
3. Claims can be tracked by accident year so as to detect changes in economic and regulatory factors or major claim events.
4. Many industry benchmarks based on accident year data exist.
(ii) ***Disadvantages*** (any one is acceptable):
1. There is still a possible mismatch between claims and exposures.
2. This method might include claims from policies issued and priced at fundamentally different times.
3. Can "mask changes in retention levels" for self-insureds with high deductibles (Friedland, p. 41).

(c) Policy year aggregation
(i) ***Advantages*** (any one is acceptable):
1. Match between claims and exposures.
2. Useful for tracking effects of underwriting or pricing changes.
3. Useful for data pertaining to self-insureds to whom only one policy applies.
(ii) ***Disadvantages*** (any one is acceptable):
1. Lengthy development timeframe and difficulty of quickly estimating ultimate claims.
2. Difficulty in isolating and comprehending the effect of a major court ruling, catastrophe, or similar large event.

(c) Report year aggregation
(i) ***Advantages*** (any one is acceptable):
1. Stability of data and development patters that are relatively easy to determine
2. No need to estimate reported claims
3. Particularly useful for claims-made policies
(ii) ***Disadvantage*** (others may be possible):
1. Pure IBNR (incurred but not reported) cannot be estimated at all, so additional methods for estimating it may be required.

Problem S6-5-5. Name two situations in which an insurer's claim development triangle might show *negative* claim development over time. (See Friedland, p. 52.)

Solution S6-5-5. The following are situations in which an insurer's claim development triangle might show *negative* claim development over time:

1. The insurer has settled some claims for less than prior case outstanding estimates.
2. The insurer has included recoveries (e.g., salvage and subrogation) with the claim data, meaning that any recovery that occurs would reduce the net amount paid on the claim.

Section 6

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 3

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Source: Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

The following conditions apply to Problems S6-6-1 and S6-6-2:

Insurer Z has an excess-of-loss reinsurance treaty that provides $500,000 in excess of $100,000 per occurrence. The following losses occurred during the term of the treaty:

Loss A: $136,000 in incurred loss, $130,000 in loss adjustment expenses

Loss B: $416,000 in incurred loss, $800,000 in loss adjustment expenses

Loss C: $700,000 in incurred loss, $20,000 in loss adjustment expenses

Problem S6-6-1. Similar to Problem 26(a) from the Fall 2009 Exam 6. If the loss adjustment expenses are included in the limit of coverage, what is the reinsurer's total obligation for the three losses?

Solution S6-6-1. Since LAE are included in the limit, the total loss amount per occurrence is the sum of incurred loss and LAE for that occurrence.

Loss A: Total loss amount is 136000 + 130000 = $266,000, of which the reinsurer covers $166,000.

Loss B: Total loss amount is 416000 + 800000 = $1,216,000, of which the reinsurer covers up to its limit of $500,000.

Loss C: Total loss amount is 700000 + 20000 = $720,000, of which the reinsurer covers up to its limit of $500,000.

The reinsurer's total obligation is thus 166000 + 500000 + 500000 = **$1,166,000.**

Problem S6-6-2. Similar to Problem 26(b) from the Fall 2009 Exam 6. If the loss adjustment expenses are shared on a pro-rata basis in addition to the incurred loss amounts, what is the reinsurer's total obligation for the three losses?

Solution S6-6-2. The proportion of the LAE that the reinsurer would pay on a pro rata basis is the ratio (Incurred loss amount covered by reinsurer)/(Total incurred loss amount).

Loss A: Incurred loss amount is $136,000, of which the reinsurer would cover $36,000, so the amount of LAE the reinsurer covers is 130000*(36000/136000) = $34,411.76, meaning that the reinsurer covers a total of $70,411.76 for Loss A.

Loss B: Incurred loss amount is $416,000, of which the reinsurer would cover $316,000, so the amount of LAE the reinsurer covers is 800000*(316000/416000) = $607,692.31, meaning that the reinsurer covers a total of $923,692.31 for Loss B.

Loss C: Incurred loss amount is $700,000, of which the reinsurer would cover $500,000, so the amount of LAE the reinsurer covers is 20000*(500000/700000) = $14,285.71, meaning that the reinsurer covers a total of $514,285.71 for Loss C.

The reinsurer's total obligation is thus 70411.76 + 923692.31 + 514285.71 = **$1,508.389.78.**

Note that it is not always the case that pro-rata sharing of LAE results in less reinsurance coverage than including LAE in the limit!

The following conditions apply to Problems S6-6-3 and S6-6-4:

You are examining a proportional reinsurance treaty (where the primary insurer and reinsurer share losses equally), with one modification: a loss corridor provision that requires the primary insurer to assume responsibility for 90% of the losses for the loss ratio layer between 50% and 80%.

You know the following:

For the loss ratio layer 0%-20%: The average loss ratio is 15%, and the probability of the loss ratio being in this layer is 11%.

For the loss ratio layer 20%-50%: The average loss ratio is 34%, and the probability of the loss ratio being in this layer is 30%.

For the loss ratio layer 50%-80%: The average loss ratio is 75%, and the probability of the loss ratio being in this layer is 40%.

For the loss ratio layer 80+%: The average loss ratio is 98%, and the probability of the loss ratio being in this layer is 19%.

Problem S6-6-3. Similar to Problem 30(a) from the Fall 2009 Exam 6. If no loss ratio corridor is applied, what is the expected loss ratio for the *primary insurer*?

Solution S6-6-3. Since losses are shared evenly between the primary insurer and the reinsurer, the expected loss ratio is the same as the weighted-average loss ratio for these data: 0.15*0.11 + 0.34*0.30 + 0.75*0.40 + 0.98*0.19 = **0.6047 = 60.47%**.

Problem S6-6-4. Similar to Problem 30(b) from the Fall 2009 Exam 6. If the loss ratio corridor *is* applied, what is the expected loss ratio for the *primary insurer*?

Solution S6-6-4. The primary insurer is obligated to retain 90% of the losses for the loss ratio layer between 50% and 80%.

For this layer, the average loss ratio percentage in the corridor is (75% - 50%) = 25%, and the primary insurer will retain 90% of this, or 22.5%.

For the 80+ layer, the average loss ratio percentage in the corridor is (80% - 50%) = 30%, and the primary insurer will retain 90% of this, or 27%.

The expected retained loss ratio is obtained by weighting the above retained loss ratios by the probability of each layer occurring: 0.225*0.40 + 0.27*0.19 = 0.1413 = 14.13%. This is the *additional* loss ratio percentage that the corridor requires the primary insurer to retain, so the *total* loss ratio percentage retained is 60.47% + 14.13% = **74.6%.**

Problem S6-6-5. Similar to Problem 1 from the Fall 2008 Exam 6. Based on the CAS *Statement of Principles Regarding Property and casualty Loss and Loss Adjustment Expense Reserves*, explain how **(a)** settlement patterns and **(b)** frequency/severity considerations could affect the determination of whether or not data in a given set are homogeneous.

Solution S6-6-5. (a) Settlement patterns - how long it takes for reported claims to settle - influence the level of reserve uncertainty. Claims that take longer to settle, such as bodily injury liability claims, have more reserve uncertainty, and the ultimate claim amount can vary considerably from the initial estimate. It may therefore be appropriate to analyze claims with long settlement patterns separately from claims with short settlement patterns, lest the actuary understate development on these claims.

(b) Frequency/severity considerations: Claims with high frequency and low severity tend to be subject to more accurate reserve estimates than claims with low frequency and high severity. The latter type of claim will often necessitate greater analysis and may therefore need to be examined separately.

Section 7

Actuarial Reserving Definitions and Principles

This section of the study guide is intended to provide practice problems and solutions to accompany the pages of the *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves*, cited below. Students are encouraged to read these pages before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

Some of the questions here ask for short written answers based on the reading. This is meant to give the student practice in answering questions of the format that will appear on Exam 5B (Old Exam 6). Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Source:
Casualty Actuarial Society, *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves*, May 1988. pp. 10-14.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-7-1.

(a) How does the *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves* (hereafter, *Statement of Principles*) distinguish between a *carried loss reserve* and an *indicated loss reserve?* Which of these two is/are likely to change over time?

(b) The *Statement of Principles* (p. 11) describes a division that is often made between two different kinds of reserves. What is this division?

Solution S6-7-1.

(a) According to the *Statement of Principles* (p. 11), the **carried loss reserve** "is the amount shown in a published statement or in an internal statement of financial condition. An **indicated loss reserve** "is the result of the application of a particular loss reserving evaluation procedure." The **indicated loss reserve will likely change over time**.

(b) The division is between (1) **reserves for claims that are known** and (2) **reserves for claims incurred but not reported (IBNR)**.

Problem S6-7-2.

(a) What is the difference between the *case reserve* and the *reserve for known claims*, as defined in the *Statement of Principles*?

(b) How does the *Statement of Principles* define a *formula reserve*?

Solution S6-7-2.

(a) According to the *Statement of Principles* (p. 11), the **case reserve** is "the sum of the values assigned to specific known claims", whereas the **reserve for known claims** consists of the case reserve *and* a "provision for future development on known claims", which a case reserve would lack.

(b) According to the *Statement of Principles* (p. 11), a **formula reserve** is a reserve "established for groups of claims for which certain classifying information is provided." As the name suggests, a formula is applied to determine reserve estimates for any claim in a group of claims with similar characteristics.

Problem S6-7-3. Briefly describe each of the three different kinds of *development* mentioned in the *Statement of Principles* (pp. 11-12).

Solution S6-7-3. The following three kinds of development are mentioned in the *Statement of Principles*:
1. **Development in the number of reported claims** for losses within a particular calendar period: More claims may get reported pertaining to the time period in question - even after that time period expires.
2. **Paid development:** Payments accumulate on particular claims over time.
3. **Incurred development:** The "difference between estimates of incurred costs at two valuation dates for a defined group of claims" (*Statement of Principles*, p. 12).

Problem S6-7-4.

(a) Fill in the blank: According to the *Statement of Principles* (p. 13), an actuarially sound loss reserve is supposed to estimate _____.

(b) If there is a range of actuarially sound estimates, the *Statement of Principles* (p. 13) articulates that the most appropriate estimate within this range is dependent on two criteria. What are these criteria?

Solution S6-7-4.

(a) According to the *Statement of Principles* (p. 13), an actuarially sound loss reserve is supposed to estimate **"the unpaid amount required to settle all claims, whether reported or not, for which liability exists on a particular accounting date."**

(b) If there is a range of actuarially sound estimates, the two criteria for selecting an estimate within the range are **(1) "the relative likelihood of estimates within the range"** and **(2) "the financial reporting context in which the reserve will be presented"** (*Statement of Principles,* p. 13).

Problem S6-7-5.

(a) According to the *Statement of Principles* (p. 14), what are the five key dates around which the organization of claim data revolves?

(b) On page 14, the *Statement of Principles* gives both aprecise definition and a broad definition of incurred but not reported (IBNR) claims. State each definition.

(c) Out of what can ambiguity regarding the definition of IBNR arise? (See *Statement of Principles,* p. 14.)

Solution S6-7-5.

(a) The organization of claim data revolves around the following five key dates:

1. Accident date
2. Report date
3. Recorded date
4. Accounting date
5. Valuation date

(b) Precise definition of IBNR (with respect to a given accounting date)**:** Claims "with report dates later than a particular accounting date and accident dates equal to or earlier than the accounting date" (*Statement of Principles,* p. 14).

Broad definition of IBNR: Provision for late reported claims, development on known claims, and reopened claims (*Statement of Principles,* p. 14).

(c) Ambiguity regarding the definition of IBNR can arise from different insurer strategies in loss reserving. The accident period approach and the report period approach are two of the most common strategies. The accident period approach does not necessitate separately treating reported and unreported claims and so lends itself open to the broad definition of IBNR. In the report period approach, claims that have not been reported during a given time period are by definition not a part of the data, and so any IBNR estimate has to be performed separately for such claims.

Section 8

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 4

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Formulas for This Section

Formula 8.1

Reported Bornhuetter-Ferguson Method (Friedland, p. 153):
Ultimate Claims = Actual Reported Claims + (Expected Claims)*(% Claims Unreported)

Formula 8.2

Paid Bornhuetter-Ferguson Method (Friedland, p. 153):
Ultimate Claims = Actual Paid Claims + (Expected Claims)*(% Claims Unpaid)

Sources:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 9, p. 153.

Past Casualty Actuarial Society exams: 2008 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-8-1. Similar to Problem 2 from the Fall 2008 Exam 6.

You know the following regarding data from policy year 2044:
Premium was $5,000,000.

It is expected that 50% of the loss would emerge after 48 months, and 70% of the loss would emerge after 60 months.

Reported loss as of the end of 2047 was $2,120,000.

The estimate of ultimate loss via the Bornhuetter-Ferguson method is $4,200,000.

(a) What was the expected loss ratio used in the Bornhuetter-Ferguson estimate of ultimate loss?

(b) Use the chain-ladder method to calculate the ultimate loss estimate for policy year 2044.

(c) Use the Bornhuetter-Ferguson method to find the expected 2048 calendar year development for losses from policy year 2044.

Solution S6-8-1.

(a) We use the Reported Bornhuetter-Ferguson method, applying Formula 8.1:

Ultimate Claims = Actual Reported Claims + (Expected Claims)*(% Claims Unreported)

Here, Ultimate Claims = 4,200,000 and Actual Reported Claims = 2,120,000. % Claims Unreported = 50% at the end of 2047, which is the point at which we have reported claim data.

Expected Claims can be expressed as (Premium)*(ELR), where the ELR is the expected loss ratio.

Thus, 4,200,000 = 2,120,000 + 5,000,000*ELR*0.5 →

2,080,000 = 2,500,000*ELR →

ELR = 2,080,000/2,500,000 = **ELR = 0.832 = 83.2%**.

(b) The chain ladder method takes the latest known reported loss figure and asks, "What percentage of the ultimate reported loss is this figure expected to be?" Here, the latest known reported loss figure is $2,120,000, and this is expected to be 50% of the ultimate loss, so the ultimate loss is 2,120,000/0.5 = **$4,240,000.**

(c) The development portion of the Bornhuetter-Ferguson method formula is the

(Expected Claims)*(% Claims Unreported) part. Here, we only want to focus on claims expected to emerge in calendar year 2048. Based on our given expected loss percentages, this is 70% - 50% = 20% of all claims. Based on part (a), we can already calculate expected claims to be 5,000,000*ELR = 5,000,000*0.832 = $4,160,000. Of this, 20% is **$832,000** - our estimate of development during CY 2048.

Problem S6-8-2. Similar to Problem 5 from the Fall 2008 Exam 6.

(a) If an insurer makes a one-time change in its policy limits applicable to all policies written after Day X, which method of data aggregation would be preferable: policy year or accident year? Why?

(b) When an insurer's business is growing rapidly within a particular year, which method of data aggregation would be preferable: accident year or accident quarter? Why?

(c) If there is a significant legal decision that changes typical amounts of damages resulting from particular incidents, which method of data aggregation would be preferable: report year or accident year? Why?

(d) What could happen to claim counts so as to make earned exposures a more reliable measure by comparison?

Solution S6-8-2.

(a) If an insurer makes a one-time change in its policy limits applicable to all policies written after Day X, the **policy year** method would be preferable, because it could separately analyze policies written before the limit change and policies written after the limit change. Accident-year aggregation could mix data on losses occurring in the same period, but pertaining both to policies written before the limit change and policies written after it.

(b) When an insurer's business is growing rapidly within a particular year, the **accident quarter** method of aggregation would be preferable, because a growing book of business would be expected to experience growing amounts of losses as well. This means that losses would be more heavily concentrated toward the end of the year, and separating data into accident quarters could segment the periods of greater losses from the periods of smaller losses.

(c) If there is a significant legal decision that changes typical amounts of damages resulting from particular incidents, the **report year** method of aggregation would be preferable, because claims reported after the decision would be subject to different likely severities than claims reported before the decision, irrespective of when the underlying losses occurred.

(d) Either the definition of what constitutes a claim or the insurer's claim-handling practices might change in such a way as to make "claim counts" non-comparable across time. In such cases, earned exposures are a more reliable measure.

Problem S6-8-3. Similar to Problem 6 from the Fall 2008 Exam 6.

During Year X, an insurer's claim-handling practices changed and each claim is now given a significantly lower initial case reserve than previously. However, the claims are also settled faster.

(a) Which of these methods would lead to definite overstatement of losses - the unadjusted reported loss development method or the unadjusted paid loss development method? Why?

(b) What changes unrelated to claims settlement could be responsible for a lowering of the initial case reserve assigned to each claim?

Solution S6-8-3.

(a) Both methods depend on development factors calculated from historical information and assuming that historical patterns of development will continue into the future. The unadjusted reported loss development method, however, will have to work with initial case reserve estimates that are lower than previously. Thus, an application of a historical loss development factor (LDF) to a lower case reserve will result in a lower estimate. However, the effect of faster claims settlement may or may not compensate for this - depending on the degree. If claims are settled significantly faster, and the historical LDF assumes a longer settlement pattern, then the effect of this would be a relative overstatement of losses. With the paid loss development method, however, the focus is only on the settlement pattern, and in this case a decrease in settlement times would produce an overstatement of ultimate losses if historical assumptions are used. So the **unadjusted paid loss development method** would lead to definite overstatement of losses.

(b) The following changes unrelated to claims settlement could be responsible for a lowering of the initial case reserve assigned to each claim:

1. Increase in deductibles on all policies - reducing the insurer's potential liability per claim.

2. Decrease in limits on all policies - reducing the insurer's potential liability per claim.

3. More rigorous underwriting standards - meaning that the insurer expects lower-risk insureds to be accepted into the program.

4. Movement of business to a different geographical area which tends to be populated by lower-risk insureds.

Problem S6-8-4. Similar to Problem 12 from the Fall 2008 Exam 6.

For a workers' compensation high-deductible policy, the full coverage premium is $120,140, and the full-coverage expected loss ratio is 0.560. The excess ratio (the ratio of losses expected to be above the deductible) is 0.222. The aggregate ratio (the proportion of losses below the deductible for which the insurer may still have to incur expenses, for instance, via administering the policy or collecting the deductible from the insured) is 0.05. For the time period in question, it is known that reported excess losses were $16,000, and the excess loss development factor is 1.11.

(a) Using the loss ratio method to set reserves, what is the estimated ultimate loss for this policy?

(b) Would the loss ratio method be more preferable for a small start-up workers' compensation insurer or a large, established insurer with plenty of its own credible data? Explain your choice.

Solution S6-8-4. (a) The loss ratio method for workers' compensation high-deductible policies *does not rely on observed losses*. Thus, reported excess losses and their associated development

factor are not going to be used. There are two components to the loss ratio method: (1) the expected excess loss and (2) the losses associated with the aggregate ratio.

The expected excess loss component is (Premium)*(Full-Coverage ELR)*(Excess Ratio) = 120140*0.560*0.222 = 14935.8048.

The losses associated with the aggregate ratio are (Premium)*(Full-Coverage ELR)*(1 - Excess Ratio)*(Aggregate Ratio) = 120140*0.560*(1-0.222)*0.05 = 2617.12976.

The total estimated ultimate loss is 14935.8048 + 2617.12976 = 17552.93456 = **$17,552.93**.

(b) The loss ratio method to set reserves would be preferable for the **small insurer** which does not have a lot of its own data to estimate ultimate losses. The approach uses industry data and therefore adds credibility. However, for a large, established insurer, the loss ratio method has the drawback of not considering that insurer's extensive loss experience data. Also, the insurer's own practices and book of business may differ from those of the overall industry, and thus a reliance on the large insurer's own data may be more appropriate.

Problem S6-8-5. Similar to Problem 16 from the Fall 2008 Exam 6.

(a) There are two claims of the exact same type. Claim A has an incurred loss amount of $60,000, while Claim B has an incurred loss amount of $6,000. An actuary is estimating unallocated loss adjustment expenses (ULAE) for these claims. He must choose between the dollar-based approach and the count-based approach. Which of these approaches would be likely to give the same estimate for ULAE for both claims?

(b) For each of the two approaches, give a diagnostic that might suggest the desirability of one approach over the other.

Solution S6-8-5.

(a) The **count-based approach** would be likely to give the same estimate for ULAE for both claims. This is because this approach assumes that ULAE does not correlate with the loss amount and is essentially the same for similar types of claims. The dollar-based approach assumes that ULAE is directly proportional to the loss amount.

(b) If a cost analysis of each claim identifies that the ULAE per claim is close to the same, irrespective of claim size, then the count-based approach can be reliable.

If the ratio of ULAE to paid loss amount is stable across all claims, then the dollar-based approach can be reliable.

Section 9

Actuarial Reserving Considerations

This section of the study guide is intended to provide practice problems and solutions to accompany the pages of the *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves*, cited below. Students are encouraged to read these pages before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

Some of the questions here ask for short written answers based on the reading. This is meant to give the student practice in answering questions of the format that will appear on Exam 5B (Old Exam 6). Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Source: Casualty Actuarial Society, *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves*, May 1988. pp. 15-19.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-9-1. According to the *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves* (hereafter, *Statement of Principles*), is it generally accepted to have the provision for reopened claims included in (i) the reserve for known claims or (ii) the IBNR reserve? Why? Under what situations might the practice that is *not* generally accepted arise?

Solution S6-9-1. It is generally accepted that the provision for reopened claims should be included in **the reserve for known claims.** This is because reopened claims are generated from claims that have been previously reported and are thus known at the time the reserve estimate is being made. But if an insurer's practices require establishing a new report date for a reopened claim, then the provision for reopened claims would be considered a part of the IBNR reserve (*Statement of Principles*, p. 15).

Problem S6-9-2.
(a) Give three examples of how data might be separated in order to achieve homogeneous groupings (*Statement of Principles*, p. 15).
(b) How might there be a tradeoff between homogeneity and credibility?

Solution S6-9-2.

(a) The following examples of how data might be separated in order to achieve homogeneous groupings are given on pp. 15-16 of the *Statement of Principles*:

1. For heterogeneous products like commercial multi-peril insurance, separating data by coverage type
2. Separating data for personal risks from data for commercial risks
3. Separating primary coverage data from excess coverage data
4. Separating data before an operational or procedural change from data after such a change
5. Separating data on claims-made policies from data on occurrence-based policies

Any three of the above suffice as an answer. Other valid answers may also be possible.

(b) The pursuit of complete homogeneity may require data to be separated into groups in each of which the amount of data is quite small. The experience within such small homogeneous groups may not be sufficient to make valid generalizations about each group, implying a lack of credibility.

Problem S6-9-3.

(a) According to the *Statement of Principles* (p. 15), with what must data reconcile when used for the analysis of reserves?

(b) How do emergence patterns typically differ between property claims and liability claims?

(c) What effect should be removed in order to evaluate development patterns correctly (*Statement of Principles*, p. 16)?

Solution S6-9-3.

(a) Data used for analysis of reserves must reconcile with **the insurer's financial records** (*Statement of Principles*, p. 15).

(b) Property claims are typically reported much more quickly than liability claims.

(c) The effect of **discounting** should be removed in order to evaluate development patterns correctly. If a reserve is established as a present value of future costs, then upward development may occur simply as a result of paying claims, and this may send a misleading signal.

Problem S6-9-4.

(a) According to the *Statement of Principles* (p. 17), what is one difference between reserving based on GAAP (Generally Accepted Accounting Principles) and reserving based on SAP (Statutory Accounting Principles)? How should insurers accommodate this difference?

(b) Name four kinds of external influences that would have an impact on reserving decisions (*Statement of Principles*, p. 18).

Solution S6-9-4.

(a) SAP reserves generally may not include deductions for anticipated salvage and subrogation recoveries, whereas GAAP reserves are typically reduced by these anticipated recoveries. According to the *Statement of Principles*, insurers should evaluate the estimated impact of such recoveries even though it may not, by law, be applicable to all officially reported reserves.

(b) The following external influences would have an impact on reserving decisions:
1. Regulatory changes
2. Judicial decisions and environment
3. Legislative changes
4. Residual/involuntary market mechanisms
5. Macroeconomic variables, such as inflation
Any four of the above suffice as an answer. Other valid answers may also be possible.

Problem S6-9-5.

(a) How might a reserve estimate account for the uncertainty inherent in reserve projections (*Statement of Principles*, p. 18)?

(b) Name four other loss-related balance sheet items for which loss reserve analysis might have implications (*Statement of Principles*, p. 18).

Solution S6-9-5.

(a) A reserve estimate might account for the uncertainty inherent in reserve projections by including (1) "a implicit provision for uncertainty due to the time value of money" or (2) "an explicit provision for uncertainty in the undiscounted amount" when there is a high degree of volatility.

(b) The following are other loss-related balance sheet items for which loss reserve analysis might have implications (*Statement of Principles*, p. 18):
1. Contingent commissions
2. Retrospective premium adjustments
3. Policyholder dividends
4. Premium deficiency reserves
5. Minimum statutory reserves
6. Deduction for unauthorized reinsurance
Any four of the above suffice as an answer. Other valid answers may also be possible.

Section 10

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 5

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:
Financial Accounting Standards Board, "Statement of Financial Accounting Standards No. 60, Accounting and Reporting by Insurance Enterprises," pp. 7, 9.

Past Casualty Actuarial Society exams: 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-10-1. Similar to Problem 5 from the Fall 2009 Exam 6.

A five-year warranty contract has its premium of $500 paid upfront. It is expected that, each year, the loss on the contract will be 20% higher than the previous year. What is the unearned premium reserve for one such contract at the end of its fourth year - under a deferral-matching accounting paradigm?

Solution S6-10-1. Under a deferral-matching accounting paradigm, the amount of premium earned is proportional to the amount of loss exposure already experienced.

Let x be the proportion of losses expected in the first year. Then we have the following:

First-year losses: x
Second-year losses: 1.2x
Third-year losses. 1.2^2*x
Fourth-year losses. 1.2^3*x
Fifth-year losses. 1.2^4*x

The sum of these values must equal 1, so

$(1 + 1.2 + 1.2^2 + 1.2^3 + 1.2^4)x = 1 \rightarrow x = 1/(1 + 1.2 + 1.2^2 + 1.2^3 + 1.2^4) = 0.1343797033$

After four years, the proportion of expected loss exposure that has not yet taken place is $1.2^4*x = 1.2^4*0.1343797033 = 0.2786497527$. This is also the proportion of premium unearned. Thus, the unearned premium reserve is 500*0.2786497527 = 139.3248764 = **$139.32.**

The following information applies to Problems S6-10-2 and S6-10-3.

You are aware of the following information for claims pertaining to accident year (AY) 2033. As of December 31, 2034, reported losses were $130. It is expected that ultimate AY 2033 losses will be $200. Based on many years of data, cumulative development factors have also been selected in the following manner:

12-months-to-ultimate factor: 1.850
24-months-to-ultimate factor: 1.628
36-months-to-ultimate factor: 1.374

As of December 31, 2035, AY 2033 reported losses are $166.

Problem S6-10-2. Similar to Problem 16(a) from the Fall 2009 Exam 6. Calculate the difference between (i)actual AY 2033 reported losses in calendar year (CY) 2035 and (ii) expected AY 2033 reported losses in CY 2035, based on the data and assumptions given.

Solution S6-10-2.

(a) The actual AY 2033 reported losses in CY 2035 are 166 - 130 = $36.
We find the expected reported losses as follows.
The expected losses yet-to-be-reported (all the way to ultimate) are 200 - 130 = $70.
We need to determine *what fraction* of this yet-to-be-reported amount is expected to be reported in CY 2035.

The 24-months-to-ultimate factor is 1.628, meaning that, as of the end of 2034, 1/1.628 of the loss is expected to have emerged. The loss yet to emerge is thus (1 - 1/1.628) of the total expected amount.

The 36-months-to-ultimate factor is 1.374, meaning that, as of the end of 2035, 1/1.374 of the loss is expected to have emerged.

During 2035, the proportion of the total expected loss that will emerge is thus (1/1.374 - 1/1.628).

Thus, the fraction of yet-to-be-reported losses assigned to CY 2035 is (1/1.374 - 1/1.628)/(1 - 1/1.628) = 0.2943657924.

The expected reported losses for CY 2035 are therefore 70*0.2943657924 = 20.60560547.

The desired (actual - expected) difference is thus 36 - 20.60560547 = 15.39439453 = **$15.39.**

Problem S6-10-3. Similar to Problems 16(b) and 16(c) from the Fall 2009 Exam 6.

(a) Using linear interpolation of the development pattern provided, what are the expected losses emerged between January 1, 2035, and September 30, 2035?

(b) Will the answer in part (a) overestimate or underestimate the projection? Explain your answer.

Solution S6-10-3.

(a) The expected losses yet-to-be-reported (all the way to ultimate) are 200 - 130 = $70.

The 24-months-to-ultimate factor is 1.628.

The 36-months-to-ultimate factor is 1.374.

We want to find, using linear interpolation, the 33-months-to-ultimate factor.

Thus value is 9/12 of the way between 1.628 and 1.374:

1.628 - (9/12)(1.628 - 1.374) = 1.4375.

Thus, the fraction of yet-to-be-reported losses assigned to the first 9 months of CY 2035 is (1/1.4375 - 1/1.628)/(1 - 1/1.628) = 0.1325217391, meaning that the expected losses are 0.1325217391*70 = 9.276521739 = **$9.28.**

(b) Even a visual comparison of the answer in part (a) to the answer in Solution S6-10-2 suggests that the linear interpolation approach would **underestimate** the projection. A real-world reason for this is that development tends to occur at a decreasing rate, with more development occurring earlier. Linear interpolation, however, presumes that development occurs at a uniform rate. The interpolated development factor thus overstates the true factor, leading to an understated estimate for the amount of development occurring up to the time in question.

Problem S6-10-4. Similar to Problem 18 from the Fall 2009 Exam 6.

Fill in the blanks in accordance with Financial Accounting Standard (FAS) #60:

(a) A liability for unpaid claims shall be accrued _____ (at what time?). (FAS #60, p. 7)

(b) Salvage and subrogation shall be evaluated _____ (how?) and _____ (added to or deducted from?) _____ (what?). (FAS #60, p. 9)

(c) Claim adjustment expenses shall be accrued when _____ (what?) is accrued? (FAS #60, p. 9)

Solution S6-10-4.

(a) A liability for unpaid claims shall be accrued **when insured events occur** (FAS #60, p. 7).

(b) Salvage and subrogation shall be evaluated **in terms of their estimated realizable value** and **deducted from the liability for unpaid claims** (FAS #60, p. 9).

(c) Claim adjustment expenses shall be accrued when **the related liability for unpaid claims** is accrued (FAS #60, p. 9).

Problem S6-10-5. Similar to Problem 23 from the Fall 2009 Exam 6.

In the context of reinsurance, what is *surplus relief* and what kind of reinsurance product typically provides it? Would an insurer with a shrinking, stable, or growing book of business have a greater need for surplus relief and why?

Solution S6-10-5. *Surplus relief* occurs when a reinsurer pays a ceding commission on the business it assumes from a primary insurer. This compensates the primary insurer for a fact that, when it acquires new business, statutory accounting principles require immediate recognition of acquisition costs but deferred recognition of premium (which must be recognized as earned over time). The ceding commissions can be recognized as earned immediately. Typically, **pro rata reinsurance**, where the primary insurer and the reinsurer share proportionally in the loss on the ceded policies,provides surplus relief. An insurer with a **growing** book of business has the greatest need of surplus relief, as its immediate acquisition expenses would be the highest.

Section 11

Development Triangles

Section 11 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 12

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 6

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Formulas for Patrik's Method:

Formula 12.1:

Credibility for IBNR =
Z*(Chain Ladder Method IBNR) + (1-Z)*(Stanard-Bühlmann Method IBNR)

Formula 12.2:

Z = (Credibility Factor)*(Report Lag Factor)

(See Gary S. Patrik, "Reinsurance.")

Sources:

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2007 Exam 6

Patrik, G.S., "Reinsurance," *Foundations of Casualty Actuarial Science (Fourth Edition)*, Casualty Actuarial Society, 2001, Chapter 7, pp. 434-464 (section on Reinsurance Loss Reserving).

Pinto, E.; and Gogol, D.F., "An Analysis of Excess Loss Development," *PCAS* LXXIV, 1987, pp. 227-255.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-12-1. Similar to Problem 1 from the 2007 CAS Exam 6.

At a particular snapshot in time, you know the following information for an insunrer:

Unearned premium liability: $150,430
Estimate of losses from runoff of unearned premium: $70,180
Estimate of expenses remaining from runoff of unearned premium: $82,012
Annual fixed and general overhead expense: $18,000

What is the magnitude of the *premium deficiency reserve* as of this snapshot in time?

Solution S6-12-1. Fixed and general overhead expenses are not considered in calculating premium deficiency reserves; only the marginal losses/expenses related to the runoff of the premium are considered.

If there is a negative difference between the unearned premium liability and the estimated losses and expenses from the runoff of unearned premium, then there is a premium deficiency reserve. Here, the difference is 150430 - 70180 - 82012 = -1762. The premium deficiency reserve is the absolute value of this amount, or **$1762.**

Problem S6-12-2. Similar to Problem 2 from the 2007 CAS Exam 6.

A property has an insured value of $560,000 and, in the period covered by a primary insurance policy, has total expected loss of $252,000. The primary insurer signs an excess-of-loss reinsurance treaty, where the reinsurer's responsibility is for the layer $224,000 in excess of $112,000. You also are aware of the following cumulative loss distribution (CLD):

Loss as Percentage of Insured Value: 10% → CLD = 23%
Loss as Percentage of Insured Value: 20% → CLD = 33%
Loss as Percentage of Insured Value: 30% → CLD = 37%
Loss as Percentage of Insured Value: 40% → CLD = 45%
Loss as Percentage of Insured Value: 50% → CLD = 55%
Loss as Percentage of Insured Value: 60% → CLD = 68%
Loss as Percentage of Insured Value: 70% → CLD = 86%
Loss as Percentage of Insured Value: 80% → CLD = 90%
Loss as Percentage of Insured Value: 90% → CLD = 98%
Loss as Percentage of Insured Value: 100% → CLD = 100%

As a result of this treaty, what is the reinsurer's expected loss?

Solution S6-12-2. First, we attempt to match the reinsurance treaty layer characteristics with the given loss percentages of insured value. The insured value is $560,000. Of this, $224,000 is 40% and $112,000 is 20%. Thus, we are looking at the layer that is 40% in excess of 20% (i.e., the layer from 20% to 60%). The loss distribution *in that layer* is CLD(60%) - CLD(20%) = 68%-

33% = 35% of the total expected loss. So the reinsurer is expected to be responsible for 0.35*252000 = **$88,200.**

Problem S6-12-3. Similar to Problem 3 from the 2007 CAS Exam 6.

You have the following information about a reinsurance contract, pertaining to calendar-accident year 2055:

Earned Risk Pure Premium: $44,430
On-Level Premium: $45,660
Aggregate Report Lag: 42%
Chain Ladder IBNR Estimate: $18,010
Stanard-Bühlmann IBNR Estimate: $20,120
Credibility Factor for Patrik's Method: 0.9

Use Patrik's method to find the credibility IBNR estimate for 2055.

Solution S6-12-3. We first use Formula 12.2 to calculate Z:

Z = (Credibility Factor)*(Report Lag Factor), where Credibility Factor = 0.9 and Report Lag Factor = 0.42, so Z = 0.42*0.9 = 0.378.

Now we apply Formula 12.1:
Credibility for IBNR =
Z*(Chain Ladder Method IBNR) + (1-Z)*(Stanard-Bühlmann Method IBNR) =
0.378*18010 + (1-0.378)*20120 = **$19,322.42.**

Problem S6-12-4. Similar to Problem 4 from the 2007 CAS Exam 6. You know the following for a particular insurer:

For a retention of $250,000 per occurrence, the percentage of ultimate loss exceeding that retention is 80%.
For a retention of $2,000,000 per occurrence, the percentage of ultimate loss exceeding that retention is 30%.

The 24-months-to-ultimate excess loss development factor above a retention of $250,000 is 1.56.
The 24-months-to-ultimate excess loss development factor above a retention of $2,000,000 is 2.78.

Use the method described by Pinto and Gogol ("An Analysis of Excess Loss Development") to find the 24-months-to-ultimate excess loss development factor (LDF) for the layer $1,750,000 excess of $250,000.

Solution S6-12-4. Assume that L is the total ground-up loss.

If the retention is $250,000, then the total ultimate loss is 0.8L and the fraction of the loss relative to the ultimate amount at 24 months is 1/1.56 = 0.641025641.

So, for the $250,000 retention, the total *developed* loss at 24 months is 0.8L*0.641025641 = 0.5128205128L.

If the retention is $2,000,000, then the total ultimate loss is 0.3L and the fraction of the loss relative to the ultimate amount at 24 months is 1/2.78 = 0.3597122302

So, for the $2,000,000 retention, the total *developed* loss at 24 months is 0.3L*0.3597122302 = 0.1079136691L.

This means that, for the layer $1,750,000 excess of $250,000, the the total *developed* loss at 24 months is 0.5128205128L-0.1079136691L = 0.4049068438L.

The *ultimate* loss for this layer is 0.8L-0.3L = 0.5L.

The LDF for the layer is (Ultimate Loss)/(Developed Loss) = 0.5L/0.4049068438L = **1.234851936.**

Problem S6-12-5. Similar to Problem 7 from the 2007 CAS Exam 6.

You are analyzing the following paid loss development triangle, where *cumulative* paid losses for each accident year (AY) are evaluated as of 12, 24, 36, and 48 months, via the following notation, where applicable: (12-month estimate, 24-month estimate, 36-month estimate, 48-month estimate)

AY 2019: (430, 450, 487, 560)
AY 2020: (243, 342, 543)
AY 2021: (1100, 1250)
AY 2022: (320)

These data are valued as of December 31, 2022. What are the losses paid in calendar year (CY) 2022?

Solution S6-12-5. The outermost diagonal of the loss development triangle indicates cumulative paid losses for each AY's experience in CY 2022. To find the *incremental* paid losses in CY 2022, we must subtract (where possible) from the CY 2022 cumulative amounts the prior year's (CY 2021's) cumulative amounts, expressed on the second-outermost diagonal.

Our incremental losses paid in CY 2022 are thus
(560 - 487) + (543 - 342) + (1250 - 1100) + 320 = **744.**

Section 13

Actuarial Triangles Involving Reported Claims, Paid Claims, and Earned Premium

This section of the study guide is intended to provide practice problems and solutions to accompany the pages of *Estimating Unpaid Claims Using Basic Techniques*, cited below. Students are encouraged to read these pages before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

Some of the questions here ask for short written answers based on the reading. This is meant to give the student practice in answering questions of the format that will appear on Exam 5B (Old Exam 6). Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Source:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 6, pp. 63-69.

Original Problems and Solutions from The Actuary's Free Study Guide

Refer to the following information for Problems S6-13-1 through S6-13-5:

You are given the following information for an insurer:

In Calendar Year (CY) 2041, earned premium was $13,135.
In CY 2042, earned premium was $31,631, and the company increased its rate level by 20%.
In CY 2043, earned premium was $24,124, and the company decreased its rate level by 5%.
In CY 2044, earned premium was $26,750, and the company decreased its rate level by 7%.

Assume all rate changes occurred on January 1 of their respective years.

Assume that reported claims for Accident Year 2041 were as follows:
As of 12 months: $8,214
As of 24 months: $10,233
As of 36 months: $11,351
As of 48 months: $14,888

Assume that reported claims for Accident Year 2042 were as follows:
As of 12 months: $12,124

As of 24 months: $16,774
As of 36 months: $20,004

Assume that reported claims for Accident Year 2043 were as follows:
As of 12 months: $15,123
As of 24 months: $16,125

Assume that reported claims for Accident Year 2044 were as follows:
As of 12 months: $14,499

Assume that paid claims for Accident Year 2041 were as follows:
As of 12 months: $6,662
As of 24 months: $8,821
As of 36 months: $9,821
As of 48 months: $13,333

Assume that paid claims for Accident Year 2042 were as follows:
As of 12 months: $10,242
As of 24 months: $13,474
As of 36 months: $18,821

Assume that paid claims for Accident Year 2043 were as follows:
As of 12 months: $10,033
As of 24 months: $14,300

Assume that paid claims for Accident Year 2044 were as follows:
As of 12 months: $12,232

Problem S6-13-1. For (i) CY 2042, (ii) CY 2043, and (iii) CY 2044, find the following:

(a) The cumulative percent rate level change from CY 2041

(b) The annual percent exposure change just for the calendar year in question

Solution S6-13-1. This problem is based on the discussion in Friedland, p. 64.

(a) We calculate the cumulative rate level change from CY 2041 for each year:
(i) For CY 2042, the rate level change is just the given value of **+20%**.
(ii) For CY 2043, the cumulative rate level change is 1.2*(1-0.05) - 1 = 0.14 = **+14%.**
(iii) For CY 2044, the cumulative rate level change is 1.2*(1-0.05)*(1-0.07) = 1.0602 = **+6.02%.**

(b) The annual exposure change is the change in earned premium that was not accounted for by the rate level change during the calendar year in question.
(i) For CY 2042, the exposure change is
((CY 2042 earned premium)/(CY 2041 earned premium))/(1+CY 2042 rate level change) - 1 =
(31631/13135)/1.2 - 1 = 1.006788479 = **+100.6788479%**.

(ii) For CY 2043, the exposure change is
((CY 2043 earned premium)/(CY 2042 earned premium))/(1+CY 2042 rate level change) - 1 = (24124/31631)/0.95 - 1 = -0.1971899652 = **-19.71899652%**.
(iii) For CY 2044, the exposure change is
((CY 2044 earned premium)/(CY 2043 earned premium))/(1+CY 2043 rate level change) - 1 = (26750/24124)/0.93 - 1 = 0.1923164011 = **+19.23164011%**.

Problem S6-13-2. Use the given information to construct a *ratio of reported claims to earned premium triangle*, in the following format for as many ratios as are applicable:

Given AY: (12-month ratio, 24-month ratio, ...).

Solution S6-13-2. This problem is based on the discussion in Friedland, p. 66.

For each accident year, we divide the reported claim figures by the earned premium *for that accident year*. Thus, we get the following:

AY 2041: (8214/13135, 10233/13135, 11351/13135, 14888/13135)
AY 2042: (12124/31631, 16774/31631, 20004/31631)
AY 2043: (15123/24124, 16125/24124)
AY 2044: (14499/26750)

Our triangle will appear as follows (with factors rounded to three decimal places):

Ratio of reported claims to earned premium triangle
AY 2041: (0.625, 0.779, 0.864, 1,133)
AY 2042: (0.383, 0.530, 0.632)
AY 2043: (0.627, 0.668)
AY 2044: (0.542)

Problem S6-13-3. Use the given information to construct a *ratio of reported claims to on-level earned premium triangle*, in the following format for as many ratios as are applicable:

Given AY: (12-month ratio, 24-month ratio, ...).

Solution S6-13-3. This problem is based on the discussion in Friedland, p. 66.

For each accident year, we divide the reported claim figures by the earned premium *for that accident year* and *then* divide the result by the cumulative rate level change factor applicable to the period from the accident year in question through 2044. Thus, we get the following:

AY 2041: (8214/(13135*1.0602), 10233/(13135*1.0602), 11351/(13135*1.0602), 14888/(13135*1.0602))
AY 2042: (12124/(31631*0.95*0.93), 16774/(31631*0.95*0.93), 20004/(31631*0.95*0.93))
AY 2043: (15123/(24124*0.93), 16125/(24124*0.93))
AY 2044: (14499/26750)

Our triangle will appear as follows (with factors rounded to three decimal places):

Ratio of reported claims to on-level earned premium triangle
AY 2041: (0.590, 0.735, 0.815, 1.069)
AY 2042: (0.434, 0.600, 0.716)
AY 2043: (0.674, 0.719)
AY 2044: (0.542)

Problem S6-13-4. Use the given information to construct a *ratio of paid claims to reported claims triangle*, in the following format for as many ratios as are applicable:

Given AY: (12-month ratio, 24-month ratio, ...).

Solution S6-13-4. This problem is based on the discussion in Friedland, p. 68.

For each accident year, we divide the paid claim figures by the reported claim figures. Thus, we get the following:

AY 2041: (6662/8214, 8821/10233, 9821/11351, 13333/14888)
AY 2042: (10242/12124, 13474/16774, 18821/20004)
AY 2043: (10033/15123, 14300/16125)
AY 2044: (12232/14499)

Our triangle will appear as follows (with factors rounded to three decimal places):

Ratio of paid claims to reported claims
AY 2041: (0.811, 0.862, 0.865, 0.895)
AY 2042: (0.844, 0.803, 0.941)
AY 2043: (0.663, 0.887)
AY 2044: (0.844)

Problem S6-13-5. Use the given information to construct a *ratio of paid claims to on-level earned premium triangle*, in the following format for as many ratios as are applicable:
Given AY: (12-month ratio, 24-month ratio, ...).

Solution S6-13-5. This problem is based on the discussion in Friedland, p. 69.

For each accident year, we divide the paid claim figures by the earned premium *for that accident year* and *then* divide the result by the cumulative rate level change factor applicable to the period from the accident year in question through 2044. Thus, we get the following:

AY 2041: (6662/(13135*1.0602), 8821/(13135*1.0602), 9821/(13135*1.0602), 13333/(13135*1.0602))
AY 2042: (10242/(31631*0.95*0.93), 13474/(31631*0.95*0.93), 18821/(31631*0.95*0.93))
AY 2043: (10033/(24124*0.93), 14300/(24124*0.93))
AY 2044: (12232/26750)

Our triangle will appear as follows (with factors rounded to three decimal places):

Ratio of reported claims to on-level earned premium triangle
AY 2041: (0.479, 0.633, 0.705, 0.957)
AY 2042: (0.366, 0.482, 0.673)
AY 2043: (0.447, 0.637)
AY 2044: (0.457)

Section 14

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 7

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Retrospective rating determines the premium for the present time period on the basis of the losses for the present time period.

Let P be the premium in retrospective rating. Then the following formula holds:

Formula 14.1
P = (B + CL*LCF)*TM, where
B = the basic premium - a constant which does not vary depending on losses;
CL = the capped loss; losses are typically capped at a certain value for the purposes of retrospective rating;
LCF = loss conversion factor - by definition, the multiplier which converts loss into premium;
TM = tax multiplier, needed to for the premium to incorporate the premium tax, which is a percentage of premium.

Due to the development of losses, multiple retrospective adjustments ("retro adjustments") are often required to accurately determine the premium.

Sources:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2007 Exam 6.

Slywotzky, A.J., and Drzik, J., "Countering the Biggest Risk of All," *Harvard Business Review*, April 2005, Harvard Business School Publishing.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-14-1. Similar to Problem 5 from the 2007 CAS Exam 6. Suppose that an insurance policy is retrospectively rated and the following information is known:

Loss conversion factor = 1.22
Tax multiplier = 1.04
Basic premium = $13,165
Loss at first retro adjustment: $15,530
Capped loss at first retro adjustment: $15,530
Loss at second retro adjustment: $25,010
Capped loss at second retro adjustment: $20,000

What is the ratio of premium development to loss development between the first and second retro adjustments?

Solution S6-14-1. We can already figure out how much loss has developed between the two adjustments: 25010 - 15530 = $9480.

To determine premium development, we need to calculate the retrospectively rated premium at each adjustment, using Formula 14.1: P = (B + CL*LCF)*TM.

At the first retro adjustment, the premium is (13165 + 15530*1.22)*1.04 = 33396.064.
At the second retro adjustment, the premium is (13165 + 20000*1.22)*1.04 = 39067.6.
Premium development is thus 39067.6 - 33396.064 = 5671.536.

The desired ratio is 5671.536/9480 = **0.5982632911**.

Problem S6-14-2. Similar to Problem 6 from the 2007 CAS Exam 6. During each particular accident year, the exposure to loss is uniform. You also know that cumulative loss development follows the function $D(t) = 1-e^{-2t}$, where t is the time in years from the *average* accident date. As of December 31, 2055, you know the following:

Incurred losses for accident year (AY) 2052 are $31,531.
Incurred losses for accident year (AY) 2053 are $12,330.

What is the ultimate loss for AY 2052 and AY 2053 combined?

Hint: This problem should be intuitive enough, but *be careful* about your choice for t!

Solution S6-14-2. The function D(t) simply represents the fraction of the ultimate loss amount that is developed to date. What is t? As defined, t is the time from the average accident date. Since exposure to loss in each year is uniform, the average accident date is the *midpoint* of the year. Thus, from the midpoint of 2052 to the end of 2055, there are 3.5 years. From the midpoint of 2053 to the end of 2055, there are 2.5 years. The rest is a matter of dividing known incurred

losses for each year by the fraction we expect to be already developed; this gives us the ultimate loss amounts for each year, and we then add those together:

$31531/(1-e^{-2*3.5}) + 12330/(1-e^{-2*2.5}) = 43973.42146 =$ **$43,973.42**.

Problem S6-14-3. Similar to Problem 8 from the 2007 CAS Exam 6. The *disposal rate* for claims measures the proportion of claims from a given report year that are settled within the specified time interval from the report year.

Consider a triangle displaying disposal rates in the following format for each report year:

(Rate at 0-24 months from the report year, rate at 25-48 months, rate from 48 months to ultimate)
Report Year 2028: (0.431, 0.352, 0.217)
Report Year 2029: (0.540, 0.260)
Report Year 2030: (0.410)

Estimate the disposal rate for the group of claims at 25-48 months from report year 2030. Use *only* in the information from the most recent available calendar year.

Solution S6-14-3. The information from the most recent available calendar year is the information pertaining to 2029 data. This shows that a 0-24 disposal rate of 0.540 corresponds to a 25-48 month disposal rate of 0.260. In 2029, after 24 months, the proportion of *unsettled* claims was 1 - 0.540 = 0.460. So the proportion of this amount that was settled was 0.260/0.460 = 0.5652173913.

In 2030, after 24 months, the proportion of *unsettled* claims was 1 - 0.410 = 0.590. If 56.52173913% of 0.590 gets settled 25-48 months from 2030, the desired disposal rate is 0.590*0.5652173913 = **0.3334782609**.

Problem S6-14-4. Similar to Problem 9 from the 2007 CAS Exam 6. You know the following:

Earned premium for Accident Year (AY) 2050 was $4000.
Earned premium for AY 2051 was $2600.
Earned premium for AY 2052 was $3600.
Earned premium for AY 2053 was $4500.

You are also given a cumulative reported loss triangle, in the following format for each accident year: (Developed loss at 12 months, developed loss at 24 months, developed loss at 36 months, ultimate loss).

Cumulative reported losses
AY 2050: (1800, 1900, 2500, 3000)
AY 2051: (1660, 1790, 2300)
AY 2052: (1900, 2200)
AY 2053: (2000)

Use the *percentage of premium method* to estimate the IBNR for accident year 2053. When calculating factors for each accident year, use a simple arithmetic average of the factors.

Solution S6-14-4. As the name suggests, the percentage of premium method assumes that the IBNR is a set percentage of the premium for the accident year in question, where the percentage is equal to the not-yet-developed losses as a fraction of premium.

First, it is helpful to convert our cumulative reported loss triangle into an *incremental* reported loss triangle:

Incremental reported losses
AY 2050: (1800, 100, 600, 500)
AY 2051: (1660, 130, 510)
AY 2052: (1900, 300)
AY 2053: (2000)

For each accident year, we can calculate the ratios of incremental reported losses to earned premium:

Ratios of incremental reported losses to earned premium:

AY 2050 - at 24 months: 100/4000 = 0.025
AY 2051 - at 24 months: 130/2600 = 0.05
AY 2052 - at 24 months: 300/3600 = 0.08333333333
Arithmetic average - at 24 months: (0.025 + 0.05 + 0.08333333333)/3 = 0.0527777778

AY 2050 - at 36 months: 600/4000 = 0.15
AY 2051 - at 36 months: 510/2600 = 0.1961538462
Arithmetic average - at 36 months: (0.15 + 0.1961538462)/2 = 0.1730769231

AY 2050 - at ultimate: 500/4000 = 0.125 - the only value, so there is no need to take an arithmetic average.

The cumulative ratio of incremental reported losses to earned premium - between 12 months and ultimate - is 0.0527777778 + 0.1730769231 + 0.125 = 0.3508547009.

Thus, the estimated IBNR is the above ratio multiplied by the AY 2053 earned premium: 4500*0.3508547009 = 1578.846154 = **$1578.85**.

Problem S6-14-5. Similar to Problem 10 from the 2007 CAS Exam 6. Briefly state two kinds of strategic risk - other than new product failure, customer priority shifts, a technology shift, and a one-of-a-kind competitor - for a company that produces physical goods - as discussed in the Slywotzky and Drzik 2005 paper. For each risk you mention, state a mitigation strategy for the risk.

Solution S6-14-5. Three other kinds of risks are mentioned in the Slywotzky and Drzik 2005 paper, and any two suffice as an answer:

Risk: Brand erosion
Mitigation strategy: Redefine the scope of brand investment - focus on quality of product and customer service instead of just marketing.

Risk: Industry margin squeeze
Mitigation strategy: Shift the collaborate/compete ratio: firms whose margins are being squeezed should cooperate and share functions more than previously.

Risk: Market stagnation
Mitigation strategy: Engage in *demand innovatio*n - focus on the customer's perspective and how the firm can help the customer beyond just offering a product, i.e., by helping the customer cut costs, improve profitability and reduce capital intensity.

Section 15

Uses of Actuarial Triangles for Claims and Claim Counts to Evaluate an Insurer's Situation

This section of the study guide is intended to provide practice problems and solutions to accompany the pages of *Estimating Unpaid Claims Using Basic Techniques*, cited below. Students are encouraged to read these pages before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

Some of the questions here ask for short written answers based on the reading. This is meant to give the student practice in answering questions of the format that will appear on Exam 5B (Old Exam 6). Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Source:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 6, pp. 70-77.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-15-1.

(a) Identify four factors that could change an insurer's ratio of closed claims to reported claims. (See Friedland, p. 71).

(b) If an insurer makes a determined effort to close claims more quickly, how would this affect the mix of open claims at a given time? Explain. (See Friedland, p. 72).

Solution S6-15-1.

(a) The following factors could change an insurer's ratio of closed claims to reported claims (Friedland, p. 71):

1. Natural disasters and other events that interrupt an insurer's systems and lead to delays in closing claims
2. "Change in the guidelines for the establishment of a claim"
3. "Delegation of a higher limit for settlement of claims to a TPA" - a third-party administrator
4. "Introduction of a new call center to handle claims"

5. "Decrease in the statute of limitations, which often accompanies major tort reform"
6. "Restructuring of the claim field offices" - e.g., acquisition of new offices or mergers of existing offices

Any four of the above suffice as an answer. Other valid answers may also be possible.

(b) If an insurer makes a determined effort to close claims more quickly, this will most likely be reflected on settlement patterns for simpler, less expensive claims, where the payout is easier to determine. Larger claims, particularly liability claims involving bodily injury, often take a long time to settle because of circumstances beyond the insurer's control, such as the workings of the court system. Faster settlement of smaller claims would likely change the mix of open claims to one comprised of a greater proportion of larger, more complex claims.

Problem S6-15-2. Various kinds of development triangles are used in analyzing claims in insurance. For each of the following values, name the kinds of triangles that could be used to calculate the value and give the formula involved in the calculation. In your answers, assume that you only have access to *reported claim and claim count triangles*, *paid claim triangles*, and *closed claim count triangles.* (See Friedland, p. 72.)

(a) Average reported claim
(b) Average paid claim
(c) Average case outstanding

Solution S6-15-2.

(a) Average reported claim = (Reported claims)/(Claim count). Use the reported claim triangle and reported claim count triangle.

(b) Average paid claim = (Paid claims)/(Closed claim count). Use the paid claim triangle and the closed claim count triangle.

(c) Average case outstanding = (Reported claims - Paid claims)/(Reported claim count - Closed claim count). Use all four triangles - for reported claims and claim counts, paid claims, and closed claim counts.

Problem S6-15-3. You have the following information for Insurer Q for Accident Year (AY) 2050, as of December 31, 2050:

Reported claims: $314,000
Paid claims: $214,000
Reported claim count: 646
Closed claim count: 441

What is the average case outstanding for AY 2050, as of December 31, 2050?

Solution S6-15-3. We use the formula Average case outstanding = (Reported claims - Paid claims)/(Reported claim count - Closed claim count) = (314000 - 214000)/(646-441) = 487.804878 = **$487.80.**

Problem S6-15-4. Friedland, on pp. 73-74, discusses a "mismatch" that occurs in average paid claim triangles. Discuss why the mismatch occurs and what it implies.

Solution S6-15-4. The mismatch can be identified by examining the formula Average paid claim = (Paid claims)/(Closed claim count). Paid claims can apply both to claims that are closed and claims that are open but on which partial payments have been made. However, the closed claim count, by definition, only identifies claims that have been fully closed. Thus, the average paid claim formula mistakenly matches partial payments on open claims to closed claims. This might lead to an overestimate of the true average payment on closed claims.

Problem S6-15-5.

(a) Suppose you are examining the average case outstanding triangle of an insurer with a stable book of business (including a stable mix of business and the same policy offerings from year to year). As you move down a particular column, representing average case outstanding at a particular claim age for multiple accident years, what do you expect to observe with regard to average case outstanding trends? (See Friedland, p. 75.)

(b) With regard to an insurer's definition of claim counts, discuss one matter that the actuary examining the insurer's book of business must clarify in order to interpret the data properly. (See Friedland, p. 73.)

Solution S6-15-5.

(a) Average case outstanding for a company with a stable book of business should be expected to change per year by the percentage of general annual inflation. (See Friedland, p. 75.)

(b) With regard to an insurer's definition of claim counts, the actuary must clarify whether or not the definition includes claims closed with no payment (CNP). If CNP claims are included, the average paid claim amount would be systematically lower than if CNP claims were not included, since the denominator is inflated without the numerator being inflated. Moreover, it is important to ascertain whether the insurer's treatment of CNP claims changed during the timeframe under observation. (See Friedland, p. 73.)

Section 16

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 8

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:

Past Casualty Actuarial Society exams: 2007 Exam 6 and 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-16-1. Similar to Problem 13 from the Fall 2009 CAS Exam 6.

For Accident Year (AY) 2120 through Accident Year 2123, you are aware of the following information, where the ratios are displayed in the format (ratio at 12 months, ratio at 24 months, ratio at 36 months, ratio at ultimate):

Ratio of Paid ALAE to Paid Claims Only
AY 2120: (0.012, 0.016, 0.020, 0.025)
AY 2121: (0.015, 0.015, 0.018)
AY 2122: (0.014, 0.018)
AY 2123: (0.013)

AY 2120 Estimated Ultimate Claims: $31,150
AY 2121 Estimated Ultimate Claims: $33,310
AY 2122 Estimated Ultimate Claims: $30,120
AY 2123 Estimated Ultimate Claims: $38,125

(a) Use the multiplicative-paid-ALAE-to-paid-claims-only method to estimate ultimate ALAE for AY 2123. When calculating age-to-age development factors, use a simple average.

(b) Briefly discuss two possible disadvantages of the multiplicative-paid-ALAE-to-paid-claims-only method.

Solution S6-16-1. (a) First, we calculate the 12-24-month, 24-36-month, and 36-month-to-ultimate age-to-age factors for each accident year where this is possible:

Age-to-Age Factors for Ratio of Paid ALAE to Paid Claims Only
AY 2120: (0.016/0.012, 0.020/0.016, 0.025/0.020)
AY 2121: (0.015/0.015, 0.018/0.015)
AY 2122: (0.018/0.014)

Age-to-Age Factors for Ratio of Paid ALAE to Paid Claims Only
AY 2120: (1.333, 1.250, 1.250)
AY 2121: (1.000, 1.200)
AY 2122: (1.28571286)

We take the simple averages of the age-to-age factors to find our estimates:

12-24 month factor estimate: (1.333333 + 1 + 1.28571286)/3 = 1.206349206.
24-36 month factor estimate: (1.250 + 1.200)/2 = 1.225.
36-month-to-ultimate factor estimate: 1.250 - this is the only value we have.

The estimated 12-month-to-ultimate factor is thus 1.206349206*1.225*1.250 = 1.8472222222. We multiply by this factor the AY 2123 ratio at 12 months of paid ALAE to paid claims only: 0.013*1.8472222222 = 0.02401388889. This is the estimated ultimate ratio of paid ALAE to paid claims only for AY 2123.

The estimated ultimate ALAE is the AY 2123 estimated claims, multiplied by the ratio derived above: 0.02401388889*38125 = 915.5295139 = **$915.53**.

(b) The following are two disadvantages of the multiplicative-paid-ALAE-to-paid-claims-only method:

1. The accuracy of the ultimate ALAE estimate is dependent on the accuracy of the ultimate claim estimate, which could be subject to considerable error.

2. If a lot of ALAE are devoted to claims that end up being closed without no payment (CNP), the determination of ultimate ALAE as a percentage of ultimate paid claims would overlook the effect that the CNP claims have on ALAE.

Problem S6-16-2. Similar to Problem 16 from the Fall 2007 CAS Exam 6.

Why is *impairment* an important concept in insurance accounting? What could happen to an insurer that does not take the possibility of impairment into consideration?

Solution S6-16-2. Impairment is a situation where an asset can no longer be expected to produce the kinds of economic benefits that were anticipated when the asset was acquired. Considering impairment is important for insurers who do not wish to overstate the value of their assets. Insurers who neglect to consider impairment might find themselves risking insolvency because of premium receivables, reinsurance recoverables, and investment assets for which these insurers would not be able to receive monetary amounts that were previously estimated to be likely.

Problem S6-16-3. Similar to Problem 14 from the Fall 2007 CAS Exam 6. Fill in the blanks on the following statements related to the qualifications for a short-duration reinsurance contract to be treated as reinsurance under Generally Accepted Accounting Principles (GAAP).

(a) One possibility is that **(i)** the reinsurer has assumed _____ and **(ii)** the reinsurer may realize _____.

(b) The other possibility is that substantially _____ has been assumed by the reinsurer from the underlying policies.

(c) Why would a *loss corridor* provision in a short-duration reinsurance contract be relevant for the purposes of determining whether the contract could be treated as reinsurance under GAAP?

(d) In what circumstances could *retrospective reinsurance* benefit the ceding insurer?

Solution S6-16-3. (a) One possibility is that **(i)** the reinsurer has assumed **significant risk from the underlying policies due to the risk transfer** and **(ii)** the reinsurer may realize **a significant loss due to the risk transfer**.

(b) The other possibility is that substantially **all of the insurance risk** has been assumed by the reinsurer from the underlying policies.

(c) A loss corridor provision states that the ceding insurer would be assuming a portion of the loss within a certain range (e.g., 10% of the loss between $200,000 and $500,000). The size of the loss corridor is important for determining the extent of the risk remaining for the reinsurer. Moreover, the loss corridor's existence implies that the reinsurer has not assumed substantially *all* of the insurance risk.

(d) Retrospective reinsurance bases the reinsurance premium on the actual loss experience of the primary insurer's book of business. If there have been fewer losses, the reinsurance premium is lower, which benefits the ceding insurer.

Problem S6-16-4. Similar to Problem 17 from the Fall 2007 CAS Exam 6. An annual-term insurance policy is sold on March 1, 2014, and has an effective date of July 1, 2014. The premium for the policy is $500 and is collected by the insurer on May 1, 2014. As of January 31, 2015, no expenses or claims were incurred in association with the policy.

(a) What would the insurer's premium-related balance sheet for this policy look like as of May 31, 2014?

(b) What would the insurer's premium-related balance sheet for this policy look like as of January 31, 2015?

Solution S6-16-4. (a) When the premium is collected but not yet earned, it is a liability - deposit premium. However, the insurer has also collected an asset - cash - of equivalent magnitude on May 1, 2014. The policy is not yet in effect on May 31, 2014, so the balance sheet appears as follows:

Asset: Cash: $500
Liability: Deposit Premium: $500.

(b) As of January 31, 2015, 7 months of premium - (7/12)*500 = $291.67 - have been earned, and the rest - 500 - 291.67 = $208.33 - are considered part of the unearned premium reserve, a liability. The cash of $500 is presumably still there, and the deposit premium liability has been eliminated, since the policy is now in effect. The balance sheet appears as follows:

Asset: Cash: $500
Liability: Unearned Premium Reserve: $208.33
Surplus (difference between assets and liabilities): $291.67

Note that the surplus here corresponds to the premium that has been earned.

Problem S6-16-5. Similar to Problem 18 from the Fall 2007 CAS Exam 6.

Zgarflzarg is a peculiar type of natural disaster against which Primary Insurer X has purchased a reinsurance treaty. The following, in thousands of dollars, are the expected losses from zgarflzarg in the year 2044:

January: 660
February: 130
March: 0
April: 0
May: 5000
June: 0
July: 0
August: 0
September: 400
October: 0
November: 2000
December: 0

The reinsurance premium for the treaty is $10,000,000.

(a) What is the difference in the unearned premium reserve as of May 31, 2044, between **(i)** the pro rata approach and **(ii)** the approach requiring recognition earned premium during a month to be proportional to expected losses during that month?

(b) Referring to part (a), is approach (i) or approach (ii) preferable here? Explain.

Solution S6-16-5. (a) (i) The pro rata approach assumes that 1/12 of the premium is earned each month. As of May 31, 2044, 7 months' premium is unearned, implying an unearned premium reserve of (7/12)*10,000,000 = $5,833,333.33.

(ii) Total expected loss in 2044 (in thousands of dollars) is 660 + 130 + 5000 + 400 + 2000 = 8190. Of this amount, 400 + 2000 = 2400 is still expected to occur after May 31, 2044. So the unearned premium reserve would be (2400/8190)*10,000,000 = $2,930,402.93.

The difference between the two results is 5,833,333.33 - 2,930,402.93 = **$2,902,930.40.**

(b) Approach (ii) from part (a) (the approach where earned premium is proportional to expected loss) is preferable here, because expected loss does not occur uniformly in each month. Zgarflzarg seems to occur primarily in May and November, and so most of the earned premium should be allocated to those months as well. The pro rata method might lead the insurer to underestimate its unearned premium liability in January-April, overestimate it in May-October, underestimate it in November, and overestimate it in December.

Section 17

Concepts Involved in the Chain Ladder Method

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Source: Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 7, pp. 84-90.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-17-1. List the seven steps of the *development method*, i.e., the *chain ladder method*. (See Friedland, p. 85.)

Solution S6-17-1. The following are the seven steps of the chain ladder method (Friedland, p. 85):
1. "Compile data in a development triangle."
2. "Calculate age-to-age factors."
3. "Calculate averages of the age-to-age factors."
4. "Select claim development factors."
5. "Select tail factor."
6. "Calculate cumulative claim development factors."
7. "Project ultimate claims."

Problem S6-17-2. For 36 to 48 months, the age-to-age factors calculated from various accident years (AYs) of data are as follows:

AY 2112:1.315
AY 2113: 1.125
AY 2114:1.128
AY 2115: 1.269

AY 2116: 1.120
AY 2117: 1.006

Calculate the following statistics:

(a) Simple average for the latest six years
(b) Simple average for the latest five years
(c) Simple average for the latest three years
(d) Medial average for the latest six years (excluding the high and low value)
(e) Geometric average for the latest six years
(f) Volume-weighted average for the latest six years, given that exposures in each subsequent year are 5% higher than in the previous year

Solution S6-17-2. (a) Simple average for the latest six years:
(1.315 + 1.125 + 1.128 + 1.269 + 1.120 + 1.006)/6 = **1.1605.**

(b) Simple average for the latest five years:
(1.125 + 1.128 + 1.269 + 1.120 + 1.006)/5 = **1.1296.**

(c) Simple average for the latest three years:
(1.269 + 1.120 + 1.006)/3 = **1.131666667**.

(d) Medial average for the latest six years:
(1.125 + 1.128 + 1.269 + 1.120)/4 = **1.1605.**

(e) Geometric average for the latest six years:
(1.315*1.125*1.128*1.269*1.120*1.006)^(1/6) = **1.155963621**.

(f) Volume-weighted average for the latest six years, given that exposures in each subsequent year are 5% higher than in the previous year:

Assume, for convenience, that there is 1 exposure unit in AY 2112. Then, each year, the number of exposure units increases by a factor of 1.05. The volume-weighted average is thus

(1*1.315 + 1.05*1.125 + (1.05^2)*1.128 + (1.05^3)*1.269 + (1.05^4)*1.120 +
(1.05^5)*1.006)/(1 + 1.05 + 1.05^2 + 1.05^3 + 1.05^4 + 1.05^5) = **1.154704947**.

Note that I have used MS Excel notation here to facilitate ease of computerized computation. Problems of this sort are not, in most real-world applications, done by hand. On the exam, the volume of computations required will, I expect, be lower per problem than it is here.

Problem S6-17-3. Friedland, p. 88, discusses five characteristics that actuaries examine when reviewing claim development factors and the experience on the basis of which they are derived. State the name of each characteristic and state one possible question related to it.

Solution S6-17-3. The following are five characteristics that actuaries examine when reviewing claim development factors and the experience on the basis of which they are derived:

1. **Smooth progression of individual age-to-age factors and average factors across development periods:** Do the age-to-age factors steadily decrease toward 1 as later time periods from the accident date are considered?

2. **Stability of age-to-age factors for the same development period:** What is the variance in the age-to-age factors for a particular time period (X months to Y months from the accident date) among the accident years considered?

3. **Credibility of the experience:** What is the volume of the underlying experience, and is it sufficiently large for the experience to be used on a stand-alone basis, or must external data be considered?

4. **Changes in patterns:** Do any systematic differences from one time period to the next suggest a different external or internal operating environment?

5. **Applicability of the historical experience:** Is it appropriate to assume that claims will develop in the future much as they have developed in the past, or must one take account of new circumstances that have not yet impacted historical claim data?

Problem S6-17-4. Briefly describe three approaches commonly used by actuaries to estimate tail development factors. (See Friedland, p. 90.)

Solution S6-17-4. The following are three approaches commonly used by actuaries to estimate tail development factors:
1. Use industry benchmark factors.
2. Extrapolate tail factors by fitting a curve - often an exponential curve - to known development factors.
3. Assume that reported development is already at ultimate and use the ratio of reported claims to paid claims as the estimate of the tail factor.

Problem S6-17-5. You know the following age-to-age factors for accident year 2033:

12-24 months: 1.134
24-36 months: 1.130
36-48 months: 1.102
48-60 months: 1.055
12 months to ultimate: 1.500

What is the tail (60-months-to-ultimate) development factor?

Solution S6-17-5. The 12-months-to-ultimate factor is the product of the given age-to-age factors *and* the tail factor. Thus, the tail factor is 1.500/(1.134*1.130*1.102*1.055) = **1.006852162.**

Section 18

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 9

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Sources:

Past Casualty Actuarial Society exams: 2007 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-18-1. Similar to Question 36 from the 2007 CAS Exam 6. Explain why, for a retrospectively rated policy, retrospective premiums will increase at a decreasing rate as reported losses increase. Also explain why this would apply, in particular, to cases where catastrophic injuries make a person eligible for lifetime benefits.

Solution S6-18-1. The determination of retrospective premium is subject to per-occurrence caps on losses and also, typically, to a maximum premium amount. Thus, not the entire amount of every subsequent loss would be added retrospective premium, and the proportion of each subsequent loss's contribution is probably going to decrease. With regard to cases where catastrophic injuries make a person eligible for lifetime benefits, the initial injury may not seem like one that will disable an individual for life. Later, as additional information is discovered, the loss would develop, and the cap and maximum premium would be quite likely to apply.

Problem S6-18-2. Similar to Question 34 from the 2007 CAS Exam 6.

(a) Why are medical malpractice paid loss data typically so sparse?

(b) What common aspects of medical malpractice settlements distort trends in loss severity for this type of insurance?

Solution S6-18-2.

(a) Medical malpractice losses are paid out very slowly compared to losses for other coverages, and this results in sparse historical paid loss data for a particular experience period.

(b) Medical malpractice settlements are often paid out with extreme irregularity, and there is considerable variation in claims closed with no payment. Assuming a regular loss trend will therefore most often not reflect reality.

Problem S6-18-3. Similar to Question 37 from the 2007 CAS Exam 6. When the underlying loss exposure is evenly distributed throughout the policy period, why is estimating earned premium for an *excess policy* using the pro rata method still inappropriate?

Solution S6-18-3. An excess policy only pays when losses exceed a certain amount. If exposure is evenly distributed, it takes some time before losses are likely to exceed the attachment point of the excess policy. As losses approach the attachment point, the risk that the excess policy will pay out anything increases, and so more premium should be earned later in the policy term. Pro rata estimates of earned premium overstate the earned premium early on.

Problem S6-18-4. Similar to Question 39 from the 2007 CAS Exam 6. Define *pure premium* by reference to its constituent terms and name three events that are external to an insurance company but which could affect pure premium. For each event, specify the component of pure premium which would be affected.

Solution S6-18-4. Pure Premium = (Frequency)*(Severity).

The following is a sample response. Many other valid answers are possible.

1. An increased tendency for juries to award higher damages for particular types of cases might increase pure premium by raising claim *severity*.

2. An increased number of uninsured motorists on the road might increase the *frequency* of claims on uninsured motorists coverage.

3. A law that requires the insurer to cover a previously excluded exposure might increase the *frequency* of claims on the policies in question.

Problem S6-18-5. Similar to Question 28 from the 2007 CAS Exam 6. A primary insurer is a party to two reinsurance treaties: Treaty A is a 65% quota-share treaty. Treaty B is a $20 million in excess of $15 million catastrophe treaty. If a loss of $40 million occurs, would the primary insurer prefer **(i)** the situation where Treaty A inures to the benefit of Treaty B or **(ii)** the situation where Treaty B inures to the benefit of Treaty A? Explain your reasoning and show your calculations.

Solution S6-18-5. When a reinsurance treaty inures to the benefit of another, it is applied to the loss amount first, leaving the other reinsurance treaty to be *benefited* by the other reinsurer having to pay less.

Situation (i): Treaty A applies first and pays 65% of $40 million, or $26 million. The remaining loss amount is $14 million, which is not above the attachment point of the catastrophe treaty. So the catastrophe treaty pays nothing, and the primary insurer pays $14 million.

Situation (ii): Treaty B applies first and pays the $20 million in excess of $15 million, since the loss is above $35 million. Of the remaining $20 million, Treaty A pays 65% or $13 million, leaving (40 - 20 - 13) million = $7 million for the primary insurer.

Situation (ii) is preferable for the primary insurer.

Section 19

Applications of the Chain Ladder Method

This section of the study guide is intended to provide practice problems and solutions to accompany the pages of *Estimating Unpaid Claims Using Basic Techniques*, cited below. Students are encouraged to read these pages before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

Some of the questions here ask for short written answers based on the reading. This is meant to give the student practice in answering questions of the format that will appear on Exam 5B (Old Exam 6). Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Source:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 7, pp. 91-97.

Original Problems and Solutions from The Actuary's Free Study Guide

The following information applies to Problems S6-19-1 through S6-19-3:

As of December 31, 2100, you are aware of the following reported loss amounts for each accident year (AY):

For AY 2097: $315,130
For AY 2098: $310,120
For AY 2099: $200,430
For AY 2100: $180,540

You also know the following age-to-age factors:

12-24 months: 1.120
24-36 months: 1.085
36-48 months: 1.030
48 months to ultimate: 1.014

Problem S6-19-1. Use the chain ladder method to estimate the ultimate losses for each accident year.

Solution S6-19-1. The farther back in time we go, the fewer age-to-age factors we need to apply. For instance, AY 2097 data are already developed to 48 months, so only the 48-months-to-

ultimate factor need apply. AY 2098 data are developed to 36 months, so we only apply the product of the 36-48-months and the 48-months-to-ultimate factors.

Ultimate losses for AY 2097: 315130*1.014 = **$319,541.82**
Ultimate losses for AY 2098: 310120*1.030*1.014 = **$323,895.53**
Ultimate losses for AY 2099: 200430*1.085*1.030*1.014 = **$227,126.41**
Ultimate losses for AY 2100: 180540*1.120*1.085*1.030*1.014 = **$229,137.61**

Problem S6-19-2. Use the chain ladder method to estimate the IBNR for each accident year as of December 31, 2100.

Solution S6-19-2. IBNR = Ultimate Claims - Reported Claims. We are given the reported claims, and, from Solution S6-19-1, we know the ultimate claims.

IBNR for AY 2097: 319541.82 - 315130 = **$4411.82**
IBNR for AY 2098: 323895.53 - 310120 = **$13,775.53**
IBNR for AY 2099: 227126.41 - 200430 = **$26,696.41**
IBNR for AY 2100: 229137.61- 180540 = **$48,597.61**

Problem S6-19-3. Based on theclaim development factors, calculate **(a)** the incremental percent of claims reported during each of the given age-to-age intervals, and **(b)** the cumulative percent of claims reported at each endpoint of the given intervals.

Solution S6-19-3. Given the age-to-age factors we have, it is actually easier to work backward from ultimate and to calculate the cumulative percent of claims reported first.

(b) For each number of months given, the cumulative percent of claims reported is 100*1/(cumulative claim development factor from that time to ultimate).

At ultimate: **100%** (by definition).
At 48 months: 100*1/1.014 = **98.61932939%**.
At 36 months: 100*1/(1.014*1.030) = **95.74692174%**.
At 24 months: 100*1/(1.014*1.030*1.085) = **88.24601082%.**
At 12 months: 100*1/(1.014*1.030*1.085*1.120) = **78.79108109%.**

(a) The incremental percent claims reported between X and Y months is
(% reported at Y months) - (% reported at X months).

For 0-12 months: 78.79108109% - 0% = **78.79108109%**.
For 12-24 months: 88.24601082% -78.79108109% = **9.4549297%**.
For 24-36 months: 95.74692174% - 88.24601082% = **7.5009109%**.
For 36-48 months: 98.61932939% - 95.74692174% = **2.8724077%**.
For 48 months to ultimate: 100% - 98.61932939% = **1.38067061%**.

Problem S6-19-4. Name two internal insurer changes and two changes external to the insurer that could invalidate the applicability of the chain ladder technique. (See Friedland, pp. 95-96.)

Solution S6-19-4. The following are possible choices. Any two from each category suffice as an answer. Many other valid choices are conceivable.

Internal changes:
1. Faster or slower claim settlement
2. Case outstanding increases or decreases
3. New claim-processing systems
4. Changes in claim management philosophy
5. Policy revisions (changes in offered limits, deductibles, etc.)

External changes:
1. Tort reforms
2. Changes in patterns regarding judicial rulings or jury awards
3. Changes in prevalent policyholder choices regarding limits and/or deductibles

Problem S6-19-5. (a) Fill in the blanks: "The chain ladder technique works best with lines of business where frequency is _____ (high or low?), severity is _____ (high or low?), and claims are reported _____ (quickly or slowly?) and _____ (concentrated toward a particular segment of the year or spread evenly throughout the year?)." (See Friedland, p. 96.)

(b) Justify your last choice in part (a).

Solution S6-19-5.

(a) The chain ladder technique works best with lines of business where frequency is **high**, severity is **low**, and claims are reported **quickly** and **spread evenly throughout the year**.

(b) For the chain ladder technique to work, it is desirable for claims to be spread evenly throughout the year, because an uneven claim spread implies that the average accident date will not be in the middle of the year, and this might render data for the given year non-comparable to historical data with an even spread of claims. (See Friedland, pp. 96-97.)

Section 20

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 10

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:
Financial Accounting Standards Board, "Interpretation No. 14," Paragraphs 1-6.

Past Casualty Actuarial Society exams: 2007 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

The following conditions apply to Problems S6-20-1 and S6-20-2.

On January 1, 2054, Insurer X had total assets in cash of $3 million.
In calendar year 2054, the insurer collected $30 million in premiums and had $9 million in losses and expenses.
If there were no reinsurance, the insurer's balance sheet as of December 31, 2054, would appear as follows:

Assets
Cash: $24 million

Liabilities
Unearned premium reserve: $15 million
Loss reserves: $15 million
Total liabilities: $30 million

Surplus
-$6 million

Insurer X, however, participated in a pro rata reinsurance treaty, where it ceded 60% of its premiums and losses to the reinsurer. The reinsurer pays the insurer a ceding commission of 40%.

Problem S6-20-1. Similar to Problem 13(a) from the 2007 CAS Exam 6. Construct a balance sheet for Insurer X as of December 31, 2054 that takes the reinsurance transaction into account.

Solution S6-20-1. Without reinsurance, the insurer would have had cash of $24 million. With reinsurance, the reinsurer collects 60% of the premium revenue, but pays the insurer 40% of that as a ceding commission, so the reinsurer really only collects 60% of 60% of premium, or 36% of premium, or 0.36*30 million = $10.8 million. At the same time, Insurer X does not have to pay 60% of its losses, meaning that its loss payment during 2054 is reduced by 0.6*9 million = $5.4 million.

So, on the assets side, cash becomes 24 million - 10.8 million + 5.4 million = $18.6 million.

On the liabilities side, both the unearned premium reserve and the loss reserves are reduced by 60% to 0.4*15 million = $6 million each.

We therefore have the following balance sheet:

Assets
Cash: $18.6 million

Liabilities
Unearned premium reserve: $6 million
Loss reserves: $6 million
Total liabilities: $12 million

Surplus
$6.6 million

Problem S6-20-2. Similar to Problem 13(b) from the 2007 CAS Exam 6. Did the reinsurance transaction described provide *surplus relief*? Explain why or why not.

Solution S6-20-2. Yes, surplus relief did occur. Because of the reinsurance transaction, the surplus was increased from -$6 million to +$6.6 million, which implies that Insurer X prevented itself from having an accounting loss. Surplus relief due to reinsurance partially remedies the accounting convention that recognizes losses as incurred immediately, while premiums must be earned over time and are considered a liability while unearned.

Problem S6-20-3. Similar to Problem 19 from the 2007 CAS Exam 6. Comment on what, according to FASB Interpretation No. 14, a company would have to present on its financial statements with regard to the following situations:

(a) A contingency which is "reasonably possible" and, if it occurs, would cost the company anywhere from $6 million to $9 million, with each outcome in the range being equally likely.

(b) A contingency which is "probable" and, if it occurs, would cost the company anywhere from $6 million to $9 million, with each outcome in the range being equally likely.

(c) A contingency which is "probable" and, if it occurs, would cost the company anywhere from $6 million to $9 million, with $8 million being the most likely amount.

Solution S6-20-3. (a) Contingencies which are not yet "probable" do not require accrual of any loss. However, a **disclosure should occurof the contingency's nature and the estimated losses' range from $6 million to $9 million**.

(b) Contingencies which are "probable" and for which no possible loss amount is more likely than any other require accrual of the minimum possible loss amount - in this case, **$6 million** - along with a **disclosure of the contingency's nature and the range of possible additional losses up to $3 million.**

(c) Contingencies which are "probable" and for which there is one most likely loss amount require accrual of the most likely loss amount - in this case, **$8 million** - along with a **disclosure of the contingency's nature and the range of possible additional losses up to $1 million.**

The following conditions apply to Problems S6-20-4 and S6-20-5.

You are aware of the following information regarding Insurer Z:

Written premium in 2034: 2100
Unearned premium reserve as of December 31, 2033: 1200
Unearned premium reserve as of December 31, 2034: 1150
Loss reserves as of December 31, 2034: 400
LAE reserves as of December 31, 2034: 200
Deferred policy acquisition costs as of December 31, 2034: 100
Office furniture as of December 31, 2034: 400
Electronic data processing equipment and software as of December 31, 2034: 100
Cash as of December 31, 2034: 3000

Problem S6-20-4. Similar to Problem 20(a) from the 2007 CAS Exam 6. What is the equity as of December 31, 2034, according to Generally Accepted Accounting Principles (GAAP)?

Solution S6-20-4. We are only considering assets and liabilities at the end of 2034, so figures as of the end of 2033 or items accruing during 2034 (such as written premium) are irrelevant.

Assets
Under GAAP, the following are permissible assets:
Cash as of December 31, 2034: 3000
Deferred policy acquisition costs as of December 31, 2034: 100

Office furniture as of December 31, 2034: 400
Electronic data processing equipment and software as of December 31, 2034: 100
Total assets: 3600

Liabilities
Under GAAP, the following are liabilities:
Unearned premium reserve as of December 31, 2034: 1150
Loss reserves as of December 31, 2034: 400
LAE reserves as of December 31, 2034: 200
Total liabilities: 1750

GAAP Equity
Assets - Liabilities: **1850**

Problem S6-20-5. Similar to Problem 20(b) from the 2007 CAS Exam 6. What is the *statutory* policyholders' surplus as of December 31, 2034?

Solution S6-20-5. Under Statutory Accounting Principles (SAP), the liabilities in this case are the same as under GAAP, so their total is 1750 (see Solution S6-20-4). However, with respect to the assets, office furniture and deferred policy acquisition costs are excluded.

The treatment of electronic data processing (EDP) equipment and software is not as intuitive. Under SAP, this can be recognized as an asset, but only up to 3% of surplus.

Let x be the permitted EDP asset. The rest of the surplus calculation is (Cash - Liabilities) = 3000 - 1750 = 1250, so the total surplus is 1250 + x.

The restriction is that $x/(1250 + x) = 0.03$, so $33.33333333x = 1250 + x \rightarrow 32.33333333x = 1250 \rightarrow x = 38.65979381$, and the statutory surplus is $1250 + 38.65979381 =$ **1288.65979381**.

Section 21

Financial Accounting Standards for Loss Contingencies

This section of the study guide is intended to provide practice problems and solutions to accompany the Financial Accounting Standards Board documents cited below. Students are encouraged to read these documents before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources:

Financial Accounting Standards Board, "Statement of Financial Accounting Standards No. 5, Accounting for Contingencies," Paragraphs 1-4, and 8-10.

Financial Accounting Standards Board, "Interpretation No. 14," Paragraphs 1-6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-21-1. According to the FASB "Statement of Financial Accounting Standards No. 5" (Hereafter, FAS 5), should depreciation be treated as a contingency? Why or why not?

Solution S6-21-1. Depreciation **should not** be treated as a contingency, as its occurrence in the future is *not uncertain*, even though it does involve a loss to the asset in question and may often need to be estimated. (See FAS 5, Paragraph 2.)

Problem S6-21-2.

(a) What are the three degrees of likelihood for a contingency used by FAS 5?

(b) Give five examples of loss contingencies, as mentioned in FAS 5.

Solution S6-21-2.

(a) The three degrees of likelihood for a contingency used by FAS 5 (Paragraph 3) are, in order of increasing likelihood, **remote, reasonably possible, and probable.**

(b) The following examples of loss contingencies are mentioned in FAS 5 (Paragraph 4):

1. The possibility of a receivable not being collected
2. The possibility of a claim
3. The possibility of having to perform on a product warranty
4. The possibility of having assets expropriated
5. The possibility of having property damaged by fire or explosion, or similar hazards
6. The possibility of catastrophe for which the exposure is assumed by an insurer
7. The possibility of litigation
8. Agreements to repurchase receivables
9. "Guarantees of indebtedness to others"
10. "Obligations of commercial banks under 'standby letters of credit'"

Any five of the above suffice as an answer. Other valid answers may also be possible.

Problem S6-21-3. According to FAS 5, what are the two conditions that must be met in order for *accrual* of an estimated loss from a loss contingency to be justified?

Solution S6-21-3. In order for accrual of an estimated loss from a loss contingency to be justified, the following two conditions must be met (See FAS 5, Paragraph 8):

1. It should be possible to reasonably estimate the amount of the loss.

2. It must be *probable* that an asset was already impaired or a liability was already incurred *as of the date of the financial statement* in question. Future events must be able to verify that the loss happened.

Problem S6-21-4.

(a) According to FAS 5, under what circumstances is *disclosure* of a loss contingency necessary?

(b) Fill in the blanks: "According to FAS 5, Paragraph 10, disclosure is not required of a loss contingency involving _____ when _____ unless _____."

Solution S6-21-4.

(a) According to FAS 5, Paragraph 10, if there is *at least a reasonable possibility* that a loss or additional loss may have occurred, the nature of the contingency should be disclosed, and either an estimate of the loss/loss range should be given, or a statement should be made that no estimate is possible.

(b) According to FAS 5, Paragraph 10, "disclosure is not required of a loss contingency involving **an unasserted claim or assessment** when **there has been no manifestation by a potential claimant of an awareness of a possible claim or assessment** unless **it is considered probable that a claim will be asserted and there is a reasonable possibility that the outcome will be unfavorable**."

Problem S6-21-5. Montezuma and Imhotep work for an insurer which will probably get an adverse court verdict handed to it soon. The insurer is barely profitable as is, and the management has hired Montezuma and Imhotep to come up with independent estimates of the likelihood of the damages accompanying the verdict.

Montezuma estimates the following probabilities:
$2,000,000 - 25%; $3,000,000 - 50%; $5,000,000 - 25%.

Imhotep estimates the following probabilities:
$2,000,000 - 25%; $4,000,000 - 25%; $6,000,000 - 25%; $8,000,000 - 25%.

The two estimators have an in-depth debate about their estimates in front of the management, and the management cannot decide conclusively that one of them is more correct than the other. In this case, explain why the management would have a financial incentive to choose a particular one of these estimates over the other, in accordance with FASB Interpretation No. 14.

Solution S6-21-5. If the management were to only consider the requirements implied in FASB Interpretation No. 14, its incentive would be to choose **Imhotep's estimate,** because it assigns equal probabilities to each of the possible outcomes, and so no one outcome is more likely than any other. Under this situation, FASB Interpretation No. 14 requires accrual of the minimum amount - $2 million - and disclosure of possible additional loss. In the case of Montezuma's estimate, even though the upper estimate is less than Imhotep's, FASB Interpretation No. 14 requires the accrual of the most likely amount - here, $3 million. If the management is concerned with making the financial statements of the already barely profitable insurer look as favorable as possible for the current time period, then Imhotep's estimate may be selected on these grounds. Unintended consequences, anyone?

Section 22

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 11

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Sources: Past Casualty Actuarial Society exams: 2007 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

The following conditions apply to Problems S6-22-1 and S6-22-2. You know the following information about a reinsurance treaty:

The reinsurer's pro rata share is 60%.
The primary insurer's pro rata share is 40%.
The reinsurer also pays the primary insurer a ceding commission of 30%.
The reinsurer's expense ratio is 5%.
The reinsurer's expected loss ratio is 75%.
The premium subject to the reinsurance treaty is $2,500,000.
The loss subject to the reinsurance treaty is $500,000.

There is also a profit-sharing commission clause which states that, if the reinsurer's profit exceeds the expected amount, the primary insurer will receive 20% of the profit *in excess of what was expected*.

Problem S6-22-1. Similar to Question 23(a) from the 2007 CAS Exam 6. Before the application of the profit-sharing commission, what is the reinsurer's profit.

Solution S6-22-1. Add reinsurer's share of premium: $2,500,000*0.60 = $1,500,000.
Subtract reinsurer's ceding commission: $2,500,000*0.30 = $750,000.
Subtract reinsurer's share of losses: $500,000*0.60 = $300,000.

Subtract reinsurer's expenses: $2,500,000*0.05 = $125,000.
Reinsurer's profit = $1,500,000 - $750,000 - $300,000 - $125,000 = **$325,000**.

Problem S6-22-2. Similar to Question 23(b) from the 2007 CAS Exam 6. What is the amount of the profit-sharing commission the reinsurer would pay the primary insurer?

Solution S6-22-2. First, we calculate the amount of the reinsurer's *expected* profit. This is the premium ceded to the reinsurer, multiplied by (1 - Expected Loss Ratio - Expense Ratio - Ceding Commission Ratio), i.e., $1,500,000*(1 - 0.75 - 0.05 - 0.30) = -$150,000. So the reinsurer can actually expect a *loss* of $150,000. The difference between the actual profit of $325,000 and this amount is $475,000, so the reinsurer must pay 20% of this, or **$95,000** as a profit-sharing commission to the primary insurer.

The following conditions apply to Problems S6-22-3 through S6-22-5. You know the following information about a reinsurance treaty:
The treaty covers 70% of the loss layer $5 million in excess of $5 million.

There is a claim subject to the treaty, with $8 million in losses and $2 million in allocated loss adjustment expenses (ALAE).

Problem S6-22-3. Similar to Questions 24(a)(i) and 24(b)(i) from the 2007 CAS Exam 6. Using the "pro rata in addition" method, calculate **(a)** the primary insurer's retained combined losses and ALAE and **(b)** the combined losses and ALAE ceded to the reinsurer.

Solution S6-22-3.

(a) To use the "pro rata in addition" method, we need to first determine how the loss is divided. The same proportions of loss that apply to each party will also apply with respect to ALAE.

The loss is $8 million, of which the primary insurer retains the first $5 million and also retains 30% of the rest, or 0.3*3 million = $900,000, meaning that the primary insurer retains $5,900,000 of $8 million, or 5.9/8 of the total loss amount. The ALAE retained by the primary insurer is thus also (2 million)*5.9/8 = $1,262,500, meaning that the primary insurer's total amount retained is $1,262,500 + $5,900,000 = **$7,162,500.**

(b) Since the combined (loss + ALAE) amount is $10 million, the reinsurer gets what the primary insurer does not retain, i.e., $10,000,000 - $7,162,500 = **$2,837,500.**

Problem S6-22-4. Similar to Questions 24(a)(ii) and 24(b)(ii) from the 2007 CAS Exam 6. Using the "included in the limit" method, calculate **(a)** the primary insurer's retained combined losses and ALAE and **(b)** the combined losses and ALAE ceded to the reinsurer.

Solution S6-22-4. (a) The "included in the limit" method treats the ALAE as part of the loss amount, and so the aggregate amount under consideration here is $10 million, of which the primary insurer must pay $5 million and then 30% of the other $5 million, for a total of 1.3*(5 million) = **$6,500,000.**

(b) Since the combined (loss + ALAE) amount is $10 million, the reinsurer gets what the primary insurer does not retain, i.e., $10,000,000 - $6,500,000 = **$3,500,000.**

Problem S6-22-5. Similar to Questions 24(c) from the 2007 CAS Exam 6. In the situation described in the given conditions, the "included in the limit" method is more advantageous to the primary insurer than the "pro rata in addition" method. By tweaking *just one item* in the given conditions, develop a scenario where the "included in the limit" method is *less* advantageous to the primary insurer.

Solution S6-22-5. Many answers are possible here. The following is a sample answer:

Change the claim being subject to the treaty to having $10 million in losses instead of $8 million. ALAE remain at $2 million.

We demonstrate that the primary insurer's losses will be greater under the "included in the limit" method.

"Pro rata in addition method": For just the losses, the primary insurer must pay $5 million and then 30% of the other $5 million, for a total of 1.3*(5 million) = $6,500,000. This means that the primary insurer is responsible for 65% of the loss amount - and, by implication, 65% of the ALAE amount, or an additional $1.3 million, for a total of $7,800,000.

"Included in the limit method": The aggregate amount under consideration is $12 million. The reinsurer's responsibility is still only for 70% of the second $5 million, leaving the primary insurer responsible for (12 million - 0.7*(5 million)) = $8,500,000 > $7,800,000.

Section 23

Concepts Involved in the Expected Claims Method

This section of the study guide is intended to provide practice problems and solutions to accompany the pages of *Estimating Unpaid Claims Using Basic Techniques*, cited below. Students are encouraged to read these pages before attempting the problems. This study guide is entirely an independent effort by Mr. Stolyarov and is not affiliated with any organization(s) to whose textbooks it refers, nor does it represent such organization(s).

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Source:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009. Chapter 8, pp. 131-138.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-23-1.

(a) What is thekey assumption of the *expected claims technique?* (See Friedland, p. 131.)

(b) Identify three types of situations in which the expected claims technique might be used. (See Friedland, p. 131.)

Solution S6-23-1.

(a) The expected claims technique's key assumption is that an a priori, initial claim estimate is more reliable than an estimate based on observed claims experience to date. (Friedland, p. 131)

(b) The following are types of situations in which the expected claims technique might be used (Friedland, p. 131):

1. An insurer's entry into a new line of business
2. An insurer's entry into a new territory
3. There is no data available to use with other methods.
4. Changes in the insurer's operations or environment render historical experience irrelevant.
5. Less mature periods cannot use the development (chain ladder) method because the

development factors to ultimate are too large (i.e., too highly leveraged, leading to large possible volatility in the ultimate projections).

Any three of the above suffice as an answer. Other valid answers may also be possible.

Problem S6-23-2.

(a) According to Friedland, p. 132, what are the two challenges of the expected claims method?

(b) If the legal climate changes such that an insurer's liability for claims of a certain kind is affected, briefly describe one advantage and one challenge that the expected claims technique would present for the insurer.

Solution S6-23-2.

(a) According to Friedland, p. 132, the two challenges of the expected claims method are as follows:
1. Determining what the appropriate exposure base is
2. Estimating how claims are to be measured relative to the appropriate exposure base.

(b) This question is based on the discussion in Friedland, p. 137.

If the legal climate changes such that an insurer's liability for claims of a certain kind is affected, one advantage of the expected claims technique is that, if historical data have been rendered irrelevant by the legal changes, this would not affect the viability of the expected claims technique. One challenge would be actually estimating the expected claim ratio in the new legal climate; the ability to estimate such a ratio reliably is an assumption of the expected claims technique.

Problem S6-23-3. The fact that the expected claims technique does not take into account experience to date for a particular group of claims simultaneously poses an advantage and a disadvantage, described by Friedland, p. 137. State both the advantage and the disadvantage. What ability can an actuary use to overcome the disadvantage?

Solution S6-23-3. Advantage: Reliance on a priori estimates ensures greater stability of projections over time.
Disadvantage: Lack of responsiveness to recent experience.

To overcome this disadvantage, an actuary can use *judgment* to determine whether recent historical experience signals a fundamental change in the insurer's external or internal environment and adjust expected claim ratios accordingly. This can make the expected claims technique more responsive to real-world changes.

Problem S6-23-4. What effect do changes in the adequacy of case outstanding have on the ultimate claim projection using the expected claims technique? Explain your answer.

Solution S6-23-4. Changes in the adequacy of case outstanding **do not** have and effect on the ultimate claim projection using the expected claims technique. This is because adequacy of case outstanding is based on actual claims experience, and this does not get incorporated into the projection using the expected claims technique (See Friedland, p. 138).

Problem S6-23-5. Identify three kinds of adjustments that might need to be made to historical data in the use of the expected claims technique (See Friedland, p. 133).

Solution S6-23-5. The following are three kinds of adjustments that might need to be made to historical data in the use of the expected claims technique:

1. **On-level adjustment for premium** to bring it to current rate levels. The premiums should be restated as if current rates were in effect during the entire historical experience period.

2. **Trend adjustment for losses** to take account of systematic influences on claim amounts, such as inflation.

3. **Adjustments for effects of major events,** such as tort reforms, other legal changes, or operational changes of the insurer.

Section 24

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 12

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:

Casualty Actuarial Society Enterprise Risk Management Committee, "Overview of Enterprise Risk Management," Casualty Actuarial Society *Forum*, Summer 2003, Section 3 and Appendix B.

Casualty Actuarial Society Valuation, Finance, and Investments Committee, "Accounting Rule Guidance Statement of Financial Accounting Standards No. 113-Considerations in Risk Transfer Testing" Casualty Actuarial Society *Forum*, Fall 2002, pp. 305-338, excluding Sections 7 and 8.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2007 Exam 6.

Patrik, G.S., "Reinsurance," *Foundations of Casualty Actuarial Science (Fourth Edition)*, Casualty Actuarial Society, 2001, Chapter 7, pp. 434-464 (section on Reinsurance Loss Reserving).

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-24-1. Similar to Question 11 from the 2007 CAS Exam 6. For each of the following pairs of variables, explain why examining them via an integrated approach in a Dynamic Financial Analysis (DFA) model is superior to using a silo approach.

(a) Underwriting profit and investment profit

(b) Loss ratio and mix of business in terms of new versus renewal business

Solution S6-24-1. In general terms, an integrated approach examines the interrelationships among variables, while a silo approach considers variables in isolation. A good resource on questions of this type is the CAS "Overview of Enterprise Risk Management".

The following are sample answers, and other valid explanations are possible.

(a) Underwriting profit and investment profit both contribute to the overall profitability of the insurer. The insurer may be able to afford to sustain slight losses in a particular year if its investments are performing well and can make up the difference. On the other hand, if the stock market is in decline and bonds are not yielding much income, the insurer may need to raise its rates and/or implement stricter underwriting standards in order to remain profitable overall.

(b) The insurer's loss experience may vary between its established customers and its new customers. If the loss ratio increases, this may be a sign of loosening underwriting standards. On the other hand, if the loss ratio increases, this may be because the insurer has been able to attract more favorable loss exposures to its book of business.

Problem S6-24-2. Similar to Question 12 from the 2007 CAS Exam 6.

The CAS "Overview of Enterprise Risk Management" discusses seven steps of enterprise risk management (ERM).

(a) The first step of ERM is to **establish the context**. What three kinds of context are typically examined as part of this step?

(b) The second step of ERM is to **identify risks.** What two broad categories of events is it important to identify here?

(c) The third step of ERM is to **analyze/quantify risks.** Name three kinds of possible analysis for this step.

(d) What three interrelated activities are part of the fourth step of ERM: **integrating risks**?

(e) Identify and briefly describe the fifth step of ERM.

(f) Identify the sixth step of ERM and some of the approaches that could be used to enact it.

(g) What is the seventh and last step of ERM, and how does it feed back into the first step?

Solution S6-24-2. Note that slight variations on these answers may also be acceptable.

(a) The three kinds of context are

1. **External context:** Relationship of the enterprise with its environment, including customers, shareholders, and other stakeholders. Evaluation of the enterprise's strengths, weaknesses, opportunities, and threats (SWOT).

2. **Internal context:** The enterprise's strategies, objectives, oversight/governance, and key performance indicators.

3. **Risk management context:** The risks relevant to the enterprise and the degree of coordination in risk management within the enterprise, including the adoption of common risk metrics.

("Overview of Enterprise Risk Management", p. 112)

(b) It is important to identify both **(i)** material threats to the enterprise's objectives and **(ii)** areas/events the enterprise could exploit for competitive advantage ("Overview of Enterprise Risk Management", p. 112).

(c) Three kinds of possible risk analysis are

1. **Sensitivity analysis:** How sensitive is a particular outcome to changes in the factors being analyzed?
2. **Scenario analysis:** What would happen under a particular set of conditions?
3. **Simulation analysis:** Examining a large number of simulated situations to see what the possibilities are.

("Overview of Enterprise Risk Management", p. 113)

(d) The integration of risks involves

1. Aggregation of the probability distributions for various risks

2. Taking into account the correlations among various risks and portfolio effects (which reduce overall variation where multiple assets or risks are involved)

3. Expression of results in terms of impact on the enterprise's key performance indicators

("Overview of Enterprise Risk Management", p. 113)

(e) The fifth step of ERM is to **assess/prioritize risks** to determine how much each risk contributes to the overall risk profile of the enterprise and to establish what the most important decisions will be with regard to the totality of risks facing the enterprise ("Overview of Enterprise Risk Management", p. 113).

(f) The sixth step of ERM is to **treat/exploit risks**. This can be done by (as the name implies) exploiting a risk (turning it into an opportunity), or retaining/financing, transferring, or reducing the risk ("Overview of Enterprise Risk Management", p. 114).

(g) The last step of ERM is to **monitor and review risks and risk management strategies.** The risk management implemented by the enterprises interacts with and contributes to the context for subsequent time periods. Thus, in monitoring and reviewing the current state of affairs with regard to risk management, the enterprise helps to establish a context (step 1 of ERM) for subsequent risk management ("Overview of Enterprise Risk Management", p. 114).

Problem S6-24-3. Similar to Question 15 from the 2007 CAS Exam 6. A reinsurance treaty has a premium of $5 million, paid in full upfront. The losses under the treaty are paid in a lump sum after 4 years, and the annual discount rate is 6%. The 90th percentile of losses is $5.5 million, and the 95th percentile of losses is $10 million.

(a) Does this treaty pass the 10-10 rule to qualify for reinsurance accounting?
(b) Should this treaty qualify for reinsurance accounting? Justify your answer.

Solution S6-24-3. This question is based on the discussion in the CAS "Accounting Rule Guidance Statement of Financial Accounting Standards No. 113-Considerations in Risk Transfer Testing".

(a) The 10-10 rule is an (essentially arbitrary) rule of thumb that a 10% probability of a 10% loss is a "reasonable probability of a significant loss" for the reinsurer, which would qualify for reinsurance accounting. Here, the 90th percentile of losses is $5.5 million, but, since losses are paid after four years, the discounted value is (5.5 million)/1.06^4 = $4,356,515.15, which is less than the reinsurance premium of $5 million. Thus, the reinsurer's expected loss ratio at the 90th percentile is even less than 100%: $4,356,515.15/($5 million) = circa 87.13%. The treaty **does not pass the 10-10 rule.**

(b) Consider the discounted loss at the 95th percentile: (10 million)/1.06^4 = $7,920,936.63. The reinsurer's expected loss ratio is thus $7,920,936.63/($5 million) =circa 158.42%. This is much more than a 10% loss and can be reasonably considered significant. Also 5% is a non-negligible probability, so per FAS 113, it is justified to conclude that there is a "reasonable probability of a significant loss" for the reinsurer, and this treaty **should qualify for reinsurance accounting.**

Problem S6-24-4. Similar to Question 21 from the 2007 CAS Exam 6. What do the Stanard-Bühlmann and Bornhuetter-Ferguson methods have in common? How are they different, in essential terms?

Solution S6-24-4. Both methods rely on the formula

Ultimate Losses = Reported Losses + (% Losses Unreported)*(Expected Losses).

In the Bornhuetter-Ferguson Method, the expected loss ratio is estimated judgmentally. Losses are compared to earned premium that is not brought to the present rate levels.

In the Stanard- Bühlmann Method, *adjusted premium* is used instead of earned premium; adjusted premium is earned premium adjusted to current rate levels. Also, the expected loss ratio

is estimated on the basis of reported claim experience from the overall time period being examined.

(See Friedland, pp. 174-175.)

Problem S6-24-5. Similar to Question 25 from the 2007 CAS Exam 6. Identify four possible problems with industry aggregates of reinsurance data as currently exist in the United States.

Solution S6-24-5. The following are possible problems with industry aggregates of reinsurance data:

1. Different reinsurance policies have different limits and attachment points, and the aggregate data are not separated by limits/attachment points.

2. Various reinsurers may choose to include or omit data about asbestos, pollution, or other environmental hazards in the numbers they submit.

3. Reinsurance contracts are unique, so the data are generally not comparable across reinsurers; there is no homogeneity on the basis of which the law of large numbers could work (Patrik, pp. 436-437).

4. Low claim frequency and extreme report lags may contribute to large fluctuations in the data (Patrik, p. 437).

Section 25

The Expected Claims Method

Section 25 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 26

The Bornhuetter-Ferguson and Benktander Methods

Section 26 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 27

The Cape Cod / Stanard-Bühlmann Method

Section 27 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 28

Frequency-Severity Approaches to Unpaid Claim Estimates - Part 1

Section 28 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 29

Frequency-Severity Approaches to Unpaid Claim Estimates - Part 2

Section 29 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 30

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 13

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Sources:

Bouska, A.S., "From Disability Income to Mega-Risks: Policy-Event Based Loss Estimation," Casualty Actuarial Society *Forum*, Summer 1996, pp. 291-320.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Harrison, C.M., *Reinsurance Principles and Practices* (First Edition), American Institute for Chartered Property Casualty Underwriters/Insurance Institute of America, 2004, Chapters 1, 2 (from beginning through page 2.21), 4, 8, 9, and 10.

Past Casualty Actuarial Society exams: 2007 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-30-1. Similar to Question 26 from the 2007 CAS Exam 6.

(a) What are the two elements of Policy-Event-Based Loss Estimation (PEBLE)? (See Bouska, p. 293.)

(b) Identify three shortcomings of traditional triangular loss analysis with regard to asbestos and pollution claims.

(c) In general terms, describe the stepwise operation of the PEBLE process.

Solution S6-30-1.

(a) The two elements of PEBLE are
1. "A loss event that might give rise to an insurance claim" and
2. "The application of the terms of an individual policy to that loss event in order to determine the insured loss" (Bouska, p. 293).

(b) The following are three shortcomings of traditional triangular loss analysis with regard to asbestos and pollution claims (See Bouska, pp. 296-297.)

1. Triangular methods rely for their accuracy on numerous, small loss events that occur with relative regularity. Asbestos and pollution claims can be rare and might often not manifest themselves until years after the insurance policies in question were issued. When they do manifest themselves, they can be immense in severity, and their magnitudes are extremely difficult to predict from prior experience.

2. In pollution/asbestos events, there is often a lack of correlation between the loss occurrence and the accounting for the loss. The original exposure may occur at a particular point in time, but its effects might be latent and develop gradually - with claims tending to follow the manifestation of the effects.

3. Courts have generally allowed all policies between first exposure to the peril and manifestation of the adverse consequences to be triggered. It is therefore not always clear to what year(s) the claims would be assigned.

Other valid answers may be possible.

(c) The following can be a description of the stepwise operation of PEBLE:
1. Acquire policy data, including the policy terms and the attributes of the loss exposure;
2. Model the loss event (e.g., a pollution or asbestos claim).
3. Derive an understanding of the losses specific to the exposure from the model of the insured event and the attributes of the loss exposure.
4. Estimate the insured cost from the policy terms and exposure-specific losses.
(See Bouska, p. 301.)

Some variations on this description may be possible.

Problem S6-30-2. Similar to Question 27 from the 2007 CAS Exam 6. You have the following data for Insurer Ω:

Policy 1: Limit is $350,000; premium is $20,000; loss is $100,000.
Policy 2: Limit is $45,000; premium is $3,000; loss is $18,000.
Policy 3: Limit is $600,000; premium is $56,000; loss is $195,000.

(a) Under a 30% quota share treaty, how much would the *reinsurer* (i) receive in premium and (ii) be liable for in losses?

(b) Under a 11-line surplus share treaty with a retained line of $50,000, how much would the *reinsurer* (i) receive in premium and (ii) be liable for in losses?

(c) For the primary insurer, what is one possible disadvantage of surplusshare treaties relative to quota share treaties?

(d) For the primary insurer, what is one possible disadvantage of quota share treaties relative to surplusshare treaties?

Solution S6-30-2.

(a) The percentage in a quota share treaty is always defined relative to the amounts ceded to the reinsurer. The reinsurer thus gets 30% of all premium and 30% of all losses.

(i) Premium: 0.3*(20000 + 3000 + 56000) = **$23,700**.
(ii) Losses: 0.3*(100000 + 18000 + 195000) = **$93,900**.

(b) For policies with a limit under $600,000, The primary insurer retains that percentage of a policy's premium and loss that corresponds to ($50,000)/(Policy limit). The rest is ceded to the reinsurer.

For Policy 1, the reinsurer's proportion is 1 - 50000/350000 = 6/7.
For Policy 2, the primary insurer retains everything, because the policy limit is under $50,000.
For Policy 3, the reinsurer's proportion is 1 - 50000/600000 = 11/12.

(i) Premium: (6/7)*20000 + (11/12)*56000 = **$68,476.19**.
(ii) Losses: (6/7)*100000 + (11/12)*195000 =**$264,464.29**.

(c) Surplus share treaties generally provide less surplus relief than quota share treaties, as less premium is ceded to the reinsurer.

(d) Quota share treaties require the primary insurer to cede X% of every loss exposure, no matter how profitable. Surplus share treaties allow the primary insurer to fully retain its lower-limit, more profitable loss exposures.

For (c) and (d), other valid answers may be possible.

Problem S6-30-3. Similar to Question 29 from the 2007 CAS Exam 6. There are two kinds of policies that Primary Insurer Π writes. Policy A has a limit of $200,000, and Policy B has a limit of $600,000. Primary Insurer Π wishes to retain all the risk on policies of type A. For policies of type B, the insurer wishes to retain only at most $300,000 on each policy. Describe the terms of two distinct kinds of reinsurance treaties that would achieve this goal.

Solution S6-30-3. This is a sample answer, and any two possibilities will suffice. Other kinds of treaties may be possible.

1. A per-policy excess-of-loss treaty: $400,000 in excess of $200,000, with a co-participation provision of 25%.
2. A one-line surplus share treaty, with the line being $300,000.
3. A variable quota share treaty, with 0% ceded on Policy A risks and 66.666666667% ceded on Policy B risks.
4. A per-policy excess-of-loss treaty: $300,000 in excess of $300,000, with no co-participation provision.

Problem S6-30-4. Similar to Question 30 from the 2007 CAS Exam 6. You know the following aggregate loss distribution for a reinsurer under a reinsurance treaty:

Loss ratio from 0% to 50%: Expected loss ratio is 30%; probability of loss in this range is 25%.
Loss ratio from 50% to 70%: Expected loss ratio is 65%; probability of loss in this range is 45%.
Loss ratio from 70% to 90%: Expected loss ratio is 80%; probability of loss in this range is 15%.
Loss ratio of 90+%: Expected loss ratio is 105%; probability of loss in this range is 15%.

The reinsurer pays the following sliding-scale commission:

Minimum of 5% for loss ratios of 90% or higher.
Slides via a 1.5:1 ratio from 90% to 70%, to 35% at 70% loss ratio.
Slides via a 0.2:1 ratio from 70% to 40%, to maximum of 41% at 40% loss ratio.

Find the reinsurer's expected technical ratio.

Solution S6-30-4. The technical ratio is (Reinsurer's loss ratio) + (Reinsurer's commission ratio).

In the range where expected loss ratio is 30%, the commission ratio is 41%, so the technical ratio is 71%.

In the range where expected loss ratio is 65%, the commission ratio is 35% + 0.2%*5 = 36%, so the technical ratio is 101%.

In the range where expected loss ratio is 80%, the commission ratio is 35% - 1.5%*10 = 20%, so the technical ratio is 100%.

In the range where expected loss ratio is 105%, the commission ratio is 5%, so the commission ratio is 110%.

The expected technical ratio is 0.25*71% + 0.45*101% + 0.15*100% + 0.15*110% = **94.7%.**

Problem S6-30-5. Similar to Question 50 from the 2007 CAS Exam 6. You have the following triangle of cumulative closed claim counts per accident year (AY), with age of

development being expressed at
(12 months, 24 months, 36 months, 48 months):

AY 2034 (13000 earned exposures): (100, 150, 175, 200)
AY 2035 (13500 earned exposures): (102, 148, 155)
AY 2036 (14000 earned exposures): (99, 130)
AY 2037 (13000 earned exposures): (80)

(a) What operational change might have occurred within the insurance company to explain the data above?

(b) How would the operational change in part (a) affect the accuracy of the calculations of ultimate losses on the basis of the corresponding paid loss triangle?

Solution S6-30-5.

(a) We can construct a triangle of claim counts per earned exposure to spot any differences:

AY 2034: (0.00769, 0.01154, 0.01346, 0.01538)
AY 2035: (0.00756, 0.01096, 0.01148)
AY 2036: (0.00707, 0.00929)
AY 2037: (0.00615)

It appears that, over the years 2035-2037, the insurer's claims department has closed increasingly fewer claims at each age of the experience, as compared to prior years. The decline is particularly evident going from AY 2036 to AY 2037. Perhaps the claims department has become less efficient or has chosen to scrutinize claims more closely.

(b) Since claims in more recent time periods are being closed at a slower rate, applying ultimate loss estimates based on the corresponding paid loss triangle, where the diagonals based on the most recent experience will give lower factors, will result in losses from earlier periods being multiplied by smaller development factors, leading to an **underestimate.**

Section 31

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 14

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that will appear on Exam 6. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Sources:

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2007 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-31-1. Similar to Question 31 from the 2007 CAS Exam 6. An excess-of-loss reinsurance treaty has a retention of $600,000 and covers losses $300,000 in excess of that retention. The losses of the experience periods subject to the treaty have been trended to current levels, and the trended ground-up values are as follows:

Year 2045 losses: Loss A: $444,603; Loss B: $747,000; Loss C: $1,000,235.
Year 2046 losses: Loss D: $556,033; Loss E: $800,430.
Year 2047 losses: Loss F: $994,100; Loss G: $650,000.

The loss development factors for excess losses subject to the treaty are as follows:

For year 2045 losses: 1.02
For year 2046 losses: 1.15
For year 2047 losses: 1.50

The on-level trended premiums subject to the treaty for each year are as follows:

Year 2045: $1,670,000
Year 2046: $2,505,200
Year 2047: $1,802,400

Determine the experience rating loss cost for this treaty.

Solution S6-31-1. The experience rating loss cost for this treaty is essentially

(Trended On-Level Losses Covered by the Treaty)/(Trended On-Level Premium Subject to the Treaty).

The premiums are already trended and brought to current rate levels. Losses are already trended, but we need to find the excess losses subject to the treaty and develop those losses.

Year 2045 losses: Loss A is below the primary insurer's retention, and the treaty covers $147,000 of Loss B and $300,000 of Loss C (which exceeds the treaty limit) - for a total of $447,000. We develop this amount: 447000*1.02 = $455,490.

Year 2046 losses: Loss D is below the primary insurer's retention, and the treaty covers $200,430 of Loss E. We develop this amount: 200430*1.15 = $230,494.50.

Year 2047 losses: The treaty covers $300,000 of Loss F (which exceeds the treaty limit) and $50,000 of Loss G - for a total of $350,000. We develop this amount: 350000*1.50 = $525,000.

The experience rating loss cost for this treaty is thus

(455490 + 230494.5 + 525000)/(1670000 + 2505200 + 1802400) = **0.2025870751.**

Problem S6-31-2. Similar to Question 32 from the 2007 CAS Exam 6.

A $500,000 excess of $500,000 reinsurance treaty covers losses from the perils of falling anvils, carnivorous rabbits, and meteorites. Falling anvils constitute 50% of all losses, carnivorous rabbits constitute 30%, and meteorites constitute 20%. You know the following cumulative loss distribution for falling anvils:

Loss is 25% of coverage limit: Falling anvil cumulative distribution is at 70%.
Loss is 50% of coverage limit: Falling anvil cumulative distribution is at 80%.
Loss is 75% of coverage limit: Falling anvil cumulative distribution is at 87%.
Loss is 100% of coverage limit: Falling anvil cumulative distribution is at 100%.

The policies covered by the treaty either have ground-up limits of $666,666.67, or $1,000,000.

Policies with $666,666.67 limits have an associated direct premium of $1,200,000.
Policies with $1,000,000 limits have an associated direct premium of $800,000.

The ceding insurer's expected loss ratio, excluding allocated loss adjustment expenses (ALAE) is 64%. The ratio of ALAE to loss is 15%. The ceding insurer's rates are inadequate by 10%, and the reinsurer's profits and expenses are 8% of the reinsurance premium.

The indicated exposure premium for the meteorite cause of loss is $230,000.

The indicated exposure rate for the entire treaty is 16%. What is the exposure rate for losses due to carnivorous rabbits?

Solution S6-31-2. We can calculate the exposure rates for falling anvils and meteorites and deduce the exposure rate for carnivorous rabbits.

We start with meteorites. The exposure rate is
(Amount needed to pay for losses and expenses)/(Total direct premium).

The total direct premium is $1,200,000 + $800,000 = $2,000,000.

For meteorites, the indicated exposure premium must be modified by multiplication by the following:

(Loss Ratio)*(1 + Ratio of ALAE to Loss)*(1/(1- Rate Inadequacy))*(1/(1-Reinsurer Profit and Expense)) = 0.64*1.15*(1/(1-0.1))*(1/(1-0.08)) = 0.8888888889 = 8/9. We can call this the modification factor for purposes of easier reference later on.

For the meteorite peril, the amount needed to pay for losses and expenses is 230000*(8/9) = 204444.44444, and the exposure rate is thus 204444.44444/2000000 = 0.10222222222.

Now we find the exposure rate for falling anvils. We consider how much of the direct premium would be part of the exposure premium by looking at the cumulative loss distribution.

For the $666,666.67 limit, 75% (500000/666666.67) of the limit - and this 87% of the loss amounts - would be below the primary insurer's retention. So only 13% of losses would be above the retention, which implies that 13% of the direct premium should be considered for this limit.

For the $1,000,000 limit, 50% of the limit - and this 80% of the loss amounts - would be below the primary insurer's retention. So 20% of the direct premium should be considered for this limit.

The exposure premium for falling anvils is thus 0.13*1200000 + 0.2*800000 = $316,000.

We multiply this by our modification factor: 316000*(8/9) = 280888.8888888, and the exposure rate for falling anvils is thus 280888.8888888/2000000 = 0.1404444444.

We now find the exposure rate for carnivorous rabbits (ERCR) by setting up the following equation in consideration of the total exposure rate of 0.16:
0.16 = 0.5*0.1404444444 + 0.2*0.10222222222 + 0.3*ERCR →

ERCR = (0.16 - 0.5*0.1404444444 - 0.2*0.10222222222)/0.3 = **0.231111111111 = 23.111111111111%**.

Problem S6-31-3. Similar to Question 33 from the 2007 CAS Exam 6. You are given the following triangles for accident years (AY) 2034 through 2036, where data is expressed in the format (Value at 12 months, Value at 24 months, Value at 36 months), where applicable.

Average Case Reserve per Open Claim
AY 2034: (230, 320, 400)
AY 2035: (260, 370)
AY 2036: (320)

Number of Open Claims
AY 2034: (110, 80, 20)
AY 2035: (140, 70)
AY 2036: (150)

Cumulative Paid Losses
AY 2034: (13000, 18900, 28000)
AY 2035: (14000, 17000)
AY 2036: (18210)

The annual severity trend is +5%. Develop the Berquist-Sherman triangle of adjusted incurred losses for this scenario.

Solution S6-31-3. The Berquist-Sherman triangle of adjusted incurred losses is developed by adjusting the case reserve estimates and de-trending the values at the latest known (outermost) diagonal by the severity trend so as to arrive at the rest of the case reserve triangle:

Adjusted Average Case Reserve per Open Claim
AY 2034: ($320/1.05^2$, 370/1.05, 400)
AY 2035: (320/1.05, 370)
AY 2036: (320)

Adjusted Average Case Reserve per Open Claim
AY 2034: (290.2494331, 352.3809524, 400)
AY 2035: (304.7619048, 370)
AY 2036: (320)

Then the adjusted incurred loss for each time period is equal to
Paid Losses + (Average Case Reserve per Open Claim)*(Number of Open Claims).

Adjusted Incurred Losses
AY 2034: (13000 + 290.2494331*110, 18900 + 352.3809524*80, 28000 + 400*20)
AY 2035: (14000 + 304.7619048*140, 17000 + 370*70)
AY 2036: (18210 + 320*150)

Our answer is
Adjusted Incurred Losses
AY 2034: (44927.44, 47090.48, 36000)
AY 2035: (56666.67, 42900)
AY 2036: (66210)

Problem S6-31-4. Similar to Question 38 from the 2007 CAS Exam 6.
(a) How would the Berquist-Sherman approach be superior to the chain ladder approach in the event of case reserve strengthening by the insurer?
(b) How would the Berquist-Sherman approach be superior to the chain ladder approach in the event of a changing claim settlement rate?
(c) If insureds are purchasing lower policy limits than before, why would it be preferable to switch from accident-year data aggregation to policy-year data aggregation?

Solution S6-31-4.

(a) In the event of case reserve strengthening by the insurer, the chain ladder method, with development factors based in part on prior experience under lower case reserves, would overstate ultimate loss results. The Berquist-Sherman approach can mitigate this by adjusting previous, lower case reserves to the level of reserve adequacy that currently exists. This is done by de-trending the most recent case reserves instead of using historical values prior to the reserve strengthening.

(b) A changing claim settlement rate could result in the chain ladder method either overstating (if the settlement rate increases) or understating (if the settlement rate decreases) ultimate losses. The Berquist-Sherman approach applies the current claim settlement rate to historical closed claims, thereby mitigating any overstatement or understatement.

(c) If insureds are purchasing lower policy limits than before, analysis using the chain ladder method and accident-year aggregation will understate the ultimate losses - in essentially the inverse fashion of what would happen under strengthening case reserves. Accident-year loss data combine losses from policies written in previous years with higher limits and policies written in later years with lower limits, whereas policy-year data are segregated by the year in which policies were written, meaning that there will not be a mix of losses from policies from years with higher limits and years with lower limits. This allows for trending of each policy year's data by any policy limit change that has been observed.

Problem S6-31-5. Similar to Question 41 from the 2007 CAS Exam 6. A large-deductible policy has the following ground-up loss amounts:
1 loss of $600,000
1 loss of $450,000
1 loss of $350,000
2 losses of $100,000
2 losses of $40,000

The per-loss deductible is $400,000, and there is no aggregate deductible. The ultimate loss development factor (ULDF) for ground-up losses is 1.5.

(a) Calculate the ULDF (i) solely for losses retained by the insured and (ii) solely for losses transferred to the insurer.
(b) If there were an aggregate deductible of $1,550,000, calculate the ULDF (i) solely for losses retained by the insured and (ii) solely for losses transferred to the insurer.

Solution S6-31-5.
(a) We first find the total ultimate loss amounts by multiplying each of the ground-up initial loss amounts of 1.5. We get the following:
1 loss of $900,000
1 loss of $675,000
1 loss of $525,000
2 losses of $150,000
2 losses of $60,000

(i) The insured's obligation for the initial losses is
$400,000
+$400,000
+$350,000
+$100,000*2
+$ 40,000*2 = $1,430,000.

The insured's obligation for the ultimate losses is
$400,000
+$400,000
+$400,000
+$150,000*2
+$ 60,000*2 = $1,620,000.

The insured's ULDF is thus $1,620,000/$1,430,000 = **1.132867133**.

(ii) The total initial losses are $1,680,000. Since the insured's obligation is $1,430,000, the insurer's obligation is the rest: $250,000.

The total ultimate losses are $2,520,000. Since the insured's obligation is $1,620,000, the insurer's obligation is the rest: $900,000. The insurer's ULDF is thus $900,000/$250,000 = **3.6.**

(b) (i) If an aggregate deductible of $1,550,000 existed, the insured's ultimate obligation would be reduced to $1,550,000, resulting in the insured's ULDF being $1,550,000/$1,430,000 = **1.083916084**.

(ii) Since the insured's ultimate obligation is $1,550,000, the insurer's ultimate obligation is $2,520,000 - $1,550,000 = $970,000. The insurer's ULDF is thus $970,000/$250,000 = **3.88.**

Section 32

Frequency-Severity Approaches to Unpaid Claim Estimates - Part 3

Section 32 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 33

The Case Outstanding Development Method

Section 33 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 34

Estimation of Salvage and Subrogation Recoveries

Section 34 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 35

Impact of Reinsurance on Unpaid Claim Estimates

Section 35 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 36

The Berquist-Sherman Method

Section 36 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 37

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 15

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2007 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-37-1. Similar to Question 42 from the 2007 CAS Exam 6. If there is a clearly identifiable trend in an insurer's loss ratio experience from one year to another, what aspects of **(a)** the Bornhuetter-Ferguson development method and **(b)** the Least-Squares development method would render such methods sub-optimal for developing ultimate loss and unpaid claim estimates?

Solution S6-37-1.

(a) The unreported component of losses under the Bornhuetter-Ferguson development method depends entirely on an expected loss ratio. Unless that expected loss ratio has already been adjusted to reflect the most recent loss ratio trends, there will be an over- or underestimation.

(b) The Least-Squares development method is designed for situations where any changes in loss ratio experience are random. If there is a clear directional trend that the insurer can identify, then this assumption would not hold, and the Least-Squares method would ignore this systematic change in the book of business.

Problem S6-37-2. Similar to Question 43 from the 2007 CAS Exam 6. You are given the following cumulative paid claim data by accident year (AY) for Insurer Λ as of December 31, 2049, expressed in the format

(Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months), where applicable.

Cumulative Paid Claims

AY 2046: (2330, 3345, 4010, 4430)
AY 2047: (2402, 3504, 4123)
AY 2048: (2403, 3450)
AY 2049: (2420)

There are two reserving methods used to determine ultimate claim amounts for each accident year. The following are the development factors to ultimate for each method:

12 months to ultimate - Method 1: 1.89
12 months to ultimate - Method 2: 1.93
24 months to ultimate - Method 1: 1.30
24 months to ultimate - Method 2: 1.35
36 months to ultimate - Method 1: 1.10
36 months to ultimate - Method 2: 1.06
48 months to ultimate - Method 1: 1.00
48 months to ultimate - Method 2: 1.00

In calendar year (CY) 2050, the following losses are actually paid out:

For AY 2046: 0
For AY 2047: 332
For AY 2048: 704
For AY 2049: 1022
For AY 2050: 2450
Total: 4508

(a) Use a retrospective test of reserve adequacy to select either Method 1 or Method 2 as the more appropriate reserving method of the two.

(b) How could the bias of the selected method be corrected via an adjustment? Explain any assumptions in your answer.

Solution S6-37-2.

(a) A retrospective test of reserve adequacy would compare the losses actually paid out in CY 2050 to the projections by each of the methods. The losses for AY 2046-2049 paid out in CY 2050 are 4508 - 2450 = 2058.

We now determine the losses projected by Method 1:

For AY 2046: 0, since losses are already at ultimate.
For AY 2047: 4123*(1.10 - 1) = 412.3
For AY 2048: 3450*(1.30/1.10 - 1) = 627.2727272727
For AY 2049: 2420*(1.89/1.30 - 1) = 1098.307692
Total: 2137.915384

Error: 2137.915384/2058 - 1 = 0.0388315784 = Overestimate of circa 3.88%.

We now determine the losses projected by Method 2:

For AY 2046: 0, since losses are already at ultimate.
For AY 2047: 4123*(1.06 - 1) = 247.38
For AY 2048: 3450*(1.35/1.06 - 1) = 943.8679245
For AY 2049: 2420*(1.93/1.35 - 1) = 1039.703704
Total: 2230.951628

Error: 2230.951628/2058 - 1 = 0.084038692 = Overestimate of circa 8.40%.

Method 1 is preferable because it has a lower overall error.

(b) To adjust for the overestimate in Method 1, one could multiply the result by

1/(1 + Error Amount) - in this case, 1/1.0388315784 = 0.962619948. This would bring the overall reserve for CY 2050 to the level of actual losses in CY 2050 and would presumably correct any bias in estimates for subsequent years. This adjustment requires the assumption that Method 1 would continue having the same bias over time, and that the insurer experiences consistent losses and has a stable book of business.

Problem S6-37-3. Similar to Question 44 from the 2007 CAS Exam 6. In accident years (AY) 2023 through 2026, the number of cumulative reported and closed claims for Insurer Σ did not vary by accident year for any particular age of maturity. Cumulative incurred losses and case loss reserves were as follows, expressed in the format

(Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months), where applicable. Assume all losses are at ultimate at 48 months.

Cumulative Incurred Losses - Data as of December 31, 2026

For AY 2023: (3033, 4044, 4505, 4606)
For AY 2024: (3185, 4246, 4730)
For AY 2025: (3344, 4459)
For AY 2026: (3511)

Case Loss Reserves - Data as of December 31, 2026

For AY 2023: (1000, 500, 200, 0)
For AY 2024: (1050, 525, 210)
For AY 2025: (1050, 525)
For AY 2026: (1103)

(a) Find the IBNR as of December 31, 2026, using the chain ladder method.

(b) What aspect of this scenario renders the IBNR estimate in part (a) inaccurate? Justify your answer by reference to the given data.

Solution S6-37-3. (a) We first calculate age-to-age development factors for incurred losses, using the format (12-24-month factor, 24-36-month factor, 36-48-month factor), where applicable.

Age-to-Age Factors for Incurred Losses
(4044/3033, 4505/4044, 4606/4505)
(4246/3185, 4730/4246)
(4459/3344)

Age-to-Age Factors for Incurred Losses
(1.333, 1.114, 1.022)
(1.333, 1.114)
(1.333)

Our selection for age-to-age factors is made simple in this scenario. We can also select factors to ultimate:
12-month-to-ultimate factor: 1.333*1.114*1.022 = 1.517631164
24-month-to-ultimate factor: 1.114*1.022 = 1.138508
36-month-to-ultimate factor: 1.022

Now we can estimate IBNR by multiplying each still-not-ultimate value on the outermost diagonal of the incurred loss triangle by (the appropriate factor to ultimate - 1) and adding these products:

4730*(1.022 - 1) + 4459*(1.138508 - 1) + 3511*(1.517631164 - 1) = 2539.070189 = **IBNR = 2539.** (Slight variations on this are possible if rounding was used at different steps of the calculation.)

(b) We consider the incurred loss trend at each age of maturity and compare it to the case reserve trend:

Cumulative Incurred Loss Trend

AY 2023 to AY 2024: (3185/3033, 4246/4044, 4730/4505)
AY 2024 to AY 2025: (3344/3185, 4459/4246)
AY 2025 to AY 2026: (3511/3344)

Cumulative Incurred Loss Trend

AY 2023 to AY 2024: (1.05, 1.05, 1.05)
AY 2024 to AY 2025: (1.05, 1.05)
AY 2025 to AY 2026: (1.05)

Case Reserve Trend

AY 2023 to AY 2024: (1050/1000, 525/500, 210/200)
AY 2024 to AY 2025: (1050/1050, 525/525)
AY 2025 to AY 2026: (1103/1050)

Case Reserve Trend

AY 2023 to AY 2024: (1.05, 1.05, 1.05)
AY 2024 to AY 2025: (1.00, 1.00)
AY 2025 to AY 2026: (1.05)

While incurred losses increased by 5% from AY 2024 to AY 2025, case reserves did not increase at all. The net result of this is reduced case outstanding strength. The chain ladder method assumes constant case outstanding strength. With reduced case outstanding strength and the same loss development factors calculated via the chain ladder method, there will be an underestimation of IBNR.

Problem S6-37-4. Similar to Question 45 from the 2007 CAS Exam 6. There are two excess-of-loss reinsurance treaties. In Treaty A, the primary insurer's retention is $500,000. In Treaty B, the primary insurer's retention is $5,000,000. For which treaty would you expect the excess loss development factors to be higher? Give two reasons justifying your answer.

Solution S6-37-4. One would expect the excess loss development factors to be higher for **Treaty B,** because it has the higher retention. Two reasons why this happens is (1) larger losses that would exceed the higher retention are more likely to be reported later, since the primary insurer may not expect certain initial claims to develop to such an extent and (2) the smaller claims that do not exceed the retention are likely to be reported sooner. More of the smaller claims would contribute to the excess loss development for a treaty with a smaller retention.

Problem S6-37-5. Similar to Question 46 from the 2007 CAS Exam 6. You have the following information about a particular insurance policy from a well-established book of business:

Premium: 200,000
Expected loss ratio: 80%
Observed loss up to December 31, 2020: 130,000
Age-to-ultimate development factor applicable at December 31, 2020: 1.60

(a) According to the Bornhuetter-Ferguson method, what is the estimated ultimate loss amount for this policy?

(b) In the answer from part (a), what is the percentage credibility assigned to the loss development projection?

(c) What is one possible shortcoming of the Bornhuetter-Ferguson method in this case, and what can be used to mitigate this shortcoming?

Solution S6-37-5.

(a) The Bornhuetter-Ferguson method uses the formula
Ultimate Claims = Actual Reported Claims + (Expected Claims)*(% Claims Unreported).
Here, we know that Actual Reported Claims = 130,000.
We calculate Expected Claims = Premium*(Expected Loss Ratio) = 200000*0.8 = 160,000.
We calculate % Claims Unreported = 1 - 1/1.60 = 0.375 = 37.5%
Thus, Ultimate Claims = 130000 + 160000*0.375 = **190,000**.

(b) For the Bornhuetter-Ferguson method, the percentage credibility assigned to the loss development projection is the percentage of claims assumed to be reported at the time as of which the data are being analyzed. This is 1/(Development Factor to Ultimate), which here is 1/1.60 = 0.625 = **62.5%**.

(c) The Bornhuetter-Ferguson method relies on a predetermined expected loss ratio that may not take into account recent changes in loss experience. To assign more credibility to the development projection, one could use the Benktander method, which is an iterative application of the Bornhuetter-Ferguson method, using the result from the first application of the Bornhuetter-Ferguson method as the "Expected Claims" component. One could also use the Stanard-Bühlmann (Cape Cod) method, which contains a systematic way of calculating the expected loss ratio.

Section 38

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 16

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Sources:

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2007 Exam 6 and 2008 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-38-1. Similar to Question 47 from the 2007 CAS Exam 6. You know the following about paid defense and cost containment (DCC) expenses as of December 31, 2047, and ultimate losses for an insurer by accident year (AY):

AY 2044: Ultimate loss: 5550; Paid DCC: 200
AY 2045: Ultimate loss: 5200; Paid DCC: 150
AY 2046: Ultimate loss: 5100; Paid DCC: 110
AY 2047: Ultimate loss: 6000; Paid DCC: 20

Ratio of Cumulative Paid DCC to Cumulative Paid Loss, expressed in the format (Ratio at 12 months, Ratio at 24 months, Ratio at 36 months, Ratio at Ultimate).

AY 2044: (0.24%, 2.22%, 3.20%, 4.25%)
AY 2045: (0.28%, 2.24%, 3.25%)
AY 2046: (0.25%, 2.30%)
AY 2047: (0.22%)

For AY 2044 through AY 2047, calculate the total DCC reserve. Show all intermediate steps contributing to the result.

Solution S6-38-1. First, we want to calculate age-to-age factors for ratio of cumulative DCC to cumulative paid loss.

Age-to-Age Factors for Ratio of Cumulative Paid DCC to Cumulative Paid Loss, expressed in the format
(Factor for 12-24 months, Factor for 24-36 months, Factor for 36 months to Ultimate).

AY 2044: (2.22%/0.24%, 3.20%/2.22%, 4.25%/3.20%)
AY 2045: (2.24%/0.28%, 3.25%/2.24%)
AY 2046: (2.30%/0.25%)

Age-to-Age Factors for Ratio of Cumulative Paid DCC to Cumulative Paid Loss

AY 2044: (9.25, 1.441441441, 1.328125)
AY 2045: (8.00, 1.450892857)
AY 2046: (9.20)

We select the simple arithmetic means of the available age-to-age factors for a given age to maturity:

12-24 months: 8.8166666667
24-36 months: 1.446167149
36 months to ultimate: 1.328125

We can now select factors to ultimate:

12-24 months: 8.8166666667*1.446167149*1.328125 = 16.93409007
24-36 months: 1.446167149*1.328125 = 1.920690745
36 months to ultimate: 1.328125

Now, for each accident year, we can project to ultimate the ratios of cumulative paid DCC to cumulative paid loss:

AY 2044: 4.25% -- Already at ultimate
AY 2045: 3.25%*1.328125 = 4.31640625%
AY 2046: 2.30%*1.920690745 = 4.417588714%
AY 2047: 0.22%*16.93409007 = 3.725499815%

This allows us to find the ultimate DCC for each accident year:

AY 2044: 5550*4.25% = 235.875
AY 2045: 5200*4.31640625% = 224.452125

AY 2046: 5100*4.417588714% = 225.2970244
AY 2047: 6000*3.725499815% = 223.5299889

Now we can find the DCC reserve for each accident year by subtracting paid DCC from ultimate DCC:

AY 2044: 235.875 - 200 = 35.875
AY 2045: 224.452125 - 150 = 74.452125
AY 2046: 225.2970244 - 110 = 115.2970244
AY 2047: 223.5299889 - 20 = 203.5299889

Total: 35.875 + 74.452125 + 115.2970244 + 203.5299889 = 429.1541383 = circa **429.15.**

Problem S6-38-2. Similar to Question 3 from the 2008 CAS Exam 6. The annual projected severity trend is +2%. All data are at ultimate at 48 months. You also know the following information by accident year (AY):

Incremental Loss and ALAE Payments on Closed Claims, expressed in thousands of dollars and in the format

(Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months).

AY 2030: (200, 250, 180, 80)
AY 2031: (250, 290, 200)
AY 2032: (300, 300)
AY 2033: (350)

Incremental Number of Claims Closed, expressed in the format

(Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months).

AY 2030: (30, 60, 50, 40) **Ultimate claims:** 180
AY 2031: (36, 72, 60) **Ultimate claims:** 216
AY 2032: (24, 48) **Ultimate claims:** 144
AY 2033: (42) **Ultimate claims:** 252

(a) Use Adler and Kline's claim closure projection method to find the projected reserve as of December 31, 2033.

(b) Describe two aspects of the calculation you performed in part (a) that would recommend it as a reserve estimation technique.

Solution S6-38-2.

(a) First we note the incremental claim closure pattern, which appears to be the same for every accident year. Let N be the number of claims closed at 12 months. Then the pattern is

(N, 2N, (5/3)N, (4/3)N). Using this formula, we can extrapolate the numbers of closed claims:

AY 2030: (30, 60, 50, 40) **Ultimate claims:** 180
AY 2031: (36, 72, 60, 48) **Ultimate claims:** 216
AY 2032: (24, 48, 40, 32) **Ultimate claims:** 144
AY 2033: (42, 84, 70, 56) **Ultimate claims:** 252

We can also figure out the incremental paid severities (Paid Amounts/Closed Claims):

Incremental Paid Severities on Closed Claims, expressed in thousands of dollars and in the format

(Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months).

AY 2030: (200/30, 250/60, 180/50, 80/40)
AY 2031: (250/36, 290/72, 200/60)
AY 2032: (300/24, 300/48)
AY 2033: (350/42)

Really, we are just interested in the outermost diagonal:
Incremental Paid Severities on Closed Claims

AY 2030: (200/30, 250/60, 180/50, 2)
AY 2031: (250/36, 290/72, 3.3333)
AY 2032: (300/24, 6.25)
AY 2033: (8.3333)

Now we can apply our annual multiplicative severity trend of 1.02:
Incremental Paid Severities on Closed Claims

AY 2030: (200/30, 250/60, 180/50, 2)
AY 2031: (250/36, 290/72, 3.3333, 2.04)
AY 2032: (300/24, 6.25, 3.40, 2.0808)
AY 2033: (8.3333, 6.375, 3.468, 2.122416)

Now we can calculate the reserve amounts for each year as the sum of 1000*(Number of Closed Claims*Closed Claim Severity) for each age to maturity. There is no reserve for AY 2030, since losses are already at ultimate.

AY 2031: 1000*(2.04*48) = 97920
AY 2032: 1000*(3.40*40 + 2.0808*32) = 202585.6
AY 2033: 1000*(6.375*84 + 3.468*70 + 2.122416*56) = 897115.296
Total: 97920 + 202585.6 + 897115.296 = 1197620.896 = circa **$1,197,620.90**.

(b) Two advantages of the calculation in part (a) are 1) explicit incorporation of claim severity trends, which could be accounted for by economic or social inflation, and 2) no reliance on incurred losses and independence from the accuracy or lack thereof of case reserve estimates.

Problem S6-38-3. Similar to Question 4 from the 2008 CAS Exam 6. You have the following information as of December 31, 2066, all expressed in the format

(Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months), where applicable.

Cumulative Reported Loss ($000)

AY 2063: (3030, 4506, 4990, 5200)
AY 2064: (3133, 4666, 5000)
AY 2065: (3002, 4556)
AY 2066: (3000)

Cumulative Paid Loss ($000)

AY 2063: (1525, 2344, 2990, 4560)
AY 2064: (1498, 2200, 3000)
AY 2065: (1555, 2660)
AY 2066: (1500)

Average Case Reserve Per Open Claim ($000)

AY 2063: (10.03, 27.15, 66.6667, 64)
AY 2064: (11.68, 35.55, 50)
AY 2065: (11.13, 24)
AY 2066: (12)

Number of Open Claims ($000)

AY 2063: (150, 80, 30, 10)
AY 2064: (140, 75, 40)
AY 2065: (130, 79)
AY 2066: (125)

(a) Assume an annual severity trend of -4% and use the Berquist-Sherman method to create an adjusted cumulative reported loss triangle, based on a severity-adjusted case reserve triangle. Round your answers to the nearest whole number.

(b) Using the adjusted cumulative reported loss triangle from part (a) and loss development factors calculated as weighted averages of all relevant years' experience, estimate the ultimate loss for AY 2066. Use a 48-month-to-ultimate factor of 1.05.

Solution S6-38-3.

(a) We take the average case reserve triangle and project backward from the outermost diagonal using the annual severity trend of -4%. We divide each subsequent vertical entry by 0.96 to get the preceding entry.

Adjusted Average Case Reserve Per Open Claim ($000)

AY 2063: (13.56336806, 26.0416667, 52.083333, 64)
AY 2064: (13.02083333, 25, 50)
AY 2065: (12.5, 24)
AY 2066: (12)

Now, to get the adjusted reported claim triangle, for each entry except the outermost diagonal (where reported claims are unchanged from what is given), we calculate

Cumulative Paid Loss + (Number of Open Claims)*(Adjusted Average Case Reserve Per Open Claim).

Sample calculation, for AY 2063 at 12 months:

1525 + 150*13.56336806 = 3559.505209 = circa 3560.

Adjusted Cumulative Reported Loss ($000)

AY 2063: (3560, 4427, 4553, 5200)
AY 2064: (3321, 4075, 5000)
AY 2065: (3180, 4556)
AY 2066: (3000)

(b) We can calculate weighted-average age-to-age factors as follows:

For 12-24 months: (4427 + 4075 + 4556)/(3560 + 3321 + 3180) = 1.297882914.
For 24-36 months: (4553 + 5000)/(4427 + 4075) = 1.123617972
For 36-48 months: 5200/4553 = 1.142104107
12-month-to-ultimate factor: 1.297882914*1.123617972*1.142104107*1.05 = 1.748836402.
Ultimate loss for AY 2066: 1000*3000*1.748836402 = **$5,246,509.21**.

Problem S6-38-4. Similar to Question 11 from the 2008 CAS Exam 6. You have the following IBNR estimates from three different methods:

Loss development method: $6000
Bornhuetter-Ferguson method: $5000
Percent of premium method: $5300

The insurer's book of business has been showing a deteriorating loss ratio, with no changes in case reserve adequacy or loss emergence patterns.

(a) Rank these methods in order of accuracy in this situation. Justify your answer.

(b) For any of these methods that are inaccurate, which are self-correcting in the long term? Why?

Solution S6-38-4.

(a) The most accurate method here is the development method. Since there are no changes in case reserve adequacy or loss emergence patterns, the loss development pattern has not altered at all, and the loss development factors based on historical losses will still fully reflect the current situation. Less accurate is the percent of premium method, where the IBNR estimate is based on the premiums and losses during the time periods in question, and only part of the experience will be based on the more recent time periods of deteriorating loss ratios. The percent of premium method would thus underestimate the true IBNR. The Bornhuetter-Ferguson method would produce an even greater underestimate, as the IBNR component of ultimate losses is based on an expected loss ratio that is determined *a priori.* This expected loss ratio would be lower than warranted by the more recent experience. The ranking in terms of accuracy would thus be

Loss development method > Percent of premium method > Bornhuetter-Ferguson method - with the ">" sign denoting greater accuracy.

(b) The **percent of premium method** would be self-correcting over time, as the earlier time periods' experience falls outside the time period being analyzed and new experience, based on more recent loss ratio behavior, would replace it. The Bornhuetter-Ferguson method would require deliberate adjustment of the expected loss ratio to reflect more current conditions.

Problem S6-38-5. Similar to Question 13(a) from the 2008 CAS Exam 6. You have the following information:

Ratios of ultimate excess loss to ground-up loss
For a retention of $100,000: 0.55
For a retention of $600,000: 0.22

Excess loss development factors, 12 months to ultimate
For a retention of $100,000: 2.24
For a retention of $600,000: 5.56

What is the 12-month-to-ultimate excess loss development factor for the layer $500,000 in excess of $100,000?

Solution S6-38-5. We consider the definition of the 12-month-to-ultimate excess loss development factor. It is (Excess loss at ultimate)/(Excess loss at 12 months).

For the layer in question, the factor is

(Loss in excess of $100,000 at ultimate - Loss in excess of $600,000 at ultimate)/
((Loss in excess of $100,000 at 12 months - Loss in excess of $600,000 at12 months).

Since we are given ratios to ultimate ground-up loss, we can express the desired factor as follows:

(Ultimate ground-up loss)*((Loss in excess of $100,000 at ultimate - Loss in excess of $600,000 at ultimate)/(Ultimate ground-up loss)/
((Loss in excess of $100,000 at 12 months - Loss in excess of $600,000 at12 months) =

((Ratio of loss in excess of $100,000 at ultimate to ultimate ground-up loss) -
(Ratio of loss in excess of $600,000 at ultimate to ultimate ground-up loss))/
((Loss in excess of $100,000 at 12 months - Loss in excess of $600,000 at12 months)/(Ultimate ground-up loss)).

We can replace ((Ratio of loss in excess of $100,000 at ultimate to ultimate ground-up loss) - (Ratio of loss in excess of $600,000 at ultimate to ultimate ground-up loss)) by 0.55 - 0.22 = 0.33, leading to

0.33/((Loss in excess of $100,000 at 12 months - Loss in excess of $600,000 at 12 months)/(Ultimate ground-up loss)).

We again consider the formula for the excess loss development factor: (Excess loss at ultimate)/(Excess loss at 12 months).

It follows that (Loss in excess of $100,000 at 12 months - Loss in excess of $600,000 at 12 months) = (Ultimate loss in excess of $100,000)/(Loss development factor in excess of $100,000) - (Ultimate loss in excess of $600,000)/(Loss development factor in excess of $600,000).

Thus we have our desired factor as

0.33/((Ultimate loss in excess of $100,000)/(Loss development factor in excess of $100,000) - (Ultimate loss in excess of $600,000)/(Loss development factor in excess of $600,000))/
(Ultimate ground-up loss) =

0.33/((Ratio of loss in excess of $100,000 at ultimate to ultimate ground-up loss)/(Loss development factor in excess of $100,000) - ((Ratio of loss in excess of $600,000 at ultimate to ultimate ground-up loss)/(Loss development factor in excess of $600,000)) =

0.33/(0.55/2.24 - 0.22/5.56) = **1.602196554**.

Section 39

Estimation of Unpaid Allocated Loss Adjustment Expenses

Section 39 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 40

Retrospective Evaluation of Unpaid Claim Estimates

Section 40 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 41

The Classical Method of Estimating Unpaid Unallocated Loss Adjustment Expenses

Section 41 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 42

The Kittel and Mango-Allen Refinements to the Classical Method of Estimating Unpaid Unallocated Loss Adjustment Expenses

Section 42 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 43

The Conger-Nolibos Method of Estimating Unpaid Unallocated Loss Adjustment Expenses

Section 43 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 44

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 17

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration – and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Formulas for Brosius's Least-Squares Method
$y = a + b*x$
$b = ((xy)^- - x^-*y^-)/((x^2)^- - (x^-)^2)$
$a = y^- - b* x^-$

This is least-squares linear regression applied to loss development. The data points x are the independent variables (earlier time periods' data), and the data points y are the dependent variables (later time periods' data). The value a is the y-intercept, and b is the slope. The $^-$ superscripts denote sample means and are equivalent to bars over the entire symbol, which are not expressed here due to notational difficulties.

Sources: Brosius, E., "Loss Development Using Credibility," CAS Study Note, March 1993.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Teng, M.T.S.; and Perkins, M.E., "Estimating the Premium Asset on Retrospectively Rated Policies," *PCAS* LXXXIII, 1996, pp. 611-647.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-44-1. Similar to Question 2 from the 2009 CAS Exam 6. You know the following information about cumulative paid losses for Insurer Ψ by accident year (AY):

Cumulative Paid Loss, expressed in the format(Amount at development year 0, Amount at development year 1, Amount at development year 2).
AY 2022: (343, 444, 500)
AY 2023: (360, 500, 555)
AY 2024: (320, 466)
AY 2025: (350)

An accident-year model of the following nature is fitted to the data above:
$y(j) = \alpha + \varepsilon_0$ for j = 0;
$y(j) = \alpha + {}_{k=1}^{j}\Sigma(\gamma_k) + \varepsilon_j$ for j = 1, 2.

Definitions:
y = ln(Incremental paid loss)
j = year of development - either 0, 1, or 2
α, γ_j (for j = 1, 2) = constants
ε_j (for each j) = error terms with means of zero

Find the median value of the estimated incremental paid loss during development year 1 for accident year 2025.

Solution S6-44-1. First we find the incremental paid losses during development year 1 for each of the other three accident years:
AY 2022: 444-343 = 101
AY 2023: 500-360 = 140
AY 2024: 466-320 = 146

For year 1, our formula becomes $y(1) = \alpha + \gamma_1 + \varepsilon_1$.

The average (arithmetic mean) of the natural logs of the known incremental paid losses should be a good approximation of $\alpha + \gamma_1$, since ε_1 has a mean of zero.

Thus, we estimate y(1) = (ln(101) + ln(140) + ln(146))/3 = 4.846789854, and the desired estimated incremental paid loss would therefore be $e^{y(1)} = e^{4.846789854}$ = **127.330982**.

Problem S6-44-2. Similar to Question 3 from the 2009 CAS Exam 6. You are again analyzing the following information about cumulative paid losses for Insurer Ψ by accident year (AY):

Cumulative Paid Loss, expressed in the format (Amount at development year 0, Amount at development year 1, Amount at development year 2).
AY 2022: (343, 444, 500)
AY 2023: (360, 500, 555)
AY 2024: (320, 466)
AY 2025: (350)

(a) Use Brosius's least-squares method to find the expected losses at development year 1 for AY 2025.

(b) Is the least-squares method proper to use in a situation such as the one in part (a)? Explain why or why not.

Solution S6-44-2.

(a) First we find the various averages necessary for the least-squares method.
Let x be experience at development year 0, and let y be experience at development year 1.
$x^- = (343 + 360 + 320)/3 = 341.$
$y^- = (444 + 500 + 466)/3 = 470$
$(xy)^- = (343*444 + 360*500 + 320*466)/3 = 160470.666667$
$(x^2)^- = (343^2 + 360^2 + 320^2)/3 = 116549.666667$
$(x^-)^2 = 341^2 = 116281$

Now we find $b = ((xy)^- - x^-*y^-)/((x^2)^- - (x^-)^2) =$
$(160470.666667 - 341*470)/(116549.666667 - 116281) = b = 0.7468982642.$

Now we find $a = y^- - b* x^- = 470-0.7468982642*341 = 215.3076919.$
For AY 2025, x = 350, so $y = a+bx = 215.3076919 + 0.7468982642*350 =$ **476.77220844**.

(b) It **is proper** to use the least-squares method in this situation, because b > 0.

Problem S6-44-3. Similar to Question 7 from the 2008 CAS Exam 6.

You are again analyzing the following information about cumulative paid losses for Insurer Ψ by accident year (AY):

Cumulative Paid Loss, expressed in the format (Amount at development year 0, Amount at development year 1, Amount at development year 2).
AY 2022: (343, 444, 500)
AY 2023: (360, 500, 555)
AY 2024: (320, 466)
AY 2025: (350)

An accident-year model of the following nature is fitted to the data above:
$y(j) = \alpha + \varepsilon_0$ for j = 0;
$y(j) = \alpha + {}_{k=1}^{j}\Sigma(\gamma_k) + \varepsilon_j$ for j = 1, 2.

Definitions:
y = ln(Incremental paid loss)
j = year of development - either 0, 1, or 2
α, γ_j (for j = 1, 2) = constants
ε_j (for each j) = error terms with means of zero

(a) Find the values of α, γ_1, and γ_2.

(b) Find the median value of the estimated incremental paid loss during development year 2 for accident year 2025.

Solution S6-44-3.

(a) Since the mean error terms are zero, the value of α is the average of the natural logarithms of the development year 0 data: $\alpha = (\ln(343) + \ln(360) + \ln(320) + \ln(350))/4 =$ **$\alpha = 5.837522157$**.

We already know from Solution S6-44-1 that $\alpha + \gamma_1 = 4.846789854$, so $\gamma_1 = 4.846789854 - 5.837522157 =$ **$\gamma_1 = -0.9907323032$**.

Now we find the known incremental losses paid for development year 2:
AY 2022: 500-444 = 56
AY 2023: 555-500 = 55

$\alpha + \gamma_1 + \gamma_2 = (\ln(56) + \ln(55))/2 = \alpha + \gamma_1 + \gamma_2 = 4.016342438$. Since $\alpha + \gamma_1 = 4.846789854$, it follows that $\gamma_2 = 4.016342438 - 4.846789854 =$ **$\gamma_2 = -0.830447416$**.

(b) Since, by our model, $y(2) = \alpha + \gamma_1 + \gamma_2 + \varepsilon_2$, and ε_2 has a mean of zero, the median incremental loss in development year 2 will be $\exp(\alpha + \gamma_1 + \gamma_2) = e^{4.016342438} =$ **55.4977477**.

Problem S6-44-4. Similar to Question 9 from the 2008 CAS Exam 6.

You are again analyzing the following information about cumulative paid losses for Insurer Ψ by accident year (AY):

Cumulative Paid Loss, expressed in the format (Amount at development year 0, Amount at development year 1, Amount at development year 2).
AY 2022: (343, 444, 500)
AY 2023: (360, 500, 555)
AY 2024: (320, 466)
AY 2025: (350)

Find the cumulative paid loss amount for AY 2024 at development year 2 using the following methods:
(a) The development method;
(b) The budgeted loss method;
(c) The least-squares method.

Solution S6-44-4. (a) We use the development method with a weighted-average loss development factor from year 1 to year 2: (500 + 555)/(444 + 500) = 1.117584746. Our answer is thus 466*1.117584746 = **520.7944915**.

(b) The budgeted loss method simply takes the expected value of the known losses at development year 2 and sets that as the loss for AY 2024: (500 + 555)/2 = **527.5**.

(c) First we find the various averages necessary for the least-squares method.

Let x be experience at development year 1, and let y be experience at development year 2.
$x^- = (444 + 500)/2 = 472.$
$y^- = (500 + 555)/2 = 527.5.$
$(xy)^- = (444*500 + 500*555)/2 = 249750$
$(x^2)^- = (444^2 + 500^2)/2 = 223568$
$(x^-)^2 = 472^2 = 222784$

Now we find $b = ((xy)^- - x^-*y^-)/((x^2)^- - (x^-)^2) = (249750 - 472*527.5)/(223568 - 222784) = b = 0.9821428571.$

Now we find $a = y^- - b* x^- = 527.5 - 0.9821428571*472 = 63.92857143.$

For AY 2024, x = 466, so $y = a+bx = 63.92857143 + 0.9821428571*466 =$ **521.6071429**.

Problem S6-44-5. Similar to Question 14 from the 2008 CAS Exam 6.

You have the following information as of December 31, 2050, for a book of retrospectively rated policies for which business first began to be written in 2047.

Expected Future Loss Emergence
For Policy Year 2047: 8000
For Policy Year 2048: 56000
For Policy Year 2049: 123000
For Policy Year 2050: 352000

Cumulative Premium Development to Loss Development (CPDLD) Ratio
For Policy Year 2047: 0.25
For Policy Year 2048: 0.66
For Policy Year 2049: 1.04
For Policy Year 2050: 1.44

Premium Booked from Prior Adjustments
For Policy Year 2047: 222000
For Policy Year 2048: 180000
For Policy Year 2049: 150000
For Policy Year 2050: 0

Premium Booked
For Policy Year 2047: 224000
For Policy Year 2048: 201000
For Policy Year 2049: 180000
For Policy Year 2050: 190000

Calculate the premium asset as of December 31, 2050, using the methodology of Teng and Perkins.

Solution S6-44-5. The CPDLD ratio - a ratio of *premium* development to *loss* development - can be used to estimate future premium emergence from expected future loss ratio.

Expected Future Premium Emergence
For Policy Year 2047: 8000*0.25 = 2000
For Policy Year 2048: 56000*0.66 = 36960
For Policy Year 2049: 123000*1.04 = 127920
For Policy Year 2050: 352000*1.44 = 506880

The expected ultimate premium is the sum of the prior booked premium and the expected future premium emergence.

Expected Ultimate Premium
For Policy Year 2047: 222000 + 2000 = 224000
For Policy Year 2048: 180000 + 36960 = 216960
For Policy Year 2049: 150000 + 127920 = 277920
For Policy Year 2050: 0 + 506880 = 506880

The premium asset is the expected ultimate premium minus the premium booked.

Premium Asset
For Policy Year 2047: 224000 - 224000 = 0
For Policy Year 2048: 216960 - 201000 = 15960
For Policy Year 2049: 277920 - 180000 = 97920
For Policy Year 2050: 506880 - 190000 = 316880

Our total premium asset is thus 0 + 15960 + 97920 + 316880 = **430760**.

Section 45

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 18

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Bayesian Credibility Formula

Z = (VHM)/(EVPV + VHM), where Z = credibility percentage, EVPV = expected value of process variance, and VHM = variance of hypothetical means. Generally, EVPV = $E_Y(Var(X \mid Y))$ and VHM = $Var_Y(E(X \mid Y))$ for random variables X and Y.

If we have as our variables Y = losses and X/Y = reporting ratio, then, in the linear approximation to the Bayesian credibility estimate, the following formulas hold:

VHM = $Var_Y(E(X/Y)*Y)$ and
EVPV = $Var(X/Y)*(Var(Y) + E(Y)^2)$.

(More discussion on this subject can be found in Brosius, pp. 13-15.)

Sources:

Blanchard, R.S., "Accounting Concepts for the Actuary," CAS Study Note, June 2003.

Brosius, E., "Loss Development Using Credibility," CAS Study Note, March 1993.

Financial Accounting Standards Board, "Statement of Financial Accounting Standards No. 5, Accounting for Contingencies" (FAS 5), Paragraphs 1-4, and 8-10.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2010.

Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Teng, M.T.S.; and Perkins, M.E., "Estimating the Premium Asset on Retrospectively Rated Policies," *PCAS* LXXXIII, 1996, pp. 611-647.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-45-1. Similar to Question 4 from the 2009 CAS Exam 6. You are given the following information for Insurer Ξ:

Cumulative Closed Claim Counts, expressed in the format
(Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months):
AY 2056: (124, 234, 304, 350)
AY 2057: (150, 225, 320)
AY 2058: (130, 240)
AY 2059: (144)

Selected Ultimate Claim Counts
AY 2056: 380
AY 2057: 400
AY 2058: 390
AY 2059: 410

Selected Cumulative Disposal Rates
At 12 Months: 0.35
At 24 Months: 0.60
At 36 Months: 0.78
At 48 Months: 0.87

Cumulative Paid Loss ($000)
AY 2056: (1000, 2150, 3340, 4400)
AY 2057: (1200, 2000, 3600)
AY 2058: (1200, 1950)
AY 2059: (1240)

Find the following using the disposal-rate frequency-severity technique and a 4% annual severity trend factor:

(a) The expected incremental claim counts for the time periods 36-48 months and 48 months to ultimate for accident year 2057;

(b) The 36-months-to-ultimate tail severity at 2059 levels;

(c) The ultimate losses for accident year 2057.

Solution S6-45-1. (a) We use the selected disposal rates to estimate the incremental claim count for 36-48 months:
(Not-Yet-Opened Claims for 2057)*(Disposal rate at 48 months - Disposal rate at 36 months)/(1 - Disposal rate at 36 months) = (400-320)*(0.87-0.78)/(1-0.78) = 32.727272727, which we round to 33. Since 80 claims remain to be opened in total, the claims opened during the time period 48 months to ultimate will be 80 - 33 = 47.

Incremental claim count for 36-48 months: **33**
Incremental claim count for 48 months to ultimate: **47**

(b) We consider the incremental severity between 36 and 48 months for AY 2056 as (Incremental Paid Loss)/(Incremental Claim Counts) = (4400 - 3340)*1000/(350-304) = 23043.47826. Since we have no information about paid losses from 48 months to ultimate, this is our best estimate regarding severity from 36 months to ultimate. Now we need to trend this answer to 2059 by using the factor 1.04^3: $23043.47826*1.04^3 = 25920.77913 = **$25,920.78.**

(c) There are 80 claims remaining to be paid in 2057. The losses that have already been paid are $3,600,000. For the remaining claims, we multiply the claim count by the estimated 2057 36-months-to-ultimate severity, which is the 2056 severity trended by one year: 23043.47826*1.04 = 24923.82609. Our answer is thus 3600000 + 80*24923.82609 = **$5,593,906.09**.

Problem S6-45-2. Similar to Question 6 from the 2009 CAS Exam 6. You have the following information on a book of retrospectively rated workers' compensation policies:

Standard premium: $260,000
Expected loss ratio to standard premium: 70%
Tax multiplier: 1.03
Loss cost factor: 1.25
Basic premium factor: 0.30

Percentages at Third Retrospective Adjustment
Loss eliminated by maximum and minimum: 16%
Loss eliminated by accident limit: 10%
Total loss emerged: 55%

Percentages at Ultimate
Loss eliminated by maximum and minimum: 24%
Loss eliminated by accident limit: 13%
Total loss emerged: 100%

What is the estimated future premium after the third retrospective adjustment?

Solution S6-45-2. The estimated future premium after the third retrospective adjustment is the difference between the estimated future premium at ultimate and the estimated premium up to the third retrospective adjustment.

The basic premium is the same irrespective of loss experience. Since it does not vary among retrospective adjustments, it can be disregarded for the purposes of this calculation. We want to find the components of premium that vary on the basis of losses. We need to convert standard premium to losses, take into account the losses eliminated, and then multiply the result by the loss cost factor and the tax multiplier to have the estimated premium take into account other elements besides losses (e.g., taxes and other expenses).

Estimated non-basic premium up to the third retrospective adjustment:
(Standard Premium)*(Expected Loss Ratio)*(% Loss Emerged)(1 - % Loss Eliminated)*(Loss Cost Factor)*(Tax Multiplier) = 260000*0.7*0.55*(1-0.16-0.1)*1.25*1.03 = 95370.275

Estimated ultimate non-basic premium:
(Standard Premium)*(Expected Loss Ratio)*(% Loss Emerged)*(1 - % Loss Eliminated)*(Loss Cost Factor)*(Tax Multiplier) = 260000*0.7*1*(1-0.24-0.13)*1.25*1.03 = 147624.75.

Thus, the estimated future premium after the third retrospective adjustment is 147624.75 - 95370.275 = 52254.475 = **$52,254.48**.

Problem S6-45-3. Similar to Question 10 from the 2008 CAS Exam 6. You are given the following information for an insurer's book of business for a particular accident year:

Earned premium: $34,350
Reported losses as of 24 months: $14,515
Expected loss ratio: 80%
Coefficient of variation of loss ratio: 0.66
Coefficient of variation of percent of loss reported: 0.88
Expected percent of loss reported at 24 months: 55%

(a) What is the linear approximation to the Bayesian credibility estimate, as of 24 months, for the ultimate loss for this accident year?

(b) Calculate estimates of ultimate loss at age 24 months using each of the following methods (i) the chain ladder method, (ii) the Bornhuetter-Ferguson method, (iii) the Benktander method.

(c) Explain, for this particular situation, how the Benktander method incorporates the idea of credibility.

Solution S6-45-3.

(a) Let Y denote losses and X/Y denote the reporting pattern.

Then E(Y) = (Expected Loss Ratio)*(Earned Premium) = 0.8*34350 = E(Y) = 27480.

The coefficient of variation (CV) is (Standard Deviation)/(Mean). So SD(Y)/E(Y) = 0.66, and thus SD(Y) = 0.66*E(Y) = 0.66*27480 = 18136.8, and Var(Y) = $SD(Y)^2 = 18136.8^2$ = 328943514.2.

E(X/Y) is given as 0.55, and SD(X/Y) = CV(X/Y)*E(X/Y) = 0.55*0.88 = 0.484. Thus, Var(X/Y) = 0.484^2 = Var(X/Y) = 0.234256.

We recall the formula for the credibility percentage Z: Z = (VHM)/(EVPV + VHM).

We need to find VHM = Var_Y(E(X/Y)*Y) = Var(0.55Y) = 0.55^2*Var(Y) = 0.55^2*328943514.2 = VHM = 99505413.06.

We also need to find EVPV = Var(X/Y)*(Var(Y) + $E(Y)^2$) = 0.234256*(328943514.2 + 27480^2) = 2539455504.

Thus, Z = (VHM)/(EVPV + VHM) = 99505413.06/(2539455504 + 99505413.06) = Z = 0.2815174416.

This credibility is being assigned to expected ultimate losses based on losses that are already developed, while the complement of credibility is assigned to E(Y), the expected losses.

Here, expected ultimate losses based on losses that are already developed, are (Losses already developed)/(% Losses reported) = 14515/0.55 = 26390.0909090909.

Thus, our estimate is 0.2815174416*26390.0909090909 + (1-0.2815174416)*27480 = 27173.40191 = **$27,173.40**.

(b) (i) Using the chain ladder method, the expected loss is (Loss already reported)/(% Loss reported) = 14515/0.55 = 26390.0909090909 = **$26,390.09**.

(ii) Using the Bornhuetter-Ferguson method, one assumes that unreported losses will be (Expected losses)*(1-% Losses already reported), and adds already reported losses to this value: 27480*(1-0.55) + 14515 = **$26,881**.

(iii) The Benktander method is an iteration of the Bornhuetter-Ferguson method, with the Bornhuetter-Ferguson estimate substituted in place of expected losses: 26881*(1-0.55) + 14515 = **$26,611.45**.

(c) The Benktander method can be seen as a credibility-weighted estimate in the sense that the percentage of credibility assigned to the chain-ladder estimate is equal to the percent of ultimate losses reported. The value 14515 in part (b)(iii) can be seen as the chain ladder estimate (26390.0909090909) multiplied by the percentage of credibility (55%). The complement of credibility is assigned to the Bornhuetter-Ferguson estimate (26881).

Problem S6-45-4. Similar to Question 17 from the 2008 CAS Exam 6.

(a) What is the purpose of an *income statement* in financial reports?

(b) What is the purpose of a *balance sheet* in financial reports?

(c) What is the purpose of a *statement of cash flows* in financial reports?

Solution S6-45-4.

(a) An income statement shows revenues, expenses, and their difference, net income, of a company during a particular period of time.

(b) A balance sheet functions as a snapshot in time of a company's financial position, showing assets, liabilities, and equity as of a particular date.

(c) A statement of cash flows describes how the company got from its position at the beginning of a time period to the end of that time period and shows the company's sources and uses of cash.

Problem S6-45-5. Similar to Question 18 from the 2008 CAS Exam 6. A workers' compensation insurer is aware that an employee of one of its insureds has had a workplace accident, but a series of medical tests is necessary to determine whether an injury has occurred and how severe it will be. The medical tests will all be paid for by the insurer, but it is not clear what the costs of treating any injury will be. A review of similar accidents has shown that injuries are incurred in most cases, and a probability distribution for the severity of possible injuries has been constructed on the basis of historical data.

(a) What is the name for this event in terms of financial accounting standards? (See FAS 5.)

(b) What are the two general criteria that this event has met to be an instance of the term from part (a)? How has each criterion been met in the situation in question?

Solution S6-45-5.

(a) The situation in question is known as a *contingency*. There is uncertainty about whether the workers' compensation insurer will suffer a loss in terms of paying for the worker's medical treatments. The uncertainty will be resolved when the medical tests are performed and the results are obtained.

(b) The two general criteria for a contingency, per FAS 5, are

1. It is probable that a liability has been incurred or an asset has been impaired as of the date of the financial statement and
2. Reasonable estimation of the amount of the loss is possible.

Criterion 1 has been met here because it is probable that an injury has occurred, per the insurer's analysis that most similar accidents result in injuries. Criterion 2 has been met because the insurer can estimate the expected loss based on its probability distribution of the severity of the likely injury.

Section 46

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 19

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that will appear on Exam 6. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources: Blanchard, R.S., "Accounting Concepts for the Actuary," CAS Study Note, June 2003.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2010.

Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-46-1. Similar to Question 15 from the 2008 CAS Exam 6. You are performing a retrospective test for reserve accuracy and find the following deviations of the reserve estimates from actual results. All differences are expressed in the format (Amount at AY+1, Amount at AY+2, Amount at AY+3, Amount at AY+4), where AY is the accident year in question.

Percentage Deviations
AY 2032: (12.5%, -2%, -8%, 4%)
AY 2033: (6%, 12.4%, -4%)
AY 2034: (-4%, 4%)
AY 2035: (6.5%)

You are also given loss reserves distributed by year of payment as follows.
AY 2033: ******** Starting with (AY + 4): (234)
AY 2034: ******** Starting with (AY + 3): (2222, 255)
AY 2035:******** Starting with (AY + 2): (3444, 2455, 300)
AY 2036: Starting with (AY + 1): (8000, 3555, 2600, 240)

(a) Find the weighted-average bias as of December 31, 2036. Use as your weights the contributions to the total reserve by *payment year*.

(b) If you used a straight arithmetic average of biases instead, identify two problems you would encounter.

Solution S6-46-1.

(a) First we select our percentage deviations for each payment year.

For AY + 1, we select the arithmetic mean of the four given percentages:
(12.5% + 6% - 4% + 6.5%)/4 = 5.25%.

We likewise select arithmetic means for the other payment years:
For AY + 2: (-2% + 12.4% + 4%)/3 = 4.8%.
For AY + 3: (-8% -4%)/2 = -6%.
For AY + 4: 4%.

We can also select our weights for each payment year by adding the reserves for that payment year.
For AY+1: 8000
For AY+2: 3444 + 3555 = 6999
For AY+3: 2222 + 2455 + 2600 = 7277
For AY+4: 234 + 255 + 300 + 240 = 1029.

Our weighted-average bias is thus
(8000*5.25% + 6999*4.8% + 7277*(-6%) + 1029*4%)/(8000 + 6999 + 7277 + 1029) = +**1.546844025%.**

(b) The following are two problems that would arise from using a straight arithmetic average of biases in this case:

1. There would be an overemphasis of the deviation for AY+4. Since the last payment year does not see nearly as much development as the others, it is appropriate to place a smaller weight on the deviation for that payment year.

2. There would be a general overestimation of the bias, as the simple arithmetic average would be (5.25% + 4.8% -6% + 4%)/4 = +2.0125%. Thus, the analyst might get the impression that the reserving method is more biased than is actually the case.

Problem S6-46-2. Similar to Question 19 from the 2008 CAS Exam 6. Match the following statements to the accounting paradigm to which they best pertain. Your choices are either (1) the deferral/matching paradigm or (2) the asset/liability paradigm.

(a) This paradigm emphasizes the income statement more than the balance sheet.

(b) This paradigm emphasizes the balance sheet more than the income statement.

(c) This paradigm focuses on temporal matching of revenues with expenses.

(d) This paradigm recognizes revenues and expenses as they are incurred, even if revenues are thereby separated from corresponding expenses.

(e) This paradigm allows for a "deferred expense" asset.

(f) This paradigm is more appropriate for a policy where coverage is provided over time in exchange for premium.

Solution S6-46-2.

(a) (1) The **deferral/matching paradigm** emphasizes the income statement more than the balance sheet.

(b) (2) The **asset/liability paradigm** emphasizes the balance sheet more than the income statement.

(c) (1) The **deferral/matching paradigm** focuses on temporal matching of revenues with expenses.

(d) (2) The **asset/liability paradigm** recognizes revenues and expenses as they are incurred, even if revenues are thereby separated from corresponding expenses.

(e) (1) The **deferral/matching paradigm** allows for a "deferred expense" asset.

(f) (1) The **deferral/matching paradigm** is more appropriate for a policy where coverage is provided over time in exchange for premium.

Problem S6-46-3. Similar to Question 20 from the 2008 CAS Exam 6. Identify whether each of the following statements applies to generally accepted accounting principles (GAAP) or statutory accounting principles (SAP):

(a) Used to evaluate an insurer's solvency.

(b) Allow for a "deferred expense" asset that is amortized over time.

(c) Require all acquisition expenses to be recognized immediately.

(d) Focus on the money that could be obtained if the firm were liquidated immediately.

(e) Used by investors and creditors to evaluate an insurer's position.

Solution S6-46-3. (a) SAP are used to evaluate an insurer's solvency.

(b)GAAP allow for a "deferred expense" asset that is amortized over time.

(c) SAP require all acquisition expenses to be recognized immediately.

(d) SAP focus on the money that could be obtained if the firm were liquidated immediately.

(e) GAAP are used by investors and creditors to evaluate an insurer's position.

Problem S6-46-4. Similar to Question 21 from the 2008 CAS Exam 6. You have the following information about an insurer as of December 31, 2040:

Bonds - market value: $5555
Bonds - amortized value: $5100
Agents' balances - under 60 days: $540
Agents' balances - 60-90 days: $400
Agents' balances - 90+ days: $300
Deferred acquisition costs: $250
Bills receivable past due: $102
Furniture: $444
Loss and LAE reserve: $1341
Unearned premium reserve: $2322

What are the insurer's **(a)** GAAP assets, **(b)** GAAP liabilities, **(c)** SAP assets, and **(d)** SAP liabilities.

Solution S6-46-4. We consider which of the above will be recognized as assets by GAAP and SAP. GAAP recognize bonds at market value, while SAP recognize bonds at their amortized value (predetermined and independent of market changes). SAP do not recognize agents' balances in excess of 90 days or bills receivable past due. Also, SAP do not allow deferral of acquisition costs and do not recognize furniture as an asset. Both sets of principles will recognize the loss/LAE reserve and unearned premium reserve as liabilities.

(a) GAAP assets:
Bonds - market value: $5555
Agents' balances - under 60 days: $540
Agents' balances - 60-90 days: $400
Agents' balances - 90+ days: $300
Deferred acquisition costs: $250
Bills receivable past due: $102
Furniture: $444
Total: **$7591**

(b) GAAP liabilities:
Loss and LAE reserve: $1341
Unearned premium reserve: $2322
Total: **$3663**

(c) SAP assets:
Bonds - amortized value: $5100
Agents' balances - under 60 days: $540
Agents' balances - 60-90 days: $400
Total: **$6040**

(d) SAP liabilities:
Loss and LAE reserve: $1341
Unearned premium reserve: $2322
Total: **$3663**

Problem S6-46-5. Similar to Question 22 from the 2008 CAS Exam 6.

(a) What is the difference between a *loss reserve* and a *premium deficiency reserve*?

(b) Calculate the premium deficiency reserve as of December 31, 2025, given the following information:

Unearned premium reserve as of December 31, 2025: $4,340,000
Expected loss ratio on the unearned premium reserve as of December 31, 2025: 90%
Loss from occurrence on December 24, 2025, reported on January 6, 2026: $600,000
Estimated marginal expenses related to the runoff of the unearned premium reserve: $500,000
Ratio of fixed and general overhead expenses: 25%

Solution S6-46-5. (a) A loss reserve is an estimate of unpaid losses as of a particular date. A premium deficiency reserve is set up relative to an unearned premium reserve, when the amount of the unearned premium reserve is not sufficient to cover all the future losses and expenses that would be associated with the unearned premium in question.

(b) This is somewhat of a trick question, because two of the given quantities are irrelevant to the premium deficiency reserve. Fixed and general overhead expenses are not used in determining whether there is a premium deficiency. Moreover, the loss occurring on December 24, 2025, will be considered in the loss reserve for 2025, irrespective of when it was reported.

Our premium deficiency is thus
Unearned premium reserve as of December 31, 2025 -
Expected losses on the unearned premium reserve -
Estimated marginal expenses related to the runoff of the unearned premium reserve =
$4,340,000 - 0.9*($4,340,000) - $500,000 = -$66,000.

Thus, the premium deficiency reserve should be **$66,000**.

Section 47

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 20

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:

Feldblum, S., Discussion of "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance", *PCAS* LXXX, 1993, pp. 380-395.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2010.

Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-47-1. Similar to Question 32 from the 2008 CAS Exam 6. A primary insurer has a 40% quota share treaty with one reinsurer, subject to a maximum reinsurance recovery of $200,000 per occurrence, and a $500,000 in excess of $200,000 per-occurrence excess-of-loss treaty with another reinsurer. The excess-of-loss treaty has a co-participation percentage of 10% above the retention.

The following occurrences are subject to the treaty:

Occurrence A has ground-up loss amount $500,000.
Occurrence B has ground-up loss amount $150,000.
Occurrence C has ground-up loss amount $750,000.
Occurrence D has ground-up loss amount $400,000.

(a) What is the primary insurer's total retained loss if neither treaty inures to the benefit of the other?

(b) What is the primary insurer's total retained loss if the quota share treaty inures to the benefit of the excess-of-loss treaty?

(c) What incentive does the co-participation provision in the excess-of-loss treaty give the primary insurer in this case?

Solution S6-47-1. (a) If neither treaty inures to the benefit of the other, then each will pay the full amount that it would pay if no other treaty existed. We can thus consider the primary insurer's total retained loss as (Total ground-up loss) - (Total loss paid by reinsurer under quota share treaty) - (Total loss paid by reinsurer under excess-of-loss treaty).

Total ground-up loss = $500,000 + $150,000 + $750,000 + $400,000 = $1,800,000.

Total loss paid under quota share treaty = 40%*($500,000 + $150,000 + $400,000) + $200,000 (since 40% of $750,000 exceeds the $200,000 maximum recovery) = $620,000.

Total loss paid by reinsurer under excess-of-loss treaty = 0.9*($300,000 + $500,000 + $200,000) = $900,000. Note that there is no recovery for Occurrence B, for which the loss is below the retention. Also, the excess recovery for Occurrence C is limited to 90% of $500,000.

Thus, the total retained loss is $1,800,000 - $620,000 - $900,000 = **$280,000**.

(b) If the quota share treaty inures to the benefit of the excess-of-loss treaty, then the quota share treaty is applied first to reduce the losses applicable to the excess-of-loss treaty. Thus, the quota share treaty still pays $620,000, as in part (a). However, after the quota share treaty, the applicable loss amounts change as follows:

For Occurrence A: $500,000*0.6 = $300,000
For Occurrence B: $150,000*0.6 = $90,000
For Occurrence C: $750,000 - 200,000 = $550,000
For Occurrence D: $400,000*0.6 = $240,000

Total loss paid by reinsurer under excess-of-loss treaty = 0.9*($100,000 + $350,000 + $40,000) = $441,000.

Thus, the total retained loss is $1,800,000 - $620,000 - $441,000 = **$739,000**.

(c) The co-participation provision gives the primary insurer the incentive to control even those claim costs which exceed the retention for the treaty, since the primary insurer will be sharing 10% of these costs.

Problem S6-47-2. Similar to Question 33 from the 2008 CAS Exam 6. A primary insurer has a 5-line surplus share reinsurance treaty with a net line of $150,000. You know the following information about policies and losses:

Policy 1: Limit is $60,000 - loss is $25,000.
Policy 2: Limit is $1,000,000 - loss is $680,000.
Policy 3: Limit is $450,000 - loss is $200,000.
Policy 4: Limit is $300,000 - loss is $40,000.

What is the primary insurer's net loss?

Solution S6-47-2. For Policy 1, the limit is less than the primary insurer's line, so the primary insurer retains the entire $25,000 loss.

For Policy 2, we need to keep in mind that the reinsurer's obligation is limited to 5 lines, or $750,000, so, of the total limit of $1,000,000, the primary insurer's obligation extends to the line of $150,000 *and* (1,000,000 - 750,000 - 150,000) = $100,000, for a total of $250,000 of the limit, and, correspondingly, a quarter of any loss. Thus, the primary insurer's obligation for this loss is 680000*0.25 = $170,000.

For Policy 3, the primary insurer's obligation is a third of any loss, since 150000/450000 = 1/3. Thus, the primary insurer's obligation for this loss is 200000/3 = $66,666.67.

For Policy 4, the primary insurer's obligation is half of any loss, since 150000/300000 = 1/2. Thus, the primary insurer's obligation for this loss is 40000/2 = $20,000.

The primary insurer's total obligation is thus $25,000 + $170,000 + $66,666.67 + $20,000 = **$281,666.67**.

Problem S6-47-3. Similar to Question 34 from the 2008 CAS Exam 6. According to the discussion of "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance" by Feldblum (p. 385), why do issues of (a) subjectivity and (b) complexity limit the usefulness of fitting curves to reinsurance data?

Solution S6-47-3. (a) Subjectivity exists in deciding which family of curves to use. In reinsurance analysis, analyzing the "tail" (greater extremes) of losses is particularly important - especially for excess-of-loss reinsurance in high layers. Different families of distributions can be similarly reasonable in estimating "usual" severities of a particular kind of loss but might diverge dramatically in their tail estimates.

(b) Complexity may occur in explaining curve-fitting techniques to representatives of the primary insurer and the reinsurance underwriter, and it is possible to derive different rates from different curve-fitting techniques, making it difficult to select rates on their basis.

Problem S6-47-4. Similar to Question 35 from the 2008 CAS Exam 6. A primary insurer has 20 claims, of which 15 will eventually develop to $80,000, while 5 will develop to $1,500,000.

The primary insurer has an excess-of-loss reinsurance treaty of $1,000,000 in excess of $500,000.

(a) Assuming that the primary insurer establishes a case reserve for each claim using the mode of claim amounts, what will be the primary insurer's IBNR reserve?

(b) What will be the reinsurer's IBNR, following the assumption in part (a)?

(c) Why does the selection of the mode for the case reserve often lead to results of the kind in parts (a) and (b)?

Solution S6-47-4. (a) Reserving at the mode would imply that each claim has a case reserve of $80,000, for a total case reserve of 20*80000 = 1600000. Of the five claims of $1,500,000 each, the primary insurer will only be responsible for the first $500,000 for each, meaning that the IBNR is 5*(500000 - 80000) = **$2,100,000.**

(b) The reinsurer will be responsible for the $1,000,000 in excess of the retention for each of the five claims of $1,500,000. Thus, the reinsurer's IBNR is 5*1000000 = **$5,000,000.**

(c) When the primary insurer reserves at the mode, that mode is typically below the primary insurer's retention. The reinsurer does not typically find out about a particular claim until that claim exceeds the retention. If all claims are assumed to be at the mode, then the reinsurer does not find out about them at all until the large losses corresponding to a few claims are actually reported. Thus, all of these claims' amounts in excess of the retention are part of the reinsurer's IBNR. By contrast, the primary insurer also has a case reserve equal to (Number of claims)*(Mode of claim amount), and the primary insurer's IBNR is limited due to the retention per the excess-of-loss treaty.

Problem S6-47-5. Similar to Question 8 from the 2009 CAS Exam 6. You have the following information about cumulative paid losses for an insurer, expressed in the format (Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months).

Cumulative Paid Losses
AY 2044: (40000, 60000, 80000, 90000)
AY 2045: (45000, 64000, 86000)
AY 2046: (51000, 69000)
AY 2047: (80000)

You also have the following information about on-level premiums and exposures:

AY 2044: On-level premium: 60000; Exposures: 300; Average premium: 200
AY 2045: On-level premium: 66000; Exposures: 327; Average premium: 201.83
AY 2046: On-level premium: 70000; Exposures: 350; Average premium: 200
AY 2047: On-level premium: 80000; Exposures: 404; Average premium: 198.02

Use two diagnostics to show why it would not be proper to use the paid development method to estimate ultimate losses for AY 2047.

Solution S6-47-5. First we consider the ratio of cumulative paid loss to on-level premium at 12 months of development:

Ratios of Cumulative Paid Loss to On-Level Premium
AY 2044: 40000/60000 = 0.666667
AY 2045: 45000/66000 = 0.681818
AY 2046: 51000/70000 = 0.728571
AY 2047: 80000/80000 = 1

We note the immense increase in the ratio of cumulative paid loss to on-level premium in AY 2047. It is possible that faster claim settlement in part contributed to this, in which case the paid development method would overestimate the claim development factors and thus the ultimate loss for AY 2047.

We can also compare the trend in exposures to the trend in paid losses at 12 months.

Exposure Trend - AY 2044 to AY 2045: 327/300 - 1 = +9%.
Exposure Trend - AY 2045 to AY 2046: 350/327 - 1 = +7.03%
Exposure Trend - AY 2046 to AY 2047: 404/350 - 1 = +15.43%

Paid Loss Trend - AY 2044 to AY 2045: 45000/40000 - 1 = +12.5%
Paid Loss Trend - AY 2045 to AY 2046: 51000/45000 - 1 = +13.33%
Paid Loss Trend - AY 2046 to AY 2047: 80000/51000 - 1 = +56.87%

The paid loss trend is higher than the exposure trend in all cases, but the difference is especially pronounced from AY 2046 to AY 2047. Since average premiums are close to constant, one can infer that a faster settlement rate has been manifested in later accident years, especially in AY 2047. Claim development factors using the chain ladder method will overestimate the ultimate loss in this situation.

Section 48

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 21

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:

Casualty Actuarial Society Enterprise Risk Management Committee, "Overview of Enterprise Risk Management," Casualty Actuarial Society *Forum*, Summer 2003, Section 3 and Appendix B.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2008 Exam 6.

Slywotzky, A.J., and Drzik, J., "Countering the Biggest Risk of All," *Harvard Business Review*, April 2005, Harvard Business School Publishing.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-48-1. Similar to Question 36 from the 2008 CAS Exam 6. You have the following information as of December 31, 2050:

Earned Premium for Calendar/Accident Year 2047: 35055
Earned Premium for Calendar/Accident Year 2048: 38899
Earned Premium for Calendar/Accident Year 2049: 37600
Earned Premium for Calendar/Accident Year 2050: 41400

Adjusted Premium for Calendar/Accident Year 2047: 34400
Adjusted Premium for Calendar/Accident Year 2048: 36011

Adjusted Premium for Calendar/Accident Year 2049: 37000
Adjusted Premium for Calendar/Accident Year 2050: 40000

Aggregate Reported Loss for Calendar/Accident Year 2047: 22222
Aggregate Reported Loss for Calendar/Accident Year 2048: 16244
Aggregate Reported Loss for Calendar/Accident Year 2049: 12522
Aggregate Reported Loss for Calendar/Accident Year 2050: 4040

Aggregate Loss Report Lag for Calendar/Accident Year 2047: 0.95
Aggregate Loss Report Lag for Calendar/Accident Year 2048: 0.75
Aggregate Loss Report Lag for Calendar/Accident Year 2049: 0.60
Aggregate Loss Report Lag for Calendar/Accident Year 2050: 0.20

Note that "report lag" here refers to the proportion of losses assumed to be *already reported*, not the proportion assumed to have yet to be reported.

Use the Cape Cod method to calculate IBNR as of December 31, 2050.

Solution S6-48-1. We can calculate the expected loss ratio (ELR) as follows:

ELR = (Sum of aggregate reported losses for each year)/(Sum of used-up premiums for each year), where the used-up premium for each year is (Adjusted Premium)*(Aggregate Loss Report Lag).

Thus, ELR = (22222 + 16244 + 12522 + 4040)/(0.95*34400 + 0.75*36011 + 0.60*37000 + 0.20*40000) = 0.6121823486.

By the Cape Cod Method, IBNR = ELR*(Sum of adjusted premiums for each year) - (Sum of aggregate reported losses for each year) =
0.6121823486*(34400 + 36011 + 37000 + 40000) - (22222 + 16244 + 12522 + 4040) =
35214.4122 = **$35,214.41**.

Problem S6-48-2. Similar to Question 37 from the 2008 CAS Exam 6. Provide two advantages and disadvantages of using a reinsurer's own experience, as opposed to using reinsurance industry data, in applying the Stanard-Bühlmann (Cape Cod) reserving technique.

Solution S6-48-2. This is a sample answer, and other valid answers are possible.

Advantages:
1. The reinsurer may have a history of rate changes that is not reflected in industry data.
2. The reinsurer may have a unique book of business that consists of a mix of treaty types not adequately represented in industry data.

Disadvantages:
1. The reinsurer's experience may be sparse and therefore not credible.

2. The reinsurer's data may be less stable than industry data, and a lot of the volatility in reinsurer data may be "noise" that would not provide a meaningful reserve estimate.

Problem S6-48-3. Similar to Question 38 from the 2008 CAS Exam 6. You have the following information about a reinsurance treaty:

The minimum ceding commission is 10% at an 80% loss ratio.
The commission then slides 1:1 to a 30% commission at a 60% loss ratio.
The commission then slides 1:4 to a maximum 40% commission at a 20% loss ratio.

The probability of a loss ratio being less than 20% is 0.3, and the average loss ratio in the range is 15%.
The probability of a loss ratio being between 20% and 60% is 0.5, and the average loss ratio in the range is 45%.
The probability of a loss ratio being between 60% and 80% is 0.15, and the average loss ratio in the range is 72%.
The probability of a loss ratio being greater than 80% is 0.05, and the average loss ratio in the range is 90%.

What is the expected ceding commissions?

Solution S6-48-3. While it is tempting to simply calculate the expected loss ratio and then calculate the corresponding ceding commission, there is not a one-to-one correspondence between loss ratio and ceding commission. The correct approach is to calculate the expected ceding commission for each loss ratio first, and then take the probability-weighted average of these expected commissions.

For the loss ratio range less than 20%, the expected ceding commission is 40%.
For the loss ratio range between 20% and 60%, the expected ceding commission is 30% + (1/4)*(60% - 45%) = 33.75%.
For the loss ratio range between 60% and 80 , the expected ceding commission is 10% + (80%-72%) = 18%.
For the loss ratio range greater than 80%, the expected ceding commission is 10%.

Thus, the overall expected ceding commission is 0.3*40% + 0.5*33.75% + 0.15*18% + 0.05*10% = **32.075%.**

Problem S6-48-4. Similar to Question 39 from the 2008 CAS Exam 6. A $400,000 in excess of $300,000 property per-occurrence excess-of-loss treaty is effective from January 1, 2044, to December 31, 2044. The annual ground-up loss trend is +2%. The loss development factors applicable to *the treaty layer* of experience are as follows:

Loss Development Factors for Treaty Layer of Losses

For AY 2039: 1.04
For AY 2040: 1.23

For AY 2041: 1.44
For AY 2042: 1.55
For AY 2043: 2.03

You are analyzing the following ground-up losses:

Loss 1 occurred on January 1, 2039, and has ground-up amount of $400,000.
Loss 2 occurred on July 1, 2041, and has ground-up amount of $250,000.
Loss 3 occurred on July 1, 2043, and has ground-up amount of $450,000.

What is the total amount of trended ultimate losses in the treaty layer?

Solution S6-48-4. First, we consider the trend factor that applies to each loss. The trend period is from the date of the loss to the midpoint of the treaty period - July 1, 2044.

For Loss 1, the trend factor is $1.02^{5.5}$, and the trended loss is thus $400000*1.02^{5.5} = 446026.781$, implying that losses in the treaty layer are 146026.781.

For Loss 2, the trend factor is 1.02^{3}, and the trended loss is thus $250000*1.02^{3} = 265302$, implying that there are no losses in the treaty layer.

For Loss 3, the trend factor is 1.02, and the trended loss is thus 450000*1.02 = 459000, implying that losses in the treaty layer are 159000.

For Loss 1, we develop losses in the treaty layer to ultimate: 146026.781*1.04 = 151867.8522.

For Loss 3, we develop losses in the treaty layer to ultimate: 159000*2.03 = 322770.

The total amount of trended ultimate losses in the treaty layer is thus 151867.8522 + 322770 = 474637.8522 = **$474,637.85**.

Problem S6-48-5. Similar to Question 40 from the 2008 CAS Exam 6. An Enterprise Risk Management approach has identified the following risks for a company:

Product obsolescence: Probability of 10%, income impact of 40%.
Customer priority shift: Probability of 20%, income impact of 8%.
Market stagnation: Probability of 5%, income impact of 30%
Entrance of a new competitor: Probability of 15%, income impact of 10%.

(a) Which of the above is the greatest risk to the company, according to the given information?

(b) Select two pairs of risks that are each correlated, either positively or negatively, and explain why they are correlated.

(c) How would the correlations in part (b) affect the selection of the second-greatest risk?

Solution S6-48-5.

(a) We consider the expected income impact of each risk (Probability*Income impact):

Product obsolescence: 10%*40% = 4%
Customer priority shift: 20%*8% = 1.6%
Market stagnation: 5%*30% = 1.5%
Entrance of a new competitor: 15%*10% = 1.5%

Product obsolescence is the greatest risk, with a 4% expected income impact.

(b) Two possible correlated pairs are as follows. (Other valid answers are possible.)

1. Product obsolescence and customer priority shift may be positively correlated. A product may become obsolete because customers no longer demand the same kind of functionality, or customers' priorities may change because more advanced technological possibilities are available for meeting the same needs.

2. Market stagnation and entrance of a new competitor are negatively correlated, since a competitor's entry suggests that the market is dynamic enough to attract competition.

(c) A risk that is positively correlated with the greatest risk is more likely to be the second-greatest risk. Even without any correlation, the customer priority shift risk has the second-highest expected income impact. Considering the correlation, one may have more evidence to suggest that the customer priority shift risk is indeed the second-greatest risk.

Section 49

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 22

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that will appear on Exam 6. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Sources:
Blanchard, R.S., "Accounting Concepts for the Actuary," CAS Study Note, June 2003.

Casualty Actuarial Society Enterprise Risk Management Committee, "Overview of Enterprise Risk Management," Casualty Actuarial Society *Forum*, Summer 2003, Section 3 and Appendix B.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2008 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-49-1. Similar to Question 23 from the 2008 CAS Exam 6.

(a) What is the "10-10" rule for risk transfer for reinsurance contracts?

(b) Give an example of a *quota share* reinsurance treaty for which the "10-10" rule does not hold, but which nonetheless entails significant risk transfer.

(c) Give an example of an *excess-of-loss* reinsurance treaty for which the "10-10" rule does not hold, but which nonetheless entails significant risk transfer.

Solution S6-49-1. (a) The "10-10" rule states that, in order for risk transfer to occur under a reinsurance contract, the reinsurer must have a 10% loss with at least a 10% probability - i.e., a 10% or greater loss exists at the 90th percentile of the loss distribution.

(b) A quota share can be with respect to a book of business of a highly profitable primary insurer that only experiences a loss less than 10% of the time. Since the reinsurer shares in the primary insurer's fortunes, even a substantial quota share can result in the reinsurer experiencing profits 90% or more of the time. Thus, the "10-10" rule fails, but there is unambiguous risk transfer, as the reinsurer is assuming responsibility for a substantial share of the primary insurer's book of business.

(c) An excess-of-loss treaty, especially a catastrophe treaty, may have a high attachment point that is rarely reached, but, when reached, is often exceeded substantially. For instance, assume a catastrophic event that occurs with 5% probability and brings about $4,000,000 in losses. If an excess-of-loss treaty has an attachment point of $1,000,000, then the reinsurer will have no losses 95% of the time and immense $3,000,000 losses 5% of the time. This is clearly a situation of risk transfer, but the "10-10" rule fails.

Problem S6-49-2. Similar to Question 24 from the 2008 CAS Exam 6. How might the "Notes and Disclosures" section of a financial report address the following topics: **(a)** uncertainty, **(b)** future developments, and **(c)** accounting practices?

Solution S6-49-2. The "Notes and Disclosures" section of a financial report may **(a)** discuss uncertainty associated with various insurance liability estimates and values that may not be possible to estimate with precision, **(b)** address estimates of future events and earnings that have not yet occurred as of the report's publication date, and **(c)** discuss what accounting policies were used in the financial statements' creation. (See Blanchard, "Accounting Concepts for the Actuary", p. 6.)

Problem S6-49-3. Similar to Question 26 from the 2008 CAS Exam 6. A quota share treaty has the following conditions:

Ceded share: 44%
Ceding commission: 30%
Subject earned premium: $500,000
Ultimate loss and ALAE on subject business: $250,000

(a) What is the net earned premium for the ceding insurer?

(b) What is the ultimate retained loss for the ceding insurer?

(c) What is the reinsurer's gain or loss?

(d) What are two reasons for which the primary insurer may be interested in this quota share treaty?

Solution S6-49-3. (a) The net earned premium for the ceding insurer is the subject earned premium multiplied by (1 - ceded share): 500000*(1-0.44) = **$280,000.**

(b) The ultimate retained loss for the ceding insurer is the ultimate loss and ALAE multiplied by (1 - ceded share): 250000*(1-0.44) = **$140,000.**

(c) The reinsurer receives 500000*0.44 = $220,000 in premium and also pays 30% of this amount as ceding commission, thus keeping only 220000*0.7 = $154,000. The reinsurer's share of losses is 250000*0.44 = $110,000, meaning that the reinsurer's net result is $154,000 - $110,000 = **a gain of $44,000.**

(d) The primary insurer may be interested in this quota share treaty to (1) obtain surplus relief for a rapidly growing book of business and (2) increase large line capacity and insure risks with a higher total insured value than would otherwise be possible.

Problem S6-49-4. Similar to Question 27 from the 2008 CAS Exam 6. You have the following information for an insurer by accident year (AY).

AY 2042
Reported loss as of December 31, 2044: 20300
Selected IBNR as of December 31, 2044: 4000
Reported loss as of December 31, 2045: 23500

AY 2043
Reported loss as of December 31, 2044: 12444
Selected IBNR as of December 31, 2044: 10340
Reported loss as of December 31, 2045: 21230

AY 2044
Reported loss as of December 31, 2044: 6005
Selected IBNR as of December 31, 2044: 18000
Reported loss as of December 31, 2045: 13000

You also have the following selected reported loss development factors to ultimate:
From 12 months: 2.602
From 24 months: 1.983
From 36 months: 1.252
From 48 months: 1.055

(a) Based on the selected IBNR and the selected reported loss development factors, what is the expected loss emergence during calendar year 2045 for AY 2042 through AY 2044?

(b) Based on the data and calculations, what conclusions can be drawn regarding the accuracy of the expected loss emergence estimate, compared to actual loss emergence?

Solution S6-49-4. (a) We calculate the expected loss emergence for each accident year as (Selected IBNR)*((LDF as of Dec. 31. 2044)/(LDF as of Dec. 31. 2045) - 1)/((LDF as of Dec. 31. 2044) - 1).

Expected loss emergence for AY 2042: 4000*(1.252/1.055 - 1)/(1.252 - 1) = **2963.965007**.
Expected loss emergence for AY 2043: 10340*(1.983/1.252 - 1)/(1.983 - 1) = **6141.579373**.
Expected loss emergence for AY 2044: 18000*(2.602/1.983 - 1)/(2.602 - 1) = **3504.340484**.

(b) We calculate the actual loss emergence.

For AY 2042: 23500 - 20300 = 3200.
For AY 2043: 21230 - 12444 = 8786.
For AY 2044: 13000 - 6005 = 6995.

For each accident year, actual loss emergence considerably exceeds expected loss emergence. Thus, we can conclude that the selected reserving method is prone to underreserving.

Problem S6-49-5. Similar to Question 41 from the 2008 CAS Exam 6.

(a) Why does a focus solely on increasing the stock price of a company inconsistent with the definition of Enterprise Risk Management (ERM)?

(b) What broad categories of risk are considered under an ERM approach?

(c) With respect to financial risks, why does a focus solely on avoiding or reducing these risks not qualify as ERM?

Solution S6-49-5. (a) The definition of ERM requires both a short-term and a long-term focus on the firm's value - and the focus is on value for all stakeholders, not just shareholders. Focus on the stock price only reflects a short-term value for stockholders.

(b) The four categories of risk considered under an ERM approach are (1) operational risk - related to business operations such as human resources, leadership, information technology, and information reporting; (2) hazard risk - physical damage and injuries and the resulting loss of revenue; (3) financial risk - including commodity risk, credit risk, foreign exchange risk, and risk posed by macroeconomic factors such as inflation; and (4) strategic risk - risks arising from such phenomena as competition, reputational damage, technological innovation, and regulatory and political trends. (See CAS Overview of Enterprise Risk Management, p. 111.)

(c) ERM focuses not just on reducing or avoiding risks, but also on exploiting them when this can raise the value of the firm. Some exposure to risk may be preferable for increasing the firm's value than no exposure at all. Calculated risks in the financial market may thus be pursued and not avoided.

Section 50

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 23

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Sources:
Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Harrison, C.M., *Reinsurance Principles and Practices* (First Edition), American Institute for Chartered Property Casualty Underwriters/Insurance Institute of America, 2004, Chapters 1, 2, 4, 8, 9, and 10.

Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-50-1. Similar to Question 28 from the 2008 CAS Exam 6. A primary insurer purchased a $500,000 in excess of $300,000 per-occurrence excess-of-loss reinsurance treaty with an annual aggregate deductible of $600,000. The following occurrences are subject to the treaty:

Occurrence A: Loss of $210,000
Occurrence B: Loss of $700,000
Occurrence C: Loss of $550,000
Occurrence D: Loss of $1,600,000
Occurrence E: Loss of $320,000
Occurrence F: Loss of $480,000

What is the net loss retained by the primary insurer?

Solution S6-50-1. The aggregate deductible of $600,000 means that the primary insurer must pay $600,000 in *excess-layer* losses before the reinsurer pays anything.

We evaluate each occurrence in turn.

Occurrence A is below the primary insurer's retention, so the primary insurer will be responsible for the entire $210,000, with no contribution to the aggregate deductible.

For Occurrence B, $400,000 is above the primary insurer's retention, and, while the primary insurer will be liable for the entire amount of the loss - $700,000 - because of the aggregate deductible, this deductible will be reduced to 600000 - 400000 = $200,000.

For Occurrence C, $250,000 is above the primary insurer's retention. $200,000 of this amount exhausts the aggregate deductible, meaning that the reinsurer will pay the rest. The primary insurer will be liable for 300000 (retention) + 200000 (rest of deductible) = $500,000.

For Occurrence D, the reinsurer is only liable for the treaty layer of losses, or $500,000. The primary insurer is liable for the remaining $1,100,000.

For Occurrences E and F each, the primary insurer is only liable for the retention of $300,000.

Thus, the total net loss retained by the primary insurer is

210000 + 700000 + 500000 + 1100000 + 300000*2 = **$3,110,000.**

Problem S6-50-2. Similar to Question 29 from the 2008 CAS Exam 6. You are analyzing a $600,000 excess of $200,000 per-occurrence excess-of-loss reinsurance treaty, with the following occurrences subject to the treaty:

Occurrence G: Loss of $400,000; Incurred ALAE of $50,000
Occurrence H: Loss of $130,000; Incurred ALAE of $200,000
Occurrence I: Loss of $800,000; Incurred ALAE of $90,000
Occurrence J: Loss of $530,000; Incurred ALAE of $400,000
Occurrence K: Loss of $1,000,000; Incurred ALAE of $120,000

(a) What is the total loss amount ceded to the treaty if incurred ALAE is to be considered a part of the loss?

(b) What is the total loss amount ceded to the treaty if incurred ALAE is to be shared pro rata with the loss?

Solution S6-50-2. (a) If incurred ALAE is to be considered a part of the loss, then the losses subject to the treaty are as follows:

Occurrence G: Loss of $450,000 → Ceded amount is $250,000.
Occurrence H: Loss of $330,000 → Ceded amount is $130,000.

Occurrence I: Loss of $890,000 → Ceded amount is $600,000.
Occurrence J: Loss of $930,000 → Ceded amount is $600,000.
Occurrence K: Loss of $1,120,000 → Ceded amount is $600,000.

Total ceded amount: **$2,180,000.**

(b) If incurred ALAE is to be shared pro rata with loss, this means that the ratio of the treaty-covered loss to the total loss is also the ratio of the treaty-covered ALAE to the total ALAE.

Occurrence G: Loss of $400,000 → Treaty-covered loss of $200,000 → Treaty-covered ALAE of $50,000*(200/400) = $25,000 → Total ceded amount of $225,000.

Occurrence H: Loss of $130,000 → Treaty-covered loss of $0 → Treaty-covered ALAE of $0 → Total ceded amount of $0.

Occurrence I: Loss of $800,000 → Treaty-covered loss of $600,000 → Treaty-covered ALAE of 90000*(600/800) = $67,500 → Total ceded amount of $667,500.

Occurrence J: Loss of $530,000 → Treaty-covered loss of $330,000→ Treaty-covered ALAE of 400000*(330/530) = $249,056.60 → Total ceded amount of $579,056.60.

Occurrence K: Loss of $1,000,000 → Treaty-covered loss of $600,000 → Treaty-covered ALAE of 120000*(600/1000) = $72,000 → Total ceded amount of $672,000.

Total ceded amount for all occurrences: **$2,143,556.60.**

Problem S6-50-3. Similar to Question 28 from the 2009 CAS Exam 6. Consider the methods of *experience rating* and *exposure rating* in reinsurance. Which of these methods can produce the problem of *free cover* and why?Which of these methods can be applied to avoid it and how?

Solution S6-50-3. Experience rating can produce the problem of free cover if the losses on the basis of which reinsurance rates are determined do not trend into the highest layers of coverage being offered. **Exposure rating** can avoid this problem when applied to these upper layers. Experience rating can still be used for the lower layers where actual loss data exists. Exposure rating can be used to develop a relativity for the upper layers which is multiplied by the experience-based rate to get the total rate.

Problem S6-50-4. Similar to Question 31 from the 2009 CAS Exam 6.

You have the following information as of December 31, 2022:

Earned Premium for Calendar/Accident Year 2020: 11500
Earned Premium for Calendar/Accident Year 2021: 12024
Earned Premium for Calendar/Accident Year 2022: 14444

Adjusted Premium for Calendar/Accident Year 2020: 11000
Adjusted Premium for Calendar/Accident Year 2021: 12000
Adjusted Premium for Calendar/Accident Year 2022: 13000

Aggregate Reported Loss for Calendar/Accident Year 2020: 9700
Aggregate Reported Loss for Calendar/Accident Year 2021: 6400
Aggregate Reported Loss for Calendar/Accident Year 2022: 4100

Aggregate Loss Report Lag for Calendar/Accident Year 2020: 0.85
Aggregate Loss Report Lag for Calendar/Accident Year 2021: 0.66
Aggregate Loss Report Lag for Calendar/Accident Year 2022: 0.42

Note that "report lag" here refers to the proportion of losses assumed to be *already reported*, not the proportion assumed to have yet to be reported.

Use the Cape Cod method to calculate IBNR as of December 31, 2022.

Solution S6-50-4. We can calculate the expected loss ratio (ELR) as follows:

ELR = (Sum of aggregate reported losses for each year)/(Sum of used-up premiums for each year), where the used-up premium for each year is (Adjusted Premium)*(Aggregate Loss Report Lag).

Thus, ELR = (9700 + 6400 + 4100)/(11000*0.85 + 12000*0.66 + 13000*0.42) = 0.8886933568.

By the Cape Cod Method, IBNR = ELR*(Sum of adjusted premiums for each year) - (Sum of aggregate reported losses for each year) = 0.8886933568*(11000 + 12000 + 13000) - (9700 + 6400 + 4100) = 11792.96084 = **$11,792.96.**

Problem S6-50-5. Similar to Question 33 from the 2009 CAS Exam 6. What is aggregate excess-of-loss reinsurance? Name one goal of reinsurance for which aggregate excess-of-loss reinsurance is the best possible option. Name two other goals of reinsurance which aggregate excess-of-loss reinsurance does not fulfill as well.

Solution S6-50-5. Aggregate excess-of-loss reinsurance is a type of reinsurance that pays for all of the primary insurer's losses in a given time period (e.g., a year) once the sum of the primary insurer's losses exceeds the aggregate treaty retention. This type of reinsurance is the best at **stabilizing loss experience,** since the primary insurer is assured that its losses will not exceed a certain magnitude. It is not as good at **providing surplus relief** (since the aggregate retention will probably be set so that exceeding it is not probable), or **facilitating withdrawal from a market**, as the primary insurer still retains losses below the aggregate retention.

Section 51

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 24

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

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Sources:
Blanchard, R.S., "Basic Insurance Accounting-Selected Topics," CAS Study Note, June 2007, pp. 1-20.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2008 Exam 6 and 2009 Exam 6.

Steeneck, L., "Commutation of Claims," CAS Study Note, 1998.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-51-1. Similar to Question 31 from the 2008 CAS Exam 6. You have the following information about a book of business subject to a quota share reinsurance treaty:

Accident Year 2024
Historical Earned Premium: 2020
Reported Losses: 1414
On-Level Factor: 1.231
Loss Development Factor: 1.024
Loss Trend Factor: 1.444
Property Value Inflation Factor: 1.234

Accident Year 2025
Historical Earned Premium: 2222
Reported Losses: 1600
On-Level Factor: 1.141
Loss Development Factor: 1.232
Loss Trend Factor: 1.255
Property Value Inflation Factor: 1.124

Accident Year 2026
Historical Earned Premium: 2500
Reported Losses:2235
On-Level Factor: 1.055
Loss Development Factor: 1.558
Loss Trend Factor: 1.088
Property Value Inflation Factor: 1.013

Of these losses, the Accident Year 2026 reported losses include 1200 in catastrophe losses.

The reinsurer also has the following expenses as a percentage of the reinsurance premium:

Administrative Expenses: 4%
Brokerage Fees: 2.5%
Unallocated Expenses: 0.5%

(a) Find the ultimate non-catastrophe loss ratio for these three years of experience.

(b) A catastrophe model suggests that a 10% catastrophe load should be added as a percentage of non-catastrophe losses. Calculate the expected ultimate loss ratio for the three-year period.

(c) If the reinsurer wishes to achieve a 94% combined ratio on the treaty, should the primary insurer's request for a 15% ceding commission be accepted? Justify your answer.

Solution S6-51-1.

(a) We need to adjust the premiums and losses. The losses for each year need to be multiplied by the development factor and the trend factor. The premiums need to be multiplied by the on-level factor and the property value inflation factor (which affects premiums only, since the loss trend factor reflects loss inflation). We do not consider catastrophe losses, so AY 2026 reported losses become 2235 - 1200 = 1035.

Total adjusted losses are 1414*1.024*1.444 + 1600*1.232*1.255 + 1035*1.558*1.088 = 6319.108224.
Total adjusted premiums are 2020*1.231*1.234 + 2222*1.141*1.124 + 2500*1.055*1.013 = 8589.956028.
The ultimate non-catastrophe loss ratio is 6319.108224/8589.956028 = **0.7356391818.**

(b) To get the ultimate loss ratio, we multiply the non-catastrophe loss ratio by 1.1: 0.7356391818*1.1 = **0.8092031.**

(c) The reinsurer's combined ratio will be the ultimate loss ratio, plus all of its expense ratios, plus the ceding commission: 0.8092031 + 0.04 + 0.025 + 0.005 + 0.15 = 1.0292031 > 0.94. Since this combined ratio exceeds the reinsurer's target ratio of 94%, the reinsurer **should not accept** the request to pay a 15% ceding commission.

Problem S6-51-2. Similar to Question 30 from the 2008 CAS Exam 6. Primary Insurer X is considering whether to commute a book of liabilities ceded to Reinsurer Y. The following information is known:

Nominal value of liabilities: $353,535
Present value of liabilities: $300,400
Current average IRS discount factor: 0.75
Present value of IRS remainder unwind: $52,424
Marginal tax rate: 35%

(a) Calculate the reinsurer's ambivalence point.

(b) Would the reinsurer regard a $260,000 offer to be better or worse than break-even?

(For a discussion of this type of situation, see Steeneck, pp. 15-16.)

Solution S6-51-2.

(a) The ambivalence point is the point at which the reinsurer would be indifferent between commuting the liability and keeping it on its books.

We calculate the IRS-discounted value of the nominal liabilities: 353535*0.75 = 265151.25.
We calculate the tax on the present value of the unwind: 0.35*52424 = 18348.4.
The *basis* for the ambivalence point is the present value of liabilities, minus the tax on the present value of the unwind: 300400 - 18348.4 = 282051.6.

The ambivalence point can be found via the equation
Ambivalence Point = (Basis for Ambivalence Point - (Nominal IRS-Discounted Liabilities)*(Tax Rate))/(1 - Tax Rate) = (282051.6 - 265151.25*0.35)/0.65 = 291151.7885 = **$291,151.79**.

(b) Since the reinsurer holds the liabilities, it would need to pay to commute them. The ambivalence point of $291,151.79 indicates the highest price the reinsurer would be willing to pay. Since $260,000 < $291,151.79, this offer is **better than break-even** for the reinsurer.

Problem S6-51-3. Similar to Question 24 from the 2009 CAS Exam 6. A primary insurer is currently undergoing liquidation due to insolvency. A $800,000 loss is outstanding, and the liquidator has determined that only 70% of the loss will be paid. The primary insurer is party to a

per-occurrence excess-of-loss reinsurance treaty of $500,000 in excess of $300,000. If the treaty has an insolvency clause without diminution, how much money would the primary insurer save compared to a situation in which the treaty does not have such a clause.

Solution S6-51-3. An insolvency clause without diminution provides that the reinsurer would pay its full obligation under the treaty, irrespective of the primary insurer's insolvency. For the outstanding $800,000 loss, the reinsurer's full obligation is $500,000. The liquidator has determined that only 0.7*800000 = $560,000 of the loss needs to be paid, so, with an insolvency clause without diminution, the primary insurer only has to pay the remaining $60,000. Without such a clause, the primary insurer still retains the amount of the adjusted loss below the attachment point, or $300,000, with the reinsurer paying the remaining $260,000. The savings for the primary insurer of having an insolvency clause without diminution is thus $300,000 - $60,000 = **$240,000.**

Problem S6-51-4. Similar to Question 22 from the 2009 CAS Exam 6. A reinsurance treaty's period is from January 1, 2044, to December 31, 2044. You are aware of the following losses:

Loss A: Renewal policy, effective October 2, 2043, date of loss of October 9, 2043, treaty-covered loss amount of $3,440
Loss B: New policy, effective November 24, 2043, date of loss of March 5, 2044, treaty-covered loss amount of $11,000
Loss C: New policy, effective December 6, 2043, date of loss of December 26, 2043, treaty-covered loss amount of $5,656
Loss D: Renewal policy, effective December 15, 2043, date of loss of February 6, 2044, treaty-covered loss amount of $7,000
Loss E: New policy, effective April 5, 2044, date of loss October 17, 2044, treaty-covered loss amount of $2,010
Loss F: Renewal policy, effective June 16, 2044, date of loss March 6, 2045, treaty-covered loss amount of $12,000

(a) Define "risks attaching basis" and find the amount of losses covered if this treaty were written on a risks attaching basis.

(b) Define "losses occurring basis" and find the amount of losses covered if this treaty were written on a losses occurring basis.

(c) Define "policies issued basis" and find the amount of losses covered if this treaty were written on a policies issued basis.

(d) Define "in-force policies basis" and find the amount of losses covered if this treaty were written on an in-force policies basis.

Solution S6-51-4.

(a) *Risks attaching basis:* Only losses on policies written or renewed during the treaty period are covered. This means that only policies written or renewed during 2044 will have losses covered.

Thus only losses E and F will be covered, for a total covered amount of $2,010 + $12,000 = **$14,010.**

(b) *Losses occurring basis:* Only losses occurring during the treaty period are covered. Thus only losses B, D, and E will be covered, for a total covered amount of $11,000 + $7,000 + $2,010 = **$20,010.**

(c) *Policies issued basis:* Only losses on new policies written during the treaty period are covered. Thus only losses on policies written in 2044 are covered. Only loss E qualifies, and the loss amount is **$2,010.**

(d) *In-force policies basis:* Only losses on policies already in force at the start of the treaty period are covered. Thus, only losses A, B, C, and D (from policies written or renewed before 2044) are covered, for a total loss amount of $3,440 + $11,000 + $5,656 + $7,000 = **$27,096.**

Problem S6-51-5. Similar to Question 19 from the 2009 CAS Exam 6.

(a) Define the *bank deposit approach,* the *prospective approach,* and the *retrospective approach* for deposit accounting.

(b) Give three general situations for which deposit accounting would be required instead of reinsurance accounting.

(See Blanchard, "Basic Insurance Accounting - Selected Topics", p. 19.)

Solution S6-51-5.

(a) Under the **bank deposit approach,** the deposit amount grows over time in accord with a fixed, predetermined interest rate and is reduced by the amount of any withdrawals. Under the **prospective approach,** the deposit's current value is equal to the value of future payments. Past payments or the amount of the initial deposit do not matter. Under the **retrospective approach,** the deposit amount depends on the initial deposit, past payments, and the current estimate of future payments. The interest rate is set such that the initial deposit is equal to the discounted value of past payments and estimated future payments (Blanchard, "Basic Insurance Accounting - Selected Topics", p. 19).

(b) Deposit accounting would be required instead of reinsurance accounting in the following situations:
1. There is no risk transfer.
2. There is only transfer of timing risk (when a payment is made) but not amount risk (how much will be paid).
3. Many situations of retroactive reinsurance (Blanchard, "Basic Insurance Accounting - Selected Topics", p. 19).

Section 52

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 25

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Sources: Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Past Casualty Actuarial Society exams: 2009 Exam 6.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-52-1. Similar to Question 10 from the 2009 CAS Exam 6. It is estimated that the expected loss rate for an insurance company is $200 per exposure unit. The company has no exposure prior to 2030. You also have the following information by accident year (AY):

AY 2030
Exposure Units: 2340
Incurred Loss: 400530
Incurred Loss Development Factor to Ultimate: 1.15

AY 2031
Exposure Units: 3000
Incurred Loss: 360470
Incurred Loss Development Factor to Ultimate: 1.45

AY 2032
Exposure Units: 3560
Incurred Loss: 350900
Incurred Loss Development Factor to Ultimate: 1.90

(a) What is the Bornhuetter-Ferguson estimate of IBNR at December 31, 2032, for all accident years?

(b) What is the Cape Cod estimate of IBNR at December 31, 2032, for all accident years?

(c) Based on the given information and your information, which method from parts (a) and (b) would produce the more accurate estimate of IBNR? Why?

Solution S6-52-1.

(a) The Bornhuetter-Ferguson estimate of IBNR does not depend on losses reported to date. For each accident year it is equal to (Expected Loss)*(1 - 1/(LDF to Ultimate). In this case, the expected loss is (Number of Exposures)*(Loss Rate Per Exposure). Thus, the Bornhuetter-Ferguson estimate of IBNR is Σ((Number of Exposures)*(Loss Rate Per Exposure)*(1 - 1/(LDF to Ultimate)) = 2340*200*(1-1/1.15) + 3000*200*(1-1/1.45) + 3560*200*(1-1/1.90) = 584513.5327 = $**584,513.53**.

(b) The Cape Cod method first derives an empirical expected loss rate per exposure by first calculating "used-up" exposures equal to (Number of Exposures)*(1/(LDF to Ultimate)) and then obtaining the loss rate as (Sum of Reported Losses)/(Sum of Used-Up Exposures).

The Cape Cod expected loss rate is thus (400530 + 360470 + 350900)/(2340/1.15 + 3000/1.45 + 3560/1.90) = 186.0163256.

The total Cape Cod IBNR is the Bornhuetter-Ferguson IBNR, multiplied by the ratio of the Cape Cod expected loss rate to the given expected loss rate: 584513.5327*186.0163256/200 = 543645.2981 = $**543,645.30**.

(c) As a diagnostic, we can calculate the expected loss per exposure for each accident year via the chain ladder method as (Reported Losses)*(LDF)/(Number of Exposures):

AY 2030: 400530*1.15/2340 = 196.8416667
AY 2031: 360470*1.45/3000 = 174.2271667
AY 2032: 350900*1.90/3560 = 187.2780899

We note that, based on losses reported to date, the a priori loss rate of $200 per exposure is too high. Changes in the recent loss reporting pattern or the nature of losses have reduced this rate, and the Cape Cod method is more responsive to such changes, as compared to the Bornhuetter-Ferguson method, which relies on predetermined expected losses for its IBNR calculation. Thus, the **Cape Cod method** is preferable.

Problem S6-52-2. Similar to Question 9 from the 2009 CAS Exam 6. The annual loss ratio trend is +3.0%. You are also given on-level earned premiums for each accident year (AY):

AY 2055: On-level premium is 55353.
AY 2056: On-level premium is 62444.
AY 2057: On-level premium is 65725.

Cumulative incurred losses are as follows, expressed in the format
(Amount at 12 months, Amount at 24 months, Amount at 36 months), where applicable:

Cumulative Incurred Losses
AY 2055: (23400, 34440, 40222)
AY 2056: (25650, 37000)
AY 2057: (28000)

The following development factors to ultimate were selected:
12 months to ultimate: 1.333
24 months to ultimate: 1.155
36 months to ultimate: 1.052

(a) Use the expected claims technique to find the IBNR for AY 2057 as of December 31, 2057.

(b) Identify three situations where it might be desirable to use the expected claims technique.

Solution S6-52-2.

(a) We want to calculate the expected ultimate loss ratio for every accident year. We develop and trend the most recent known incurred losses (outermost diagonal of the triangle) and divide them by the on-level earned premium for each accident year.

AY 2055: Expected loss ratio is $40222*1.052*1.03^2 / 55353 = 0.8109847493$.
AY 2056: Expected loss ratio is $37000*1.155*1.03/62444 = 0.7049043943$.
AY 2057: Expected loss ratio is $28000*1.333/65725 = 0.5678813237$.

To get our total expected loss ratio, we can take the arithmetic mean of the three accident-year expected loss ratios: $(0.8109847493 + 0.7049043943 + 0.5678813237)/3 = 0.6945901558$.

Expected ultimate losses for AY 2057 are the AY 2057 on-level earned premium, multiplied by the expected loss ratio: $65725*0.6945901558 = 45651.93799$. To get IBNR, we subtract already reported losses from expected losses: $45651.93799 - 28000 = 17651.93799 =$ **17651.94**.

(b) The expected claims method might be useful in the following situations:

1. A new book of business for which prior experience is not available;
2. A long-tailed book of business at the early stages of development, where there is too much volatility in reported losses to use methods that rely on them;
3. A book of business subject to recent macroeconomic or regulatory changes which render past data largely irrelevant.

Problem S6-52-3. Similar to Question 11 from the 2009 CAS Exam 6. You are given the following information, expressed in the format

(Number at 12 months, Number at 24 months, Number at 36 months) by accident year (AY):

Cumulative Paid Losses
AY 2044: (5505, 6666, 7044)
AY 2045: (5880, 6900)
AY 2046: (5400)

Number of Open Claims
AY 2044: (72, 38, 32)
AY 2045: (70, 34)
AY 2046: (66)

Average Case Reserve
AY 2044: (100, 194, 202)
AY 2045: (99, 180)
AY 2046: (103)

The annual case reserve severity trend is selected to be +2.0%.
The 36-month-to-ultimate incurred loss development factor is selected to be 1.04.

(a) What does the Berquist-Sherman case reserve adjustment do, and what is its purpose?
(b) Use the Berquist-Sherman case reserve adjustment to arrive at ultimate losses for AY 2046.

Solution S6-52-3.

(a) The Berquist-Sherman case reserve adjustment takes the most recent (outermost diagonal) known average case reserves and de-trends them by the annual case reserve severity trend to arrive at average case reserve estimates for the same ages of maturity for experience of prior accident years. This done in order to facilitate the assumption of the same case outstanding adequacy for all calendar years as exists in the current calendar year.

(b) We first de-trend average case outstanding to get the adjusted average case outstanding:

Adjusted Average Case Reserve
AY 2044: $(103/1.02^2, 180/1.02, 202)$
AY 2045: (103/1.02, 180)
AY 2046: (103)

Adjusted Average Case Reserve
AY 2044: (99, 176.47, 202)
AY 2045: (100.98, 180)
AY 2046: (103)

Now we can estimate adjusted reported claims for each time period as (Paid Claims) + (Adjusted Average Case Reserve)*(Number of Open Claims):

Adjusted Reported Claims
AY 2044: (5505 + 99*72, 6666 + 176.47*38, 7044 + 202*32)
AY 2045: (5880 + 100.98*70, 6900 + 180*34)
AY 2046: (5400 + 103*66)

Adjusted Reported Claims
AY 2044: (12633, 13371.86, 13508)
AY 2045: (12948.6, 13020)
AY 2046: (12198)

Using this information, we can calculate weighted-average age-to-age factors for adjusted reported claims:
Factor for 12 months to 24 months: (13371.86 + 13020)/(12633 + 12948.6) = 1.031673547
Factor for 24 months to 36 months: 13508/13371.86 = 1.010181082
Factor for 36 months to ultimate: 1.04 (given)

Factor for 12 months to ultimate: 1.031673547*1.010181082*1.04 = 1.083864183.

Our estimate of ultimate losses for AY 2046 is thus 12198*1.083864183 = 13220.97531 = **13220.98.** (Note that some minor discrepancies with this answer may arise due to rounding.)

Problem S6-52-4. Similar to Question 12 from the 2009 CAS Exam 6. You know that the following incremental paid losses occurred by accident year:

AY 2034: Valuation Date: December 31, 2034; Incremental paid loss: 1140
AY 2034: Valuation Date: December 31, 2035; Incremental paid loss: 240
AY 2035: Valuation Date: December 31, 2035; Incremental paid loss: 210
AY 2034: Valuation Date: December 31, 2036; Incremental paid loss: 140
AY 2035: Valuation Date: December 31, 2036; Incremental paid loss: 1240
AY 2036: Valuation Date: December 31, 2036; Incremental paid loss: 1000

You also know that the 36-month-to-ultimate paid loss development factor is 1.03.

(a) Develop a triangle of cumulative paid losses on the basis of this information.
(b) Use the chain ladder method and volume-weighted-average development factors to estimate the unpaid claim liability for AY 2036 as of December 31, 2036.

Solution S6-52-4. (a) We develop our cumulative paid loss triangle by accident year in the format (Number at 12 months, Number at 24 months, Number at 36 months):

Cumulative Paid Losses
AY 2034: (1140, 1140+240, 1140+240+140)
AY 2035: (210, 210+1240)
AY 2036: (1000)

Cumulative Paid Losses
AY 2034: (1140, 1380, 1520)
AY 2035: (210, 1450)
AY 2036: (1000)

(b) We find the volume-weighted age-to-age factors:

Factor for 12 months to 24 months: (1380 + 1450)/(1140 + 210) = 2.096296296
Factor for 24 months to 36 months: 1520/1380 = 1.101449275
Factor for 36 months to ultimate: 1.03 (given)

Factor for 12 months to ultimate: 2.096296296*1.101449275*1.03 = 2.378232958.

The estimated unpaid claims for AY 2036 are thus 1000*2.378232958 - 1000 = 1378.232958 = **1378.23**.

Problem S6-52-5. Similar to Question 20 from the 2009 CAS Exam 6. You have the following information about Primary Insurer Q, which started business on January 1, 2044.

Calendar year 2044 written premium: $6660
Calendar year 2044 policy acquisition cost: $2000

Here is the primary insurer's balance sheet as of December 31, 2044, *before* any consideration of reinsurance:

Assets
Cash: $8000
Total assets: $8000

Liabilities and Surplus
Unearned premiums: $3350
Loss reserves: $2500
Total liabilities: $5850
Surplus:$2150
Total liabilities and surplus: $8000

The primary insurer enters into a 60% quota share reinsurance treaty on a risk attaching basis. The reinsurer pays the primary insurer a ceding commission of 25%. There are no claim payments as of December 31, 2044.

(a) Create the statutory balance sheet of Primary Insurer Q *net* of reinsurance as of December 31, 2044.

(b) What is the change in Primary Insurer Q's ratio of net written premium to surplus as a result of the reinsurance treaty.

Solution S6-52-5.

(a) The reinsurer receives 60% of all premiums and losses. This means that the reinsurer gets 0.6*6600 = 3960 of written premiums. But the reinsurer pays 25% of this amount, or 0.25*3960 = 990, to Primary Insurer Q as ceding commission. The resulting amount of the primary insurer's cash is thus 8000 - 3960 + 990 = 5030.

Unearned premiums are reduced to 0.4*3350 = 1340, and loss reserves are reduced to 0.4*2500 = 1000.

Thus, we have the following statutory balance sheet:

Assets
Cash: $5030
Total assets: $5030

Liabilities and Surplus
Unearned premiums: $1340
Loss reserves: $1000
Total liabilities: $2340
Surplus:$2690
Total liabilities and surplus: $5030

(b) Before the treaty, the written-premium-to-surplus ratio is 6660/2150 = 3.097674419.

The net written premium from the reinsurance treaty is 6660*0.4 = 2664, and so the post-treaty net-written-premium-to-surplus ratio is 2664/2690 = 0.9903345725.

The change in the ratio is 0.9903345725/3.097674419 - 1 = **-68.02973978%.**

Section 53

Principles and Criteria of Insurance Accounting

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Source:

Blanchard, R.S., "Accounting Concepts for the Actuary," CAS Study Note, June 2003.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-53-1. (a) Identify and briefly describe the six criteria for accounting information discussed by Blanchard in "Accounting Concepts for the Actuary", pp. 1-4.

(b) Discuss a possible tradeoff that might exist between two of these criteria.

(c) Why might application of the principle of conservatism be difficult in some circumstances?

Solution S6-53-1. (a) The six criteria for accounting information discussed by Blanchard are as follows:

1. **Relevance:** The information should be timely, should have predictive value, and should provide useful feedback on past decisions of the users of the information.
2. **Reliability:** The information should represent what it claims to represent, be independently verifiable, and contain all relevant material facts that assure its completeness.
3. **Comparability and Consistency:** The information must allow comparisons over time and competing interests. For instance, an accounting paradigm should enable comparisons of performance between a company and its competitor in the same industry.
4. **Lack of Bias:** Either there is no bias or the bias is clearly disclosed, and users with a different bias can adjust the information accordingly. Also, a "prudent" or "conservative" bias is possible.
5. **Cost-Benefit Effectiveness:** The value of the information should exceed the cost in producing it.
6. **Understandability:** The intended users of the information should be able to readily comprehend it.

(b) There may be a tradeoff between **relevance** and **reliability.** For relevant information, there may be too much uncertainty to determine exact values or reliable predictions. Reliable, easily estimated figures may not be relevant to future events or the user's decisions.

(c) It may be difficult to apply the principle of conservatism when overstating or understating a particular estimate could both inflate and deflate the company's results, depending on the situation. For instance, a overstating a claim liability might be conservative when predicting future paid claims, but not conservative when predicting future reinsurance receivables.

Problem S6-53-2. Identify **(a)** three distinct broad accounting paradigms and **(b)** four broad categories of users of accounting information. (See Blanchard, "Accounting Concepts for the Actuary", pp. 4-5.)

Solution S6-53-2. (a) Three distinct broad accounting paradigms are (1) **Generally Accepted Accounting Principles (GAAP)**, (2) **Statutory Accounting Principles (SAP)**, and (3) **specialized tax accounting paradigms.**

(b) Four broad categories of users of accounting information are (1) **current and potential investors and creditors**, (2) **insurance regulators and supervisors**, (3) **management**, and (4) **tax authorities**.

Problem S6-53-3. Describe the GAAP hierarchy of accounting rules. (See Blanchard, "Accounting Concepts for the Actuary", p. 7.)

Solution S6-53-3. In the GAAP hierarchy of accounting rules, the highest authority is the securities regulator of the given jurisdiction, which has the power to add rules and requirements and to veto designated standard setters' rules.

The next-highest authorities are the organizations specified as the accounting standard-setters - such as the International Accounting Standards Board (IASB) in the European Union and the Financial Accounting Standards Board (FASB) in the United States . For privately owned firms, these organizations are at the top of the hierarchy of accounting rules.

Lower on the hierarchy are interpretations of accounting standards, which are not themselves accounting standards. These are issued by the standard-setting organizations when prompt clarification and guidance are needed.

Problem S6-53-4. Briefly describe each of the key issues in each following concepts and/or distinctions discussed by Blanchard, "Accounting Concepts for the Actuary", pp. 8-12:

(a) Fair Value versus Historical Cost

(b) Recognition versus Measurement

(c) Deferral/Matching versus Asset/Liability

(d) Impairment

(e) Revenue Recognition

(f) Reporting Segment

(g) Liquidation versus Going Concern

(h) Change in Accounting Principle versus Change in Accounting Estimate

(i) Principle-based versus Rule-based

Solution S6-53-4. (a) Fair Value versus Historical Cost: Is the asset/liability valued at the price for which it was originally obtained (historical cost) or at the amount for which it could be exchanged today between knowledgeable, willing parties in an arm's length transaction (fair value)?

(b) Recognition versus Measurement: When does an asset or liability first get recorded (recognition) and what amount is assigned to it once it is recorded (measurement)?

(c) Deferral/Matching versus Asset/Liability: Are expenses and revenues recognized as occurring at the same time (deferral-matching) with a focus on the income statement, or are they recognized at the time they occur, without a necessary matching and with a focus on the balance sheet?

(d) Impairment: An impaired asset is one that no longer produces the economic benefits expected upon its acquisition. If one paradigm is used for the income statement and another is used for the balance sheet, the differences may result in impaired assets.

(e) Revenue Recognition: When should revenue be recognized - as a service is rendered (deferral/matching paradigm) or when the control of the asset representing the revenue is obtained (asset/liability paradigm)?

(f) Reporting Segment: Are the financial statements to be produced for the entire consolidated reporting entity, or separately for each legal entity comprising the reporting entity?

(g) Liquidation versus Going Concern: Is it being assumed that the company is going to continue operating and creating value (going concern), or is the focus on what the assets and liabilities would be if the company were sold off by its components today (liquidation)?

(h) Change in Accounting Principle versus Change in Accounting Estimate: Is a fundamental assumption about the accounting rules being changed (change in accounting principle) or is there simply a revision in a particular value, without a change in the underlying analytical framework (change in accounting estimate)?

(i) Principle-based versus Rule-based: Does the accounting standard rely on interpretation and judgment to be implemented (principle-based), or is flexibility limited and the use of the standard defined in detail in advance (rule-based)?

Problem S6-53-5. (a) Sometimes a distinction is made between reinsurance *recoverables* and reinsurance *receivables*. Explain the difference.

(b) Define *deferred acquisition costs* and state whether they are used under a deferral/matching paradigm or an asset/liability paradigm.

(c) Broadly describe three options for discounting a liability.

(See Blanchard, "Accounting Concepts for the Actuary", pp. 12-14.)

Solution S6-53-5. (a) Reinsurance *recoverables* are contra-liabilities representing amounts expected to be due from reinsurers as a result of incurred but currently not paid losses. Reinsurance *receivables* are assets representing amounts already billed to and due from reinsurers as a result of ceded losses that have already been paid by the primary insurer.

(b) Deferred acquisition costs can be an asset under the deferral/matching accounting paradigm, used to enable expenses to be recognized at the same time as revenues. Instead of being recognized immediately as expenses, acquisition costs are expensed over time, and the portion that has not yet been expensed is part of the deferred acquisition cost asset.

(c) Blanchard, on p. 14, describes the following three options for discounting a liability:

1. Treat the discount as an asset and report the liability on an undiscounted basis.
2. Report the liability with the discount built in.
3. Report the undiscounted liability and report the discount as a contra-liability (i.e., a negative entry under liabilities).

Section 54

Assorted Exam-Style Questions for Actuarial Exam 6 – Part 26

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Some of the problems in this section were designed to be similar to problems from past versions of Exam 6, offered by the Casualty Actuarial Society. They use original exam questions as their inspiration - and the specific inspiration is cited to give students an opportunity to see the original. All of the original problems are publicly available, and students are encouraged to refer to them. But all of the values, names, conditions, and calculations in the problems here are the original work of Mr. Stolyarov.

Sources:

Conger, R.F.; and Nolibos, A., "Estimating ULAE Liabilities: Rediscovering and Expanding Kittel's Approach," CAS *Forum*, Fall 2003, pp. 94-139, excluding appendices.

Friedland, Jacqueline F. *Estimating Unpaid Claims Using Basic Techniques*. Casualty Actuarial Society. July 2009.

Ludwig, S.J., "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance," *PCAS* LXXVIII, 1991, pp. 110-145.

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Problem S6-54-1. Similar to Question 14 from the 2009 CAS Exam 6. You are given the following information about an insurer's book of business by calendar year (CY):

CY 2024: Paid ULAE: 4444; Paid Loss & ALAE: 24000; Reported Loss & ALAE: 33000; Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year: 40000

CY 2025: Paid ULAE: 3800; Paid Loss & ALAE: 18000; Reported Loss & ALAE: 29000; Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year: 38000

CY 2026: Paid ULAE: 5190; Paid Loss & ALAE: 30000; Reported Loss & ALAE: 34000; Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year: 42000

You also know the following information as of December 31, 2026, for all accident years combined:

Case Reserves: 90990
IBNR: 40440
Ultimate Loss & ALAE: 120111

(a) Find the ULAE reserve as of December 31, 2026, using the classical method.

(b) Find the ULAE reserve as of December 31, 2026, using Kittel's refinement to the classical method.

(c) Find the ULAE reserve as of December 31, 2026, using the Conger-Nolibos generalized approach with the Bornhuetter-Ferguson method. Assume that 75% of work is expended when opening a claim, and 25% of the work is expended when maintaining the claim. No work is expended when closing the claim.

(d) How does the Conger-Nolibos generalized approach (i) more satisfactorily address growing books of business and (ii) make possible more realistic assumptions regarding the distribution of ULAE over the lifetime with regard to opening, closing, and maintaining claims, as compared to the classical method and Kittel's refinement?

Solution S6-54-1.

(a) In the classical method, we first calculate the ratio of paid ULAE to paid claims. We can select as our total ratio the sum of paid ULAE over all the given years, divided by the sum of paid claims (loss and ALAE) over all the given years: (4444 + 3800 + 5190)/(24000 + 18000 + 30000) = 0.186583333.

The classical method's estimate of the ULAE reserve is (ULAE Ratio)*(0.5*Case Reserve + IBNR) = 0.186583333*(0.5*90990 + 40440) = 16034.03872 = **16034.04**.

(b) Using the Kittel refinement, the ULAE ratio is calculated as (Paid ULAE)/(Claims Basis), where in the case of the Kittel refinement, the Claims Basis is the average of reported claims and paid claims. Here, our claims basis for all of the years is ((24000 + 33000)/2 + (18000 + 29000)/2 + (30000 + 34000)/2) = 84000, and our ULAE ratio is (4444 + 3800 + 5190)/84000 = 0.1599285714.

The Kittel refinement preserves the classical method's estimate of the ULAE reserve: (ULAE Ratio)*(0.5*Case Reserve + IBNR) = 0.1599285714*(0.5*90990 + 40440) = 13743.46178 = **13743.46**.

(c) Using the Conger-Nolibos approach, the ULAE ratio is calculated as (Paid ULAE)/(Claims Basis). The total Claims Basis is calculated as follows (assuming no work is expended closing claims):
(% of Work Expended Opening a Claim)*(Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year) + (% of Work Maintaining a Claim)*(Paid Claims) = 0.75*(40000 + 38000 + 42000) + 0.25*(24000 + 18000 + 30000) = 108000.

The ULAE ratio is thus (4444 + 3800 + 5190)/108000 = 0.124388889.

Applying the Bornhuetter-Ferguson method to the Conger-Nolibos approach, the ULAE Reserve is (ULAE Ratio)*(Ultimate Loss & ALAE - Total Claims Basis) = 0.124388889*(120111 - 108000) = 1506.473835 = **1506.47**.

(d) The Conger-Nolibos generalized approach (i) more satisfactorily addresses growing books of business than approaches that depend on comparing paid ULAE solely to paid claims or to the average of paid and reported claims, because the ULAE for a particular calendar year may not match to the paid claims of that calendar year, and the mismatch may be material if the volume of business written is changing. The Conger-Nolibos approach allows more flexibility in its claims basis and ties the claims basis to a more realistic assessment of how ULAE are distributed throughout the life of the claim.

The Conger-Nolibos generalized approach (ii) also relaxes the assumption that 50% of ULAE are spent opening a claim, and the other 50% are spent opening a claim. The generalized approach allows any mathematically legitimate percentage distribution of costs among opening, maintaining, and closing claims.

Problem S6-54-2. Similar to Question 15 from the 2009 CAS Exam 6.

(a) If it is known that the payment rate for claims slowed down over a particular time period, which of the following methods would it be inappropriate to use, and why?

(i) The Unadjusted Reported Development Method
(ii) The Unadjusted Paid Development Method
(iii) The Both-Case-and-Payment-Rate-Adjusted Reported Development Method
(iv) The Payment-Rate-Adjusted Paid Development Method

(b) You are given the following values for Accident Year 2033 as of December 31, 2033:

Claims Reported: 55555
Claims Paid: 14244
Earned Premium: 130555
Estimated Ultimate Claim Count: 57
Open and IBNR Count: 50
Ultimate Claims, Using Unadjusted Reported Development Method: 60000
Ultimate Claims, Using Unadjusted Paid Development Method: 52222
Ultimate Claims, Using Both-Case-and-Payment-Rate-Adjusted Reported Development

Method: 80220
Ultimate Claims, Using Payment-Rate-Adjusted Paid Development Method: 81232

Using each of the methods that were *not* rejected in part (a), calculate (i) the ultimate claim ratio, (ii) the ultimate severity, and (iii) the unpaid severity for Accident Year 2033.

(c) Describe one conclusion that might be drawn from the diagnostic results in part (b).

Solution S6-54-2.

(a) The **(ii) Unadjusted Paid Development Method** should be rejected, as it does not take into account the change in paid development as a result of the slowed payment rate. As a result, the unadjusted paid development will understate ultimate losses for years after the slowdown of the payment rate.

(b)

Using Unadjusted Reported Development Method
Claim Ratio: (Ultimate Claims/Earned Premium) = 60000/130555 = **0.459576424**.
Ultimate Severity: (Ultimate Claims/Ultimate Claim Count) = 60000/57 = **1052.631579**.
Unpaid Severity: (Ultimate Claims - Paid Claims)/(Open & IBNR Count) = (60000-14244)/50 = **915.12**.

Using Both-Case-and-Payment-Rate-Adjusted Reported Development Method
Claim Ratio: (Ultimate Claims/Earned Premium) = 80220/130555 = **0.614453679**.
Ultimate Severity: (Ultimate Claims/Ultimate Claim Count) = 80220/57 = **1407.368421**.
Unpaid Severity: (Ultimate Claims - Paid Claims)/(Open & IBNR Count) = (80220-14244)/50 = **1319.52**.

Using Payment-Rate-Adjusted Paid Development Method
Claim Ratio: (Ultimate Claims/Earned Premium) = 81232/130555 = **0.622205201**.
Ultimate Severity: (Ultimate Claims/Ultimate Claim Count) = 81232/57 = **1425.122807**.
Unpaid Severity: (Ultimate Claims - Paid Claims)/(Open & IBNR Count) = (81232-14244)/50 = **1339.76**.

(c) It can be seen that the Unadjusted Reported Development Method gives a dramatically lower estimate of the claim ratio and the severities than the other two methods. This might be due to other changes that unadjusted methods do not capture, such as case outstanding adequacy, which would affect a method based on reported claim development, even though payment rates would not.

Problem S6-54-3. Similar to Question 27 from the 2009 CAS Exam 6. You are given the following distribution of an insurer's premium by Coverage A limit:

Limit of $100,000 - Premium of $30 million
Limit of $200,000 - Premium of $20 million

Limit of $400,000 - Premium of $40 million
Limit of $800,000 - Premium of $10 million

You are also given the following Salzmann table of cumulative loss distributions corresponding to total losses as percentages of Coverage A:

Cumulative Loss Distribution for 10% of Coverage A: 8%
Cumulative Loss Distribution for 20% of Coverage A: 13%
Cumulative Loss Distribution for 30% of Coverage A: 15%
Cumulative Loss Distribution for 40% of Coverage A: 18%
Cumulative Loss Distribution for 50% of Coverage A: 24%
Cumulative Loss Distribution for 60% of Coverage A: 29%
Cumulative Loss Distribution for 70% of Coverage A: 38%
Cumulative Loss Distribution for 80% of Coverage A: 44%
Cumulative Loss Distribution for 90% of Coverage A: 52%
Cumulative Loss Distribution for 100% of Coverage A: 70%
Cumulative Loss Distribution for 120% of Coverage A: 75%
Cumulative Loss Distribution for 140% of Coverage A: 81%
Cumulative Loss Distribution for 160% of Coverage A: 89%
Cumulative Loss Distribution for 180% of Coverage A: 96%
Cumulative Loss Distribution for 200% of Coverage A: 100%

This primary insurer enters into an excess-of-loss reinsurance treaty covering $600,000 in excess of $200,000.

(a) What is the percentage of the insurer's expected losses covered by the reinsurance treaty? Use linear interpolation where needed.
(b) Salzmann tables consider only which type of peril?
(c) Salzmann tables consider only which lines of business?
(d) Salzmann tables consider only which types of coverage (typically)?

Solution S6-54-3.

(a) We first consider the percentage of expected losses covered by policy limit.

For the limit of $100,000, 0% is covered by the treaty, since losses never exceed the attachment point, which is 200% of the limit.

For the limit of $200,000, only losses in excess of 100% of the limit are covered. This means that only 100%-70% = 30% of losses are covered.

For the limit of $400,000, only losses between 50% of the limit and 200% of the limit are covered. This means that 100%-24% = 76% of losses are covered.

For the limit of $800,000, only losses between 25% of the limit and 100% of the limit are covered. Thus, only 70% - 14% = 56% of losses are covered. (The 14% was obtained by linear interpolation between the cumulative loss distributions for 20% and 30% of Coverage A.)

The overall percentage of losses covered is the average of the percentages above, weighted by the written premium for each coverage limit:
0%*30/100 + 30%*20/100 + 76%*40/100 + 56*10/100 = **42%.**

(b) Salzmann tables only consider the **fire** peril.

(c) Salzmann tables only consider the **homeowners' insurance** line of business.

(d) Salzmann tables only typically consider **Coverage A** and assume that losses for other coverages are proportional to losses for Coverage A.

Problem S6-54-4. Similar to Question 29 from the 2009 CAS Exam 6. You are given the following information regarding a primary insurer's experience for Accident Year 2028. The primary insurer is party to a quota share reinsurance treaty written on a losses-occurring basis for a term of 12 months. There are no reported catastrophe losses.

Earned premium: $15150
Incurred loss & ALAE: $10120
Premium on-level factor: 1.023
Loss & ALAE development factor: 1.102
Annual loss trend: +4%
Annual premium trend: +7%
Ceding commission: 15% of premium
Brokerage fees: 4% of premium
Administrative expenses: 6% of premium
ALAE: 10% of loss
Catastrophe load: 8% of non-catastrophe loss and ALAE
ULAE: 5% of total loss and ALAE.

For the 2030 treaty renewal period, calculate the projected combined ratio, on the basis of the experience for Accident Year 2028.

Solution S6-54-4. Because the treaty renewal period will occur two years after Accident Year 2028, our trend factors will be $(1 + \text{Annual Trend})^2$.

We conduct the necessary adjustments to premium:
(Earned premium)*(Premium on-level factor)*(Premium trend factor) = $15150*1.023*1.07^2$ = 17744.17541.

We conduct the necessary adjustments to losses and ALAE:
(Incurred loss & ALAE)*(Loss & ALAE development factor)*(Loss trend factor) = $10120*1.102*1.04^2$ = 12062.26278.

Our adjusted non-catastrophe loss ratio is 12062.26278/17744.17541 = 0.6797871699.

We multiply this by the catastrophe load factor: 0.6797871699*1.08 = 0.7341701435.

We multiply this by the ULAE factor: 0.7341701435*1.05 = 0.7708786507. This is our adjusted loss, ALAE, and ULAE ratio, including catastrophes.

We ignore the ALAE factor, because ALAE is already considered in the existing loss figures.

Now we add to our adjusted ratio the various reinsurer expenses and ceding commission, thereby getting our combined ratio: 0.7708786507 + 0.15 + 0.04 + 0.06 = **1.020878651.**

Problem S6-54-5. Similar to Question 32 from the 2009 CAS Exam 6. Suppose you are faced with a large set of reinsurance data from various sources.

(a) Identify three ways in which you might partition the data.
(b) Identify four considerations you might examine in deciding how to partition the data.

Solution S6-54-5. The following is a sample answer, and other valid answers are possible.

(a) One might partition the data
1. By reinsurance treaty type;
2. By type of underlying primary coverage;
3. By type of exposure being insured (e.g., reinsurance for policies written on manufacturing businesses might be treated separately from reinsurance for policies written on service businesses, if enough credible data exist).

(b) One might examine the following considerations:
1. How similar or different the reporting patterns are for the various categories of data into which partitions are contemplated;
2. How much data would exist in each category post-partition, and whether that is a credible amount of data;
3. How similar or different the underlying insured risks are;
4. How the treaty terms affect the reinsurer's exposure to loss.

Section 55

Basic Principles of Premium Accounting

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources:
Blanchard, R.S., "Premium Accounting," CAS Study Note, May 2005.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-55-1. According to Blanchard in "Premium Accounting" (p. 1), what are five possibilities for determining the time at which premium is recognized as revenue? Which of these possibilities is most commonly used in the life insurance industry? Which is most commonly used in the property/casualty insurance industry?

Solution S6-55-1. Blanchard lists the following five possibilities for determining the time at which premium is recognized as revenue:

1. The signing of the insurance contract
2. The due date of the premium from the policyholder
3. The date the premium is received by the insurer
4. The date the insurance policy becomes effective.
5. As the risk covered by the policy runs off over time.

The life insurance industry most often recognizes premium as revenue at the due date of the premium from the policyholder (Item 2 above).

The property and casualty insurance industry most often recognizes premium as revenue as the risk covered by the policy runs off over time (Item 5 above).

Problem S6-55-2. Assume an insurance policy sold on June 1, 2030, effective August 1, 2030, and having a term of one year. The risk insured does not vary over time. The premium charged is $2400. The insurer collects all the premium upfront at the sale date.

(a) Construct a balance sheet entry, from the insurer's perspective, of the assets and liabilities associated with this policy as of July 1, 2030, assuming no insured losses.

(b) Construct a balance sheet entry, from the insurer's perspective, of the assets and liabilities associated with this policy as of May 1, 2031, assuming no insured losses.

Solution S6-55-2.

(a) On July 1, 2030, the policy is still not in effect, so no premium has been earned. The insurer has already collected the $2400 from the insured in cash, but this cash asset needs to be balanced by an equivalent liability of $2400, called *Deposit Premium.* Thus, our balance sheet looks as follows:

Balance Sheet As of July 1, 2030:
Assets
Cash: $2400
Liabilities
Deposit Premium: $2400
Surplus
$0

(b) As of May 1, 2031, 9 of 12 months' premium have been earned, making the unearned premium (3/12)*2400 = $600. This is the liability known as the *Unearned Premium Reserve*. The insurer still has the $2400 cash asset. The surplus, the difference between assets and liabilities, represents the amount of policy premium that has already been earned. (In this case, that is all the surplus represents, as there are no insured losses.)

Balance Sheet As of May 1, 2031:
Assets
Cash: $2400
Liabilities
Unearned Premium: $600
Surplus
$1800

Problem S6-55-3.

(a) Describe the concept of "premium receivable", when it applies, and how it is treated on an insurer's balance sheet. (See Blanchard, "Premium Accounting", p. 4.)

(b) Under GAAP accounting, how would an insurer treat premium that is ultimately never collected? (See Blanchard, "Premium Accounting", p. 5)

Solution S6-55-3.

(a) "Premium receivable" is an asset that appears on the balance sheet when the policy to which a premium applies has already become effective, but the premium has not yet been paid by the insured. This premium is considered "premium receivable" until it is paid, at which time this asset is eliminated, and the equivalent amount becomes a cash asset.

(b) GAAP accounting treats premium that is ultimately never collected as an underwriting expense for the amount of uncollected premium. Treating it as negative premium is another option.

Problem S6-55-4.

(a) If premium revenue were recognized under an asset/liability paradigm rather than a deferral/matching paradigm, what would be the primary difference? (See Blanchard, "Premium Accounting", p. 7.)

(b) Identify five phenomena that may make it difficult to estimate premium upfront for an insurance policy. (See Blanchard, "Premium Accounting", pp. 7-8.)

Solution S6-55-4.

(a) If premium revenue were recognized under an asset/liability paradigm rather than a deferral/matching paradigm, then there would be no Unearned Premium Reserve liability. Instead of considering unearned premium, all premium would be recorded as revenue at the policy effective date or the policy sale date. *However,* the asset/liability paradigm also requires an estimate of expected losses and expenses for the portion of the policy than has not expired. These would be liabilities on the insurer's balance sheet.

(b) The following six phenomena are described by Blanchard, pp. 7-8. Any five would constitute a satisfactory answer.

1. **Deposits:** The "binder" premium, or initial deposit, may differ from the premium ultimately agreed upon once the details of an insurance policy are finalized.

2. **Estimates:** Premium may need to be estimated in cases where actual exposure to loss is not known. The estimates would need to be adjusted later as more information arises.

3. **Audits:** An insurer's audit of the insured's risk may revise the premium for a particular policy period.

4. **Endorsements/cancellations:** Endorsements selected by the insured or insurer may change the amount of premium, while cancellations of the policy imply the need for the insurer to issue a premium refund in most cases.

5. **Reinstatements:** For certain catastrophe excess-of-loss reinsurance treaties, a reinstatement premium is required to maintain coverage after a catastrophic event has occurred. This reinstatement premium is contingent on the catastrophe actually happening.

6. **Retrospective premium adjustments:** Premium for a retrospectively rated insurance policy depends on the insured's actual loss experience, which cannot be known in advance.

Problem S6-55-5. You have the following information about an insurer's book of business:

Written premium: $400
Earned premium: $360
Incurred losses: $320
Underwriting Expenses: $30
Policyholder Dividends: $15

(a) What is the loss ratio if the dividends are treated as expenses?
(b) What is the loss ratio if the dividends are treated as premium?
(c) What is the expense ratio if the dividends are treated as premium?
(d) What is the expense ratio if the dividends are treated as premium?

Solution S6-55-5.

(a) If dividends are treated as expenses, then the loss ratio is simply
(Incurred Losses)/(Earned Premium) = 320/360 = **0.8888888889** .

(b) If dividends are treated as (negative) premium, then the loss ratio is
(Incurred Losses)/(Earned Premium - Dividends) = 320/(360 - 15) = **0.9275362319.**

(c) If dividends are treated as expenses, then the expense ratio is
(Underwriting Expenses + Dividends)/(Written Premium) = (30 + 15)/400 = **0.1125.**

(d) If dividends are treated as (negative) premium, then the expense ratio is
(Underwriting Expenses)/(Written Premium - Dividends) = 30/(400-15) = **0.0779220779.**

Section 56

Intermediate Principles of Premium Accounting

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Source: Blanchard, R.S., "Premium Accounting," CAS Study Note, May 2005.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-56-1. In "Premium Accounting" (p. 10), Blanchard describes two distinct roles that unearned premium may be seen as having. Describe these two roles and identify a situation in which they may result in different values for unearned premium.

Solution S6-56-1. The two roles described by Blanchard for unearned premium are
1. Estimating the insurer's refund liability in the event the policy is cancelled and
2. Deferring revenue so that the timing of the insurer's revenue matches the timing of the associated expenses.

If an insurer charges the policyholder a penalty when the policyholder initiates cancellation of the policy, then the unearned premium under approach 1 above would be lower than under approach 2, since the insurer would not be liable to refund the amount of the penalty to the policyholder.

Problem S6-56-2. Describe four situations in which it would probably not be appropriate to consider premium as being earned over time in a pro rata fashion. (See Blanchard, "Premium Accounting", p. 10.)

Solution S6-56-2. In the following situations, it would probably not be appropriate to consider premium as being earned over time in a pro rata fashion:

1. Seasonal risks, where the exposure to risk varies depending on the time of year.
2. Aggregate excess-of-loss reinsurance policies, where the probability of exceeding the attachment point is higher at the end of the policy period than at the beginning.
3. Warranty products, where the risk is greater when the product subject to the warranty is older.
4. Performance bonds, where the risk of non-performance increases with the age of the contract.
(See Blanchard, "Premium Accounting", p. 10.)

Problem S6-56-3. (a) Define how the premium deficiency reserve is calculated and comment regarding the *only* situations in which a premium deficiency reserve would be established. (See Blanchard, "Premium Accounting", p. 12.)

(b) If a balance sheet is established as of January 1, 2034, but events on March 1, 2034, substantially change the insurer's estimate of future losses and expenses, should the unearned premium reserve and the premium deficiency reserve be effected? (See Blanchard, "Premium Accounting", p. 13.)

(c) Give a numerical example of how the level of aggregation of an insurer's operations can materially affect the premium deficiency reserve calculated for the insurer.

Solution S6-56-3. (a) A premium deficiency reserve is the difference between (1) "the losses and expenses expected from the runoff of the unexpired policy term" and (2) "the unearned premium liability already held with respect to the unexpired policy term" (Blanchard, "Premium Accounting", p. 12). Note that fixed expenses are not considered. The only time a premium deficiency reserve is established is when the difference above is *positive*, i.e., losses and expenses expected from the runoff of the unexpired policy term exceed the unearned premium liability already held with respect to the unexpired policy term. Otherwise, no premium deficiency reserve is established.

(b) No, the unearned premium and premium deficiency reserves should not be affected by events that occur after the balance sheet date; these reserves are estimates of the insurer's liability *as of the balance sheet date* and so should only be based on information known up to that date or anticipated as of that date.

(c) Many examples are possible. Here is a sample answer.
An insurer has two operations, 1 and 2, with the following characteristics:

Operation 1
Unearned Premium Liability of $4000
Estimated losses from the unearned premium runoff of $3610
Estimated expenses remaining from the runoff of the unearned premium liability: $300

Operation 2
Unearned Premium Liability of $5000
Estimated losses from the unearned premium runoff of $4800
Estimated expenses remaining from the runoff of the unearned premium liability: $500

There are two ways to aggregate: by operation or in total.
If the aggregation is performed by operation, then the premium deficiency reserve for Operation 1 is max(0, (3610 + 300) - 4000) = max(0, -90) = 0, and the premium deficiency reserve for Operation 2 is max(0, (4800 + 500) - 5000) = max(0, 300) = 300, for a total premium deficiency reserve of 0 + 300 = 300.

If the aggregation is performed for the entire insurer, then the premium deficiency reserve is max(0, (3610 + 300 + 4800 + 500) - (4000 + 5000)) = max(0, 210) for a total premium deficiency reserve of 210.

Problem S6-56-4. (a) On what basis are expected deficiencies in reserves typically allocated between the unearned premium reserve and the loss reserve? (See Blanchard, "Premium Accounting", p. 14.)
(b) Explain the distinction between "finance charges" and "service charges" in U.S. regulatory accounting (See Blanchard, "Premium Accounting", p. 15.)
(c) Assume an inflationary environment in which the annual inflation rate for expected losses is 30%.What fraction of the premium should be considered earned on a four-year insurance policy for the second year of the policy? Assume that the insurer wants an exact match between expected exposure to loss and earned premium.

Solution S6-56-4. (a) Expected deficiencies associated with events that have not yet occurred are typically reflected in the unearned premium reserve. Expected deficiencies associated with events that have already occurred are typically reflected in the loss reserve.

(b) In U.S. regulatory accounting, "finance charges" are payments pertaining to an installment plan that are a function of the amount of premium paid (and therefore have a similar role to that of interest). "Service charges" on the other hand are fixed monetary amounts that do not vary based on the amount of premium paid.

(c) We can assume that the expected loss is as follows:
1 in Year 1, 1.3 in year 2, 1.3^2 in year 3, 1.3^3 in year 4. If premium is earned in proportion to expected loss, then the fraction earned in year 2 would be $1.3/(1 + 1.3 + 1.3^2 + 1.3^3) =$ **0.2101179893 = 1300/6187**.

Problem S6-56-5. (a) Explain the concept of "earned but unbilled" (EBUB) or "earned but not reported" (EBNR) premium and where it would be applied. (See Blanchard, "Premium Accounting", pp. 16-17.)

(b) Explain how U.S. regulatory accounting requires the insurer's liability for extended reporting periods to be treated. (See Blanchard, "Premium Accounting", p. 17.)

Solution S6-56-5. (a) EBUB or EBNR premium occurs when the insurer can reasonably expect (on aggregate) that a certain amount of audit premium or reinstatement premium would be due to it, but has not established the exact amounts for specific insureds - as the audits or the events requiring reinstatement have yet to occur. The specific parties from which such premiums are to be due have not yet been identified, but the effect on the insurer's accounting figures can nonetheless be considerable.

(b) U.S. regulatory accounting requires that the insurer's liability for *finite* extended reporting periods be treated as *unearned premium reserves* and the liability for *indefinite* extended reporting periods be treated as *loss reserves*.

Section 57

Calculation of Premium-Development-to Loss-Development Ratios for Retrospectively Rated Insurance Policies

Formula 57.1: Retrospective Premium Calculation
$P_n = (BP+(CL_n *LCF))*TM$

Definitions of Variables
P_n = Premium at the nth retro adjustment
BP = Basic premium
CL_n = Capped loss at the nth retro adjustment
LCF = Loss conversion factor
TM = Tax multiplier

Formula 57.2: Premium Development to Loss Development (PDLD) Ratio at First Retro Adjustment
$PDLD_1 = ((BP/L_1)*TM)+((CL_1/L_1)*LCF*TM)$

Formula 57.3: Approximation of Premium Development to Loss Development (PDLD) Ratio at First Retro Adjustment
$PDLD_1 \approx BP*TM/(SP*ELR*\%Loss_1)$

Definitions of Variables
SP = Standard premium
ELR = Expected loss ratio
$\%Loss_1$ = Percent of loss emerged for the first retro adjustment

Formula 57.4: Premium Development to Loss Development (PDLD) Ratio for Second and Subsequent Retro Adjustments
$PDLD_n = ((CL_n - CL_{n-1})/(L_n - L_{n-1}))*LCF*TM$

Definitions of Variables
L_n = Uncapped loss at the nth retro adjustment

Sources: Teng, M.T.S.; and Perkins, M.E., "Estimating the Premium Asset on Retrospectively Rated Policies," *PCAS* LXXXIII, 1996, pp. 611-647, excluding Section 5.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-57-1. Consider a retrospectively rated book of business. The premium at the 3rd retro adjustment is \$142,140, the basic premium is \$70,600, and the capped loss at the 3rd retro adjustment is \$58,250. The tax multiplier is 1.04. What is the loss conversion factor?

Solution S6-57-1. We use Formula 57.1: $P_n = (BP+(CL_n * LCF))*TM$, rearranging it in terms of LCF: $P_3/TM = BP+(CL_3 * LCF) \rightarrow P_3/TM - BP = CL_3 * LCF \rightarrow$
$LCF = (P_3/TM - BP)/CL_3 = (142140/1.04 - 70600)/58250 =$ **LCF = 1.13430175**.

Problem S6-57-2. Consider a retrospectively rated book of business. At the first retro adjustment, the uncapped loss is \$130,000, and the capped loss is \$110,000. The tax multiplier is 1.08, and the loss conversion factor is 1.2. The basic premium is \$45,000. What is the PDLD ratio at the first retro adjustment?

Solution S6-57-2. We use Formula 57.2: $PDLD_1 = ((BP/L_1)*TM)+((CL_1/L_1)*LCF*TM) = ((45000/130000)*1.08)+((110000/130000)*1.2*1.08) =$ **$PDLD_1$ = 1.470461538**.

Problem S6-57-3. Consider a retrospectively rated book of business. At the first retro adjustment, the basic premium is \$55,000, and the standard premium is \$65,000. The expected loss ratio is 80%, and the expected percentage of loss emerged for the first retro adjustment is 56%. The tax multiplier is 1.05. Estimate the PDLD ratio at the first retro adjustment.

Solution S6-57-3. We use Formula 57.3: $PDLD_1 \approx BP*TM/(SP*ELR*\%Loss_1) = 55000*1.05/(65000*0.8*0.56) =$ **$PDLD_1$ =1.983173077**.

Problem S6-57-4. Consider a retrospectively rated book of business. At the first retro adjustment, the uncapped loss is \$130,000, and the capped loss is \$110,000. The tax multiplier is 1.08, and the loss conversion factor is 1.2. At the second retro adjustment, the uncapped loss is \$150,000, and the capped loss is \$125,000. The basic premium is \$45,000. What is the PDLD ratio at the second retro adjustment?

Solution S6-57-4. We use Formula 57.4: $PDLD_n = ((CL_n - CL_{n-1})/(L_n - L_{n-1}))*LCF*TM$. Here, $PDLD_2 = ((CL_2 - CL_1)/(L_2 - L_1))*LCF*TM = ((125000 - 110000)/(150000 - 130000))*1.2*1.08 =$ **$PDLD_2$= 0.972**.

Problem S6-57-5. The PDLD ratio at the third retro adjustment is 0.555, while the PDLD ratio at the second retro adjustment is 1.222. There are no taxes, and the loss conversion factor is 1. It is known that 20000 in uncapped losses emerged between the first and second retro adjustments, and 60000 in uncapped losses emerged between the second and third retro adjustments. Capped losses at the first retro adjustment were 12200. What was the magnitude of capped losses at the third retro adjustment?

Solution S6-57-5. We use Formula 57.4: $PDLD_n = ((CL_n - CL_{n-1})/(L_n - L_{n-1}))*LCF*TM$. Here, $LCF*TM = 1$, so $PDLD_n = ((CL_n - CL_{n-1})/(L_n - L_{n-1}))$. We are given the following:
$L_2 - L_1 = 20000$

$L_3 - L_2 = 60000$
$PDLD_2 = 1.222$
$PDLD_3 = 0.5555$
$CL_1 = 12200$
We want to find CL_3.
$1.222 = (CL_2 - CL_1)/(20000)$, so $CL_2 - CL_1 = 24440$.
$0.5555 = (CL_3 - CL_2)/(60000)$, so $CL_3 - CL_2 = 33330$.
$(CL_3 - CL_2) + (CL_2 - CL_1) = CL_3 - CL_1 = 33330 + 24440 = 57770$, so $CL_3 = 57770 + CL_1 = 57770 + 12200 =$ **$CL_3 = 69970$.**

Section 58

Calculation of Cumulative Premium-Development-to-Loss-Development Ratios and Premium Assets for Retrospectively Rated Insurance Policies

Source: Teng, M.T.S.; and Perkins, M.E., "Estimating the Premium Asset on Retrospectively Rated Policies," *PCAS* LXXXIII, 1996, pp. 611-647, excluding Section 5.

Original Problems and Solutions from The Actuary's Free Study Guide

The following conditions apply to all the problems in this section:
A particular book of retrospectively rated insurance policies has the following characteristics:

First Retro Adjustment
Emerged Losses (Incremental): 66600
Booked Premium (Incremental): 124000

Second Retro Adjustment
Emerged Losses (Incremental): 15500
Booked Premium (Incremental): 10300

Third Retro Adjustment
Emerged Losses (Incremental): 12220
Booked Premium (Incremental): 10000

Fourth Retro Adjustment
Emerged Losses (Incremental): 8000
Booked Premium (Incremental): 2000

There are no subsequent retro adjustments.

Problem S6-58-1. On the basis of the given information, calculate the empirical premium development to loss development (PDLD) ratios for each retro adjustment.

Solution S6-58-1. The PDLD ratio for each retro adjustment (in the absence of any additional data) is best estimated by dividing incremental booked premium by incremental emerged losses at each retro adjustment.

First Retro Adjustment
PDLD = 124000/66600 = **1.861861862**

Second Retro Adjustment
PDLD = 10300/15500 = **0.664516129**

Third Retro Adjustment
PDLD = 10000/12220 = **0.818330606**

Fourth Retro Adjustment
PDLD = 2000/8000 = **0.25**

Problem S6-58-2. As of each retro adjustment, calculate the incremental percent of the total losses emerged.

Solution S6-58-2. As of each retro adjustment, the percent of total losses emerged is the incremental emerged losses for that adjustment, divided by the ultimate emerged losses.

First Retro Adjustment
% Losses Emerged = 66600/(66600 + 15500 + 12220 + 8000) = 0.65089914 = **65.089914%**.

Second Retro Adjustment
% Losses Emerged = 15500/(66600 + 15500 + 12220 + 8000) = 0.151485536 = **15.1485536%**.

Third Retro Adjustment
% Losses Emerged = 12220/(66600 + 15500 + 12220 + 8000) = 0.119429242 = **11.9429242%**.

Fourth Retro Adjustment
% Losses Emerged = 8000/(66600 + 15500 + 12220 + 8000) = 0.078186083 = **7.8186083%**.

Problem S6-58-3. As of each retro adjustment, calculate the cumulative premium development to loss development (CPDLD) ratio.

Solution S6-58-3. Calculating the CPDLD ratio from the PDLD ratio and the percentage of losses emerged as of each retro adjustment takes four steps:

(1) For each retro adjustment, multiply the PDLD ratio by the incremental percentage of losses emerged.
(2) At each retro adjustment, calculate the cumulative ratio based on the results in step (1) (i.e., the sum of the ratio from step (1) for that adjustment *and all subsequent* adjustments).
(3) At each retro adjustment, calculate the cumulative percentage of loss emerging either *at that* retro adjustment *or at subsequent* retro adjustments.
(4) Divide the ratios from step (2) by the respective percentages from step (3) to get the CPDLD ratios.

In the displayed calculations below, it is intended that step (1) be performed first for *each* retro adjustment, followed by step (2) for each retro adjustment, etc.

First Retro Adjustment
(1) PDLD*(% Losses Emerged) = 1.861861862*65.089914% = 1.211884285
(2) Cumulative (PDLD*(% Losses Emerged)) = 1.211884285 + 0.100664582 + 0.097732604 + 0.019546521= 1.429827992
(3) Cumulative % Losses to Emerge = 100%
(4) CPDLD = (2)/(3) = 1.429827992/100% = **1.429827992**

Second Retro Adjustment
(1) PDLD*(% Losses Emerged) = 0.664516129*15.1485536% = 0.100664582
(2) Cumulative (PDLD*(% Losses Emerged)) = 0.100664582 + 0.097732604 + 0.019546521= 0.217943707
(3) Cumulative % Losses to Emerge = 100% - 65.089914% = 34.910086%
(4) CPDLD = (2)/(3) = 0.217943707/34.910086% = **0.624300115**

Third Retro Adjustment
(1) PDLD*(% Losses Emerged) = 0.818330606*11.9429242%=0.097732604
(2) Cumulative (PDLD*(% Losses Emerged)) = 0.097732604 + 0.019546521= 0.117279125
(3) Cumulative % Losses to Emerge = 100% - 65.089914% - 15.1485536% = 19.7615324%
(4) CPDLD = (2)/(3) = 0.117279125/19.7615324% = **0.593471815**

Fourth Retro Adjustment
(1) PDLD*(% Losses Emerged) = 0.25*7.8186083% = 0.019546521
(2) Cumulative (PDLD*(% Losses Emerged)) = 0.019546521
(3) Cumulative % Losses to Emerge = 7.8186083%
(4) CPDLD = (2)/(3) = 0.019546521/7.8186083% = **0.25**

Problem S6-58-4. Now you apply the CPDLD ratios derived in Solution S6-58-3 to the following related book of business with similar characteristics:

Policies at First Retro Adjustment
Expected future loss emergence of 40660
Premiums booked from prior adjustments of 0
Booked premium as of the present: 80210

Policies at Second Retro Adjustment
Expected future loss emergence of 22030
Premiums booked from prior adjustments of 60560
Booked premium as of the present: 90800

Policies at Third Retro Adjustment
Expected future loss emergence of 12300
Premiums booked from prior adjustments of 86000
Booked premium as of the present: 72040

Policies at Fourth Retro Adjustment
Expected future loss emergence of 6900
Premiums booked from prior adjustments of 96000
Booked premium as of the present: 80200

Find the estimated total (ultimate) premium for each set of policies above.

Solution S6-58-4. To find the estimated total (ultimate) premium for each set of policies above, we take the following steps:

(1) Multiply the expected future loss emergence by the CPDLD ratio to get the expected future premium.
(2) Add the premium booked from prior adjustments to the expected future premium to get the estimated total (ultimate) premium.

Policies at First Retro Adjustment
(1) Expected Future Premium = (Expected Future Loss Emergence)*CPDLD = 40660*1.429827992 = 58136.80615
(2) Estimated Total Premium = Expected Future Premium + Premium Booked from Prior Adjustments = 58136.80615 + 0 = circa **58136.81**

Policies at Second Retro Adjustment
(1) Expected Future Premium = (Expected Future Loss Emergence)*CPDLD = 22030*0.624300115 = 13753.33153
(2) Estimated Total Premium = Expected Future Premium + Premium Booked from Prior Adjustments = 13753.33153 + 60560 = circa **74313.33**

Policies at Third Retro Adjustment
(1) Expected Future Premium = (Expected Future Loss Emergence)*CPDLD = 12300*0.593471815 = 7299.703325
(2) Estimated Total Premium = Expected Future Premium + Premium Booked from Prior Adjustments = 7299.703325 + 86000 = circa **93299.70**

Policies at Fourth Retro Adjustment
(1) Expected Future Premium = (Expected Future Loss Emergence)*CPDLD = 6900*0.25 = 1725
(2) Estimated Total Premium = Expected Future Premium + Premium Booked from Prior Adjustments = 1725 + 96000 = **97725**

Problem S6-58-5. Using the information in Problem S6-58-4, estimate the *premium asset* for this book of business, for each set of policies and in total.

Solution S6-58-5. The premium asset is the difference between the estimated total premium and the premium booked as of the present.

Policies at First Retro Adjustment
Premium Asset = Estimated Total Premium - Premium Booked at Present = 58136.81 - 80210 = **-22073.19**

Policies at Second Retro Adjustment
Premium Asset = Estimated Total Premium - Premium Booked at Present = 74313.33 - 90800 = **-16486.67**

Policies at Third Retro Adjustment
Premium Asset = Estimated Total Premium - Premium Booked at Present = 93299.70 - 72040 = **21259.70**

Policies at Fourth Retro Adjustment
Premium Asset = Estimated Total Premium - Premium Booked at Present = 97725 - 80200 = **17525**

Total Premium Asset: -22073.19 - 16486.67 + 21259.70 + 17525 = **224.84**.

Section 59

Analysis of the PDLD Approach to Estimating the Premium Asset for Loss-Sensitive Contracts

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources:
Teng, M.T.S.; and Perkins, M.E., "Estimating the Premium Asset on Retrospectively Rated Policies," *PCAS* LXXXIII, 1996, pp. 611-647, excluding Section 5. Discussion by Feldblum, S., *PCAS* LXXXV, 1998, pp. 274-315, Sections 1 and 2 only.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-59-1. Consider the Teng and Perkins approach to estimating the premium asset for books of retrospectively rated insurance policies.

(a) Why is it sometimes the case that the losses emerged for a time period such as 0 to 18 months would correspond to premium booked at, for instance, 27 months for the first retro adjustment? That is, why is there a mismatch between the period for emerged losses and the period for booked premiums?

(b) Why is it common for the premium development to loss development (PDLD) ratio to exceed 1 at the first retro adjustment and to be less than 1 for subsequent retro adjustments?

Solution S6-59-1.

(a) It takes time to perform the retro adjustment calculations and to convert emerged losses into retrospective premium - especially if one has to do this with a large number of insureds. This is known as the booking lag. For losses emerged from 0 to 18 months, it is possible for the corresponding emerged premium to only be known and booked at 27 months. (See Teng and Perkins, p. 619.)

(b) It is common for the premium development to loss development (PDLD) ratio to exceed 1 at the first retro adjustment because (1) the basic premium is considered as part of premium development, and (2) fewer losses are likely to reach the amount of the loss cap, meaning that, in their conversion to retrospective premium, these losses are fully affected by the tax multiplier

and the loss conversion factor. Losses emerging for subsequent retro adjustments are much likelier to be affected by the cap, meaning that there is a decreased likelihood of each incremental dollar of uncapped loss translating into premium. (See Teng and Perkins, p. 622.)

Problem S6-59-2. Teng and Perkins (pp. 627-628) discuss three further issues to consider when calculating the premium asset for loss-sensitive rating plans. What are these issues as they relate to the diversity possible among such plans?

Solution S6-59-2. Teng and Perkins mention the following issues:

1. Are allocated loss adjustment expenses (ALAE) considered part of the loss for the purposes of retrospective rating? This affects whether ALAE are also considered in calculating the PDLD ratios.

2. Have there been changes in the mix of business - e.g., by state, geographical region, or industry group? If so, this may affect both loss emergence and premium sensitivity to loss.

3. How collectible are the premiums considered to be a part of the premium asset? Are they secured, or is there a need for some consideration of the possibility of bad debt?

Problem S6-59-3.

(a) Feldblum mentions three distinct advantages that the Teng and Perkins PDLD method has over other procedures. Briefly describe these advantages. (See Feldblum, pp. 274-275.)

(b) Feldblum discusses a difficulty of applying a simple chain-ladder method to estimating development for premium on loss-sensitive contracts. Identify such a difficulty and how specialized methods like the PDLD method and Fitzgibbon's method overcome it. (See Feldblum, pp. 276-277.)

Solution S6-59-3.

(a) The following are the advantages of the Teng and Perkins PDLD method mentioned by Feldblum:

1. It is based directly on the retrospective rating formula and can be easily explained to underwriters and claims personnel.

2. There is correspondence with statutory reporting of accounting information (in particular, Part 7 of Schedule P). This arises because the PDLD approach emphasizes premium sensitivity to losses.

3. Other methods' indications may be distorted by changes in retrospective rating plans' parameters, whereas the PDLD method would take these changes into account.

(b) The chain-ladder method relies on direct estimates of retrospective premiums, which take much longer to emerge than incurred losses, resulting in a lag of about 9 months in many cases. The PDLD and Fitzgibbon approaches rely on emerged losses, from which retrospective premiums can be estimated, resulting in more expeditious calculations.

Problem S6-59-4.

(a) Describe the central assumption of the Fitzgibbon approach. How does linear regression relate to this assumption? (See Feldblum, pp. 280-281.)

(b) Why might using regression even be desirable in addressing retrospectively rated policies?

Solution S6-59-4.

(a) The central assumption of the Fitzgibbon approach is that retrospective premium linearly relates to incurred losses, via the formula (Retrospective Premium) = C + B*Losses. This translates into the formula (Retro Adjustment) = A+B *(Standard Loss Ratio), where the constant A and the slope B are estimated via linear regression on historical data for loss ratios and retro adjustments from mature policy years (Feldblum, pp. 280-281).

(b) There can be much variation in the details of specific retrospective rating plans - for instance, in the basic premiums, in the maximum premiums set, and in the premium taxes and involuntary market loads by jurisdiction. It may be extremely challenging and time-consuming to collect and analyze all these data for individual retrospective rating plans. Instead, average characteristics could be estimated using regression. Moreover, because the parameters established for a particular retrospective plan by its pricing actuary might not correspond to the parameters actually used in practice, regression analysis of empirical data might give a more accurate understanding of what is actually happening than inferences from how the retrospective plans "ought" to be applied. (See Feldblum, pp. 281-282.)

Problem S6-59-5.

(a) Identify two weaknesses of the Fitzgibbon approach.

(b) How does the Teng and Perkins PDLD approach depart from the fundamental assumption of the Fitzgibbon method?

(c) Feldblum identifies two factors that correlate with the decline of premium responsiveness on loss-sensitive contracts. What are they? (See Feldblum, pp. 288-289.)

(d) What, according to Feldblum, are the two assumptions of the PDLD method? (See Feldblum, p. 291.)

Solution S6-59-5.

(a) The Fitzgibbon approach has the following weaknesses:

1. In assuming a linear relationship between premium development and loss development, the Fitzgibbon approach tends to overstate premium development at later retro adjustments.

2. The Fitzgibbon approach does not distinguish, in its projections, between the impact of many small losses and one large loss of the same magnitude. However, because of loss limits and maximum premiums in the retrospective rating plan, this can make a material difference in the retrospective premium. (See Feldblum, pp. 284-285.)

(b) The Fitzgibbon approach assumes a linear relationship between premium development and loss development. The Teng and Perkins PDLD approach, instead, posits a decreasing slope over time for premium development as compared to loss development. The slope between every two subsequent retro adjustments is less steep than the slope preceding it, which counteracts the Fitzgibbon method's tendency to overestimate premium development at later retro adjustments.

(c) Premium responsiveness on loss-sensitive contracts with (1) **increase in maturity (age) of a book of business** and (2) **higher reported loss ratios** (Feldblum, pp. 288-289).

(d) The PDLD method assumes

1. The independence of premium development for subsequent retro adjustments from premium development for prior retro adjustments and

2. Dependence on the slope of premium development versus loss development on the time period, not on the beginning loss ratio or retrospective premium ratio (Feldblum, p. 291).

Section 60

Reinsurance Pricing Concepts

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources:
Clark, D.R., "Basics of Reinsurance Pricing," CAS Study Note, 1996.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-60-1. What is the paradox of reinsurance pricing described by Clark?

Solution S6-60-1. The paradox of reinsurance pricing is that a ceding company will not want to buy a reinsurance contract that can be precisely priced (Clark , p.1).

Problem S6-60-2. Fill in the blanks:

(a) If a reinsurance treaty is written on a "risks attaching" basis, use _____ (type of premium) and _____ (type of losses).

(b) If a reinsurance treaty is written on a "losses occurring" basis, use _____ (type of premium) and _____ (type of losses).

Solution S6-60-2. This question is based on the discussion by Clark , p. 3.

(a) If a reinsurance treaty is written on a "risks attaching" basis, use **written premium** and **losses covered by the attaching policies**.

(b) If a reinsurance treaty is written on a "losses occurring" basis, use **earned premium** and **accident-year losses.**

Problem S6-60-3. In adjusting reinsurance experience to an ultimate level, describe two adjustments to premium and two adjustments to losses that might be made.

Solution S6-60-3. This question is based on the discussion by Clark , pp. 3-4.

Two adjustments to premium are (i) on-level factors to bring premiums to current rate levels and (ii) exposure inflation factors if the premium base is inflation-sensitive.

Two adjustments to losses are (i) loss development factors to ultimate and (ii) loss trend factors.

Problem S6-60-4. Clark (pp. 4-5) discusses four ways of estimating catastrophe loads for proportional property reinsurance treaties. What are these four ways?

Solution S6-60-4. The four ways of estimating catastrophe loads for proportional property reinsurance treaties are as follows (Clark, pp. 4-5):

1. Basing the estimate on the ceding company's expected distribution of premium by state;
2. Basing the estimate on the average number of times per year that the treaty's occurrence limit is expected to be exceeded;
3. Basing the estimate on historical catastrophe losses spread evenly over a longer time period, adjusted to current cost and exposure levels;
4. Basing the estimate on the output from a catastrophe simulation model.

Problem S6-60-5. In situations where there is a sliding-scale ceding commission that varies on the basis of loss ratio, Clark (pp. 9-10) distinguishes between a "naïve" approach of estimating the expected ceding commission and a more accurate approach. Conceptually describe each and explain why the latter approach is more accurate.

Solution S6-60-5. The "naïve" approach works as follows:

1. Calculate the expected loss ratio.
2. The expected ceding commission is the ceding commission corresponding to the expected loss ratio.

The improved approach works as follows:

1. Calculate the ceding commission associated with the average loss ratio for each range in an aggregate loss distribution model.
2. Calculate a probability-weighted average ceding commission, based on the ceding commissions in each range of the aggregate loss distribution model.

The second approach is preferable because it takes into account the possibility (indeed, the likelihood) that the distribution of ceding commissions may differ from the distribution of loss ratios (i.e., a ceding commission may "slide" differently depending on the range of loss ratios being considered). Thus, simply calculating an expected loss ratio fails to capture the impact of loss ratio variation on the ceding commission. Using an aggregate loss distribution at least enables one to account for the differential "slides" of the ceding commission within each selected loss ratio range.

Problem S6-60-6. What is a *carryforward provision* for a ceding commission? Give a brief numerical example of how such a provision would work. (See Clark, p. 10)

Solution S6-60-6. A carryforward provision is a clause in the reinsurance contract that allows subsequent years' ceding commissions to be modified by any of the primary insurer's prior-year loss amounts in excess of the loss ratio corresponding to the minimum ceding commission.

For instance, if the minimum ceding commission is 10%, corresponding to an 80% loss ratio, and the ceding company's loss ratio in Year X is 85%, then the loss amount corresponding to the excess 5% of losses may be used in calculating the Year (X+1) loss ratio for ceding commission purposes.

Problem S6-60-7. Clark (pp. 10-12) describes two ways of addressing carryforward provisions in estimating expected ceding commissions for proportional reinsurance contracts. Conceptually describe each approach and identify a shortcoming of each.

Solution S6-60-7. The following are two ways of addressing carryforward provisions in estimating expected ceding commissions for proportional reinsurance contracts:

1. Shift the ceding commission "slide" by the amount of the carryforward from prior years. For instance, if the ceding commission slides 1:1 from 10% at an 80% loss ratio to 30% at a 60% loss ratio, and the carryforward from prior years is +3%, then one could adjust the slide to 1:1 from 10% at an 77% loss ratio to 30% at a 57% loss ratio.

Shortcoming: Only carryforward for the current year is addressed; the possibility of carryforward for subsequent years is not taken into account.

2. Look at the long-run circumstances of the contract and, instead of applying the sliding scale to just the current year, apply it to a longer block of years. This allows for reduced aggregate variance in losses.

Shortcomings (any one will suffice): (1) The approach assumes that the reinsurance contract will be renewed over many years - which is often not a certainty. (2) There is no way of addressing situations where only ceding commission deficits, but not credits, can be carried forward. (3) There is no unambiguous way of estimating the variation reduction achieved as a result of this approach.

Section 61

Reinsurance Pricing Concepts – Part 2

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources:
Clark, D.R., "Basics of Reinsurance Pricing," CAS Study Note, 1996.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-61-1. According to Clark (p. 19), why is the actual number of claims during the historical period in question not a good basis for credibility of reinsurance data? What is a superior basis?

Solution S6-61-1. The actual number of claims during the historical period in question not a good basis for credibility, because this measure implies that fortuitously worse time periods in terms of loss are necessarily assigned more credibility, thus making the reinsurer's experience appear worse than it actually is. The number of claims *expected* during the historical period is a better basis for credibility, because it does not bias the data by assigning more credibility to less favorable experience.

Problem S6-61-2. Describe the three general categories of casualty per-occurrence excess-of-loss reinsurance treaties (Clark, p. 22).

Solution S6-61-2. The three general categories of casualty per-occurrence excess-of-loss reinsurance treaties are as follows (Clark, p. 22):
1. **Working cover:** The attachment point is low and is expected to be penetrated multiple times per treaty period.

2. **Exposed excess:** The excess layer of reinsurance coverage attaches below the policy limits for some of the policies in the book of business. It is possible for the reinsurer to incur a loss on the treaty if losses on enough underlying policies are close to their limits.

3. **Clash cover:** The attachment point is typically above the limits of any one policy, and the treaty is expected to apply only in cases where multiple policies (including policies of different types) are triggered, or extra-contractual obligations or damages in excess of policy limits are required of the primary insurer.

Problem S6-61-3. Clark (p. 23) discusses a theoretical problem with regard to capping trended losses at applicable policy limits. Explain the difficulty in determining the right approach to follow on this matter.

Solution S6-61-3. It is theoretically desirable to cap trended losses at the limits at which the policy would have been issued, had it been issued during the later time period under consideration. This means that applying the policy's historical limit may not be appropriate, as insurers generally increase policy limits over time in an inflationary environment. If no capping is used, this assumes that policy limits increase at the rate of inflation, which may also not correspond to reality; this approach would also require an adjustment to the subject premium, or else the loss costs will be overstated (Clark, p. 23).

Problem S6-61-4. Clark (p. 25) describes four areas of caution to be exercised with regard to using development data from the Reinsurance Association of America (RAA). Describe these areas.

Solution S6-61-4. The following cautions should be taken with regard to development data from the Reinsurance Association of America (RAA) (Clark, p. 25):

1. The RAA data are from various companies, and the reporting lag between the event's occurrence and the reinsurer's establishment of a case reserve for it varies by company. Also, the reporting lag for retrocessional business exceeds the reporting lag for ordinary reinsurance.

2. The RAA data are not sufficiently segmented by limits and attachment points, and, even when some segmentation is attempted, the stability of the data is compromised as a result.

3. It is not clear whether the reporting companies consistently adhere to the RAA's standard of excluding asbestos and environmental claims. Also, some long-term exposure claims pertaining to certain products are not excluded, even though they are not relevant to all reinsurance business.

4. It is not clear whether reporting workers' compensation reinsurers consistently treat the tabular discount on large claims.

Problem S6-61-5.

(a) What are two shortcomings of using a single continuous distribution function to model reinsurance losses? (See Clark, p. 36.)

(b) Briefly explain how a collective risk model differs from using a single continuous distribution function. (See Clark, pp. 39-40.)

Solution S6-61-5.

(a) Two shortcomings of using a single continuous distribution function to model reinsurance losses are that (1) the distribution does not permit a scenario where the loss amount is zero (since

x = 0 is the smallest independent variable, the cumulative distribution function is zero at x= 0, implying a zero probability) and (2) the impact on the distribution of changing the treaty limits (e.g., the per-occurrence limits) is not easy to determine.

(b) A collective risk model employs a probability distribution to model the severity of each loss, and the number of losses also follows its own probability distribution. This enables explicit recognition of both frequency and severity of losses.

Problem S6-61-6.

(a) According to Clark (p. 40), what assumption do most collective risk models make regarding loss occurrences?

(b) Clark (p. 40) distinguishes between "process variance" and "parameter variance". In a collective risk model, which of these is always reflected by the aggregate distribution, and which might not be?

Solution S6-61-6.

(a) Most collective risk models assume that loss occurrences are independent of one another. This may or may not be true in reality.

(b) "Process variance" is "the random fluctuation of actual results about the expected value" (Clark, p. 40). "Parameter variance" can also be referred to as "model risk" and is uncertainty about whether the model's own design and parameters are appropriate for describing the situation in question (in Clark's words, "whether you are in the right model"). The aggregate distribution of a collective risk model always reflects process variance, but not necessarily parameter variance.

Problem S6-61-7. You have the following information about a catastrophe excess-of-loss reinsurance treaty for the annual term encompassing the entire year 2022:

Annual premium: $4,000,000
Occurrence limit: $50,000,000
Date of loss: September 1, 2022
Loss amount: $35,000,000
Reinstatement provision: 120%

(a) Calculate the reinstatement premium after the loss if the reinstatement provision is pro rata as to amount, but not pro rata as to time.

(b) Calculate the reinstatement premium after the loss if the reinstatement provision is pro rata as to amount *and* pro rata as to time.

(c) Why are most reinstatement premiums for catastrophe excess-of-loss treaties *not* pro rata as to time? (See Clark, p. 41.)

Solution S6-61-7.

(a) If the reinstatement provision is pro rata as to amount, but not pro rata as to time, then we only need to consider the fraction of the annual premium corresponding to the proportion of the loss amount to the occurrence limit, multiplied by the percentage in the reinstatement provision.

Reinstatement premium = ($35,000,000/$50,000,000)*$4,000,000*120% = **$3,360,000.**

(b) If the reinstatement provision is pro rata as to amount and to time, then a further reduction of the reinstatement premium is needed to account for the time during which the new coverage will be effective - i.e., from September 1, 2022, until the end of 2022, or 4/12 = 1/3 years. Thus, the reinstatement premium is $3,360,000*(1/3) = **$1,120,000.**

(c) Most reinstatement premiums for catastrophe excess-of-loss treaties are not pro rata as to time because many catastrophes, such as hurricanes, occur seasonally, so a pro rata approach to time does not take into account the actual exposure to risk during the remainder of the treaty period.

Section 62

Exposure Rating for Property Excess-of-Loss Reinsurance

Section 62 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 63

Approaches to Reserving for Workers' Compensation High-Deductible Policies

Section 63 is a Microsoft Excel file. Clicking on the link will prompt a download.

Section 64

Characteristics of Excess Loss Development

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources:
Pinto, E.; and Gogol, D.F., "An Analysis of Excess Loss Development," *PCAS* LXXIV, 1987, pp. 227-255.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-64-1. Pinto and Gogol (pp. 230-233) analyzed excess loss development factors using various treatments of ALAE: (1) adding ALAE to the loss amount, (2) assigning ALAE to the excess layer on a pro rata basis, and (3) not including ALAE in the loss amount. What did Pinto and Gogol observe regarding the effect of the treatment of ALAE on excess loss development factors?

Solution S6-64-1. Pinto and Gogol observed that, after 39 months, the treatment of ALAE has no material effect on the excess loss development factors (pp. 230-233).

Most of the factors displayed on pp. 231-233 are roughly the same after 39 months, irrespective of how ALAE is treated.

Problem S6-64-2. Fill in the blanks regarding the curve-fitting approach used by Pinto and Gogol. (See p. 234.)

(a) For each development interval, a curve was fitted to the excess loss development factors as a function of _____.

(b) Separate curves were fitted for each _____.

(c) The curve formula selected was y = _____. (Give the variables on other side of the equation, with definitions as appropriate.)

(d) The distribution whose qualities motivated the selection of the function in (c) was the _____ distribution.

Solution S6-64-2. (a) For each development interval, a curve was fitted to the excess loss development factors as a function of **retention**.

(b) Separate curves were fitted for each **line of business**.

(c) The curve formula selected was $y = \mathbf{ax^b}$, where x is the retention divided by $10,000, and a is the value for development in excess of $10,000.

(d) The distribution whose qualities motivated the selection of the function in (c) was the **single-parameter Pareto** distribution. (In the function, b acts as the parameter.)

Problem S6-64-3. For any one of the curves $y = a_n * x^{b_n}$ used by Pinto and Gogol, explain in general terms the procedure by which the values for a_n and b_n were determined. (See p. 234.)

Solution S6-64-3. Pinto and Gogol applied natural logarithms to each side of the equation $y = a_n * x^{b_n}$ to get $\ln(y) = \ln(a_n) + b_n * \ln(x)$. This is a linear function, where the values $\ln(a_n)$ and b_n can be determined using least-squares linear regression.

Problem S6-64-4. Pinto and Gogol compare development factors for excess losses between those based on data from the Reinsurance Association of America (RAA) and those based on data from the Insurance Services Office (ISO), fitted using the methods of Pinto and Gogol. It is observed that, after 99 months, the RAA data show materially higher development factors than the fitted ISO data. What are the possible explanations offered by Pinto and Gogol for this observation? (See pp. 239-240.)

Solution S6-64-4. Pinto and Gogol offer the following explanations for this observation:

1. Aggregate-basis reinsurance coverage may show its effects later on in the loss development. When reinsurance treaties are written on an aggregate basis, it takes a longer period for losses to accumulate so as to breach the aggregate retention.

2. There may be unidentified longer-tailed medical malpractice losses present in the RAA data.

3. The ISO data have a fixed retention of $250,000. The RAA data have various retentions and limits, and the distribution of those may be more conducive to larger excess loss development factors later on.

4. The RAA data include information on umbrella, excess, and surplus lines business that the ISO data exclude.

5. RAA development factors are based on actual empirical data, with no fitting. The ISO factors beyond 99 months are based on a curve fitted to the data through 99 months.

Problem S6-64-5. Pinto and Gogol also analyzed excess *paid* loss and ALAE development and compared it to excess *reported* loss and ALAE development (pp. 241-242), as a function of the retention. What did they observe?

Solution S6-64-5. Pinto and Gogol observed that the ratios of paid excess losses to reported excess losses at each age of maturity do not materially vary as a function of the retention (p. 242).

Problem S6-64-6. (a) In a single sentence, summarize the main theoretical insight of the Pinto and Gogol paper.

(b) What are the two principal influences on excess loss development, according to Pinto and Gogol, and which one of these influences is indispensable to the main theoretical insight in part (a) above?

Solution S6-64-6. (a) Excess loss development increases as the retention increases (p. 245).

(b) The two principal influences on excess loss development are

1. The change in the reporting pattern of claims over time; and
2. Change in the characteristics of the distribution for loss size at successive reports (p. 245).

Influence 2 above is the one indispensable to Pinto and Gogol's observation. The lack of this influence would produce excess loss development factors that do not vary by retention.

Section 65

Calculation of Loss Development Factors for Excess Loss Layers

Formula 65.1:
LDF from nth report to ultimate for excess layer from c to d = $(f(c) - f(d))/(e_{c,n} - e_{d,n})$

c = Lower bound of excess layer
d = Upper bound of excess layer
f(c) = ratio of ultimate losses in excess of c to ultimate ground-up losses
f(d) = ratio of ultimate losses in excess of d to ultimate ground-up losses
$e_{c,n}$ = ratio of nth-report losses in excess of c to ultimate ground-up losses
$e_{d,n}$ = ratio of nth-report losses in excess of d to ultimate ground-up losses

Source:
Pinto, E.; and Gogol, D.F., "An Analysis of Excess Loss Development," *PCAS* LXXIV, 1987, pp. 227-255.

Original Problems and Solutions from The Actuary's Free Study Guide

The following information applies to all problems in this section.

Ultimate ground-up losses: $3,400,000
Ultimate losses in excess of $25,000: $3,000,000
Ultimate losses in excess of $50,000: $2,100,000
Ultimate losses in excess of $100,000: $1,200,000
Ultimate losses in excess of $200,000: $500,000
Ultimate losses in excess of $500,000: $350,000

Excess loss development factor from 39 months to ultimate for losses in excess of $25,000: 2.55
Excess loss development factor from 39 months to ultimate for losses in excess of $50,000: 2.89
Excess loss development factor from 39 months to ultimate for losses in excess of $100,000: 3.35
Excess loss development factor from 39 months to ultimate for losses in excess of $200,000: 4.13
Excess loss development factor from 39 months to ultimate for losses in excess of $500,000: 5.56

Problem S6-65-1. What is the 39-month-to-ultimate excess loss development factor for the layer $25,000 in excess of $25,000?

Solution S6-65-1. We use Formula 65.1:
LDF from nth report to ultimate for excess layer from c to d = $(f(c) - f(d))/(e_{c,n} - e_{d,n})$

In this case, the desired LDF is $(f(25000) - f(50000))/(e_{25000,39\ months} - e_{50000,39\ months})$
We calculate f(25000) = \$3,000,000/\$3,400,000 = 15/17.
We calculate f(50000) = \$2,100,000/\$3,400,000 = 21/34

We recall that the $e_{c,n}$ and $e_{d,n}$ are ratios of nth-report excess losses to ultimate ground-up losses. To use the excess loss development factors (nth report excess loss)/(ultimate excess loss), we need to multiply f(c) and f(d) by (nth report excess loss)/(ultimate excess losses), or, equivalently, divide by the excess loss development factors.

Thus, $e_{25000,39\ months} = (15/17)/2.55$
$e_{50000,39\ months} = (21/34)/2.89$

Thus, the desired LDF is (15/17 - 21/34)/((15/17)/2.55 - (21/34)/2.89) = **2.000769231**

Problem S6-65-2. What is the 39-month-to-ultimate excess loss development factor for the layer \$75,000 in excess of \$25,000?

Solution S6-65-2. We use Formula 65.1:
LDF from nth report to ultimate for excess layer from c to d = $(f(c) - f(d))/(e_{c,n} - e_{d,n})$

In this case, the desired LDF is $(f(25000) - f(100000))/(e_{25000,39\ months} - e_{100000,39\ months})$
We calculate f(25000) = \$3,000,000/\$3,400,000 = 15/17.
We calculate f(100000) = \$1,200,000/\$3,400,000 = 6/17.

We recall that the $e_{c,n}$ and $e_{d,n}$ are ratios of nth-report excess losses to ultimate ground-up losses. To use the excess loss development factors (nth report excess loss)/(ultimate excess loss), we need to multiply f(c) and f(d) by (nth report excess loss)/(ultimate excess losses), or, equivalently, divide by the excess loss development factors.

Thus, $e_{25000,39\ months} = (15/17)/2.55$
$e_{100000,39\ months} = (6/17)/3.35$

Thus, the desired LDF is (15/17 - 6/17)/((15/17)/2.55 - (6/17)/3.35) = **2.199785408**

Problem S6-65-3. What is the 39-month-to-ultimate excess loss development factor for the layer \$100,000 in excess of \$100,000?

Solution S6-65-3. We use Formula 65.1:
LDF from nth report to ultimate for excess layer from c to d = $(f(c) - f(d))/(e_{c,n} - e_{d,n})$

In this case, the desired LDF is $(f(100000) - f(200000))/(e_{100000,39\ months} - e_{200000,39\ months})$
We calculate f(100000) = \$1,200,000/\$3,400,000 = 6/17.
We calculate f(200000) = \$500,000/\$3,400,000 = 5/34.

We recall that the $e_{c,n}$ and $e_{d,n}$ are ratios of nth-report excess losses to ultimate ground-up losses. To use the excess loss development factors (nth report excess loss)/(ultimate excess loss), we need to multiply f(c) and f(d) by (nth report excess loss)/(ultimate excess losses), or, equivalently, divide by the excess loss development factors.

Thus, $e_{100000,39\text{ months}} = (6/17)/3.35$
$e_{200000,39\text{ months}} = (5/34)/4.13$
Thus, the desired LDF is (6/17 - 5/34)/((6/17)/3.35 - (5/34)/4.13) = **2.951798232**

Problem S6-65-4. What is the 39-month-to-ultimate excess loss development factor for the layer $400,000 in excess of $100,000?

Solution S6-65-4. We use Formula 65.1:
LDF from nth report to ultimate for excess layer from c to d = $(f(c) - f(d))/(e_{c,n} - e_{d,n})$

In this case, the desired LDF is $(f(100000) - f(500000))/(e_{100000,39\text{ months}} - e_{500000,39\text{ months}})$
We calculate f(100000) = $1,200,000/$3,400,000 = 6/17.
We calculate f(500000) = $350,000/$3,400,000 = 7/68.

We recall that the $e_{c,n}$ and $e_{d,n}$ are ratios of nth-report excess losses to ultimate ground-up losses. To use the excess loss development factors (nth report excess loss)/(ultimate excess loss), we need to multiply f(c) and f(d) by (nth report excess loss)/(ultimate excess losses), or, equivalently, divide by the excess loss development factors.

Thus, $e_{100000,39\text{ months}} = (6/17)/3.35$
$e_{500000,39\text{ months}} = (7/68)/5.56$
Thus, the desired LDF is (6/17 - 7/68)/((6/17)/3.35 - (7/68)/5.56) = **2.878825348**

Problem S6-65-5. What is the 39-month-to-ultimate excess loss development factor for the layer $300,000 in excess of $200,000?

Solution S6-65-5. We use Formula 65.1:
LDF from nth report to ultimate for excess layer from c to d = $(f(c) - f(d))/(e_{c,n} - e_{d,n})$

In this case, the desired LDF is $(f(200000) - f(500000))/(e_{200000,39\text{ months}} - e_{500000,39\text{ months}})$
We calculate f(200000) = $500,000/$3,400,000 = 5/34.
We calculate f(500000) = $350,000/$3,400,000 = 7/68.

We recall that the $e_{c,n}$ and $e_{d,n}$ are ratios of nth-report excess losses to ultimate ground-up losses. To use the excess loss development factors (nth report excess loss)/(ultimate excess loss), we need to multiply f(c) and f(d) by (nth report excess loss)/(ultimate excess losses), or, equivalently, divide by the excess loss development factors.

Thus, $e_{200000,39\text{ months}} = (5/34)/4.13$
$e_{500000,39\text{ months}} = (7/68)/5.56$
Thus, the desired LDF is (5/34 - 7/68)/((5/34)/4.13 - (7/68)/5.56) = **2.581056575**

Section 66

Characteristics of Excess Loss Development – Part 2

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that may appear on the exam. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources: Pinto, E.; and Gogol, D.F., "An Analysis of Excess Loss Development," *PCAS* LXXIV, 1987, pp. 227-255. Including discussions of paper: Levine, G.M., *PCAS* LXXIV, 1987, pp. 256-271; and Bear, R.A., *PCAS* LXXIX, 1992, pp. 134-148.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-66-1. In what ways does Bear generalize/extrapolate upon the Pinto-Gogol formula for computation of industry loss development factors for unbounded layers as a function of the retention? Give a qualitative answer. (See Bear, pp. 136-137.)

Solution S6-66-1. Bear generalizes the Pinto-Gogol formula to apply to large-account primary pricing and account-specific reinsurance pricing. It is possible, using Bear's approach, to estimate the account-specific development pattern for a higher layer based on the pattern for a lower layer, if the two layers have the same ratios of gross limit to retention (Bear, pp. 136-137).

Problem S6-66-2. You are given the following definitions:

q_i = estimated value of single-parameter Pareto distribution parameter during the valuation at i
q_j = estimated value of single-parameter Pareto distribution parameter during the valuation at j
$e = q_i - q_j$
k_1 = Lower bound of excess layer k
k_2 = Upper bound of excess layer k
x_1 = Lower bound of excess layer x
x_2 = Upper bound of excess layer x
$c = x_1/k_1 \geq 1$
d = Incurred loss development factor from valuation at i to valuation at j for losses in layer k

Then Bear's generalized formula for the incurred loss development factor from valuation i to valuation j for losses in the layer x is as follows (fill in the blanks):

_____, where _____.

Solution S6-66-2. Bear's generalized formula for the incurred loss development factor from valuation i to valuation j for losses in the layer x is $\mathbf{dc^e}$, where $\mathbf{x_2/x_1 = k_2/k_1}$.

Problem S6-66-3. You are using Bear's generalized formula to compare the layer from $400,000 to $600,000 with the layer from $500,000 to $750,000. The estimated value of the single-parameter Pareto distribution parameter during the valuation at 12 months is 1.56. The estimated value of this parameter during the valuation at ultimate is 1.05. These parameter estimations were made on the basis of losses in the layer in excess of $400,000.

The incurred loss development factor from 12 months to ultimate, for the layer from $400,000 to $600,000, is estimated to be 2.45. Find the incurred loss development factor from 12 months to ultimate, for the layer from $500,000 to $750,000.

Solution S6-66-3. Our desired factor is dc^e, where $x_2/x_1 = k_2/k_1$.Here, $k_1 = 400000$, $k_2 = 600000$, $x_1 = 500000$, $x_2 = 750000$, so $750000/500000 = 600000/400000 = 1.5 \rightarrow x_2/x_1 = k_2/k_1$.

Here, d = 2.45, the LDF for the layer from $400,000 to $600,000. Also, $c = x_1/k_1 = 500000/400000 = 1.25$. Finally, $e = q_i - q_j = q_{12\ months} - q_{ultimate} = 1.56 - 1.05 = 0.51$.

Thus, $dc^e = 2.45*1.25^{0.51}$ = the desired LDF = **2.745302409**.

Problem S6-66-4. Levine used the Pinto-Gogol technique to fit development factors to empirical reinsurance data. What did he find regarding the error between the actual excess loss development factors and the fitted excess loss development factors? (See Levine, pp. 258-263.)

Solution S6-66-4. Levine found that, for retentions under $250,000, the error between the actual and fitted excess loss development factors is generally small (under ± 2%), but the error becomes considerably larger for retentions in excess of $250,000. Levine concludes that the Pinto-Gogol technique provides a poor fit to actual data for retentions in excess of $250,000.

Problem S6-66-5. Explain the "catch-up" theory described by Levine on pp. 263-264. Why does this theory suggest that, in using the Pinto-Gogol technique, it would be desirable to exclude very high retentions?

Solution S6-66-5. The "catch-up" theory attempts to explain the reversal observed by Pinto and Gogol for very high retentions ($500,000 and $1,000,000) of the tendency for excess loss development to increase as retention increases. The theory posits that, many times, the initial estimate of a large claim is held at the same level for many years, until an actual jury trial is held, after which there is a dramatic revision. Because of this dramatic revision, considering development at late maturities would actually *introduce* error into one's estimate. The "catch-up" theory suggests that different reserving practices are associated with claims that have very high retentions, so applying the Pinto-Gogol technique to those claims may not be optimal.

Problem S6-66-6. What did Levine observe regarding the application of the Pinto-Gogol technique to *primary* loss development? What could explain this observation? (See Levine, pp. 265-267.)

Solution S6-66-6. Levine observed "disappointing" results regarding the application of the Pinto-Gogol technique to primaryloss development. Using curves fitted to a single-parameter Pareto distribution has been known to underestimate trends for small sizes of losses, and this tendency may apply to loss development as well. The Pinto-Gogol techniques works better with excess losses than with primary losses because excess losses, by definition, exclude small sizes of losses.

Section 67

Reserve Estimation Using the Extended Link Ratio Family and the Probabilistic Trend Family of Models

Some of the questions here ask for short written answers. This is meant to give the student practice in answering questions of the format that will appear on Exam 6. Students are encouraged to type their own answers first and then to compare these answers with the solutions given here. Please note that the solutions provided here are not necessarily the only possible ones.

Sources: Barnett, G.; and Zehnwirth, B, "Best Estimates for Reserves," *PCAS* LXXXVII, 2000, pp. 245-303.

Original Problems and Solutions from The Actuary's Free Study Guide

Problem S6-67-1.
(a) What is the central objective of the Barnett and Zehnwirth paper? (See Barnett and Zehnwirth, p. 245.)
(b) Why do Barnett and Zehwirth pursue this objective (i.e., what problem are they trying to remedy)? (See Barnett and Zehnwirth, p. 245.)
(c) According to Barnett and Zehwirth, what are the "three critical stages" of arriving at a reserve figure? (See Barnett and Zehnwirth, pp. 245-246.)

Solution S6-67-1.

(a) Barnett and Zehnwirth seek to explain how to use the "extended link ratio family" (ELRF) of regression models to test the assumptions of standard link ratio techniques and to "compare their predictive power with modeling trends in the incremental data" (Barnett and Zehnwirth, p. 245).

(b) Barnett and Zehnwirth state that data in most loss arrays do not fulfill the assumptions of standard link ratio techniques, and ELRF approach is more consistent with empirical data. But the ELRF approach is itself deficient and offers a bridge from standard link ratio techniques to more thoroughly statistical modeling approaches (Barnett and Zehnwirth, p. 245).

(c) According to Barnett and Zehwirth, what are the "three critical stages" of arriving at a reserve figure are as follows:

1. Extracting trend and stability information from the data, including distributions about the trends;
2. Formulating assumptions about the future → forecasting paid loss distributions;

3. Considering correlations between lines and how they affect the desired security level (Barnett and Zehnwirth, pp. 245-246).

Problem S6-67-2.

(a) Fill in the blank: Barnett and Zehnwirth propose a statistical modeling framework based on the analysis of the _____ of the incremental data. (See Barnett and Zehnwirth, p. 247.)

(b) What are the four components of interest in the Barnett/Zehnwirth statistical modeling framework? (See Barnett and Zehnwirth, p. 247.)

(c) What is the name of the family of models based on the four components of interest in part (b)? (See Barnett and Zehnwirth, p. 247.)

(d) Fill in the blanks: The statistical nature of the Barnett/Zehnwirth modeling framework allows for the separation of _____ and _____. (See Barnett and Zehnwirth, p. 247.)

(e) What five beneficial functions does the Barnett/Zehnwirth modeling framework enable, aside from the answer to part (d)? (See Barnett and Zehnwirth, p. 247.)

Solution S6-67-2.

(a) Barnett and Zehnwirth propose a statistical modeling framework based on the analysis of the **logarithms** of the incremental data.

(b) The four components of interest in the Barnett/Zehnwirth statistical modeling framework are as follows (Barnett and Zehnwirth, p. 247):

1. Trend in the development period
2. Trend in the accident period
3. Trend in the payment/calendar period
4. Distribution of data about the trends

(c) The name of the family of models based on the four components of interest in part (b) is the **Probabilistic Trend Family (PTF)**.

(d) The statistical nature of the Barnett/Zehnwirth modeling framework allows for the separation of **parameter uncertainty** and **process variability**.

(e) The Barnett/Zehnwirth modeling framework enables the following five beneficial functions (Barnett and Zehnwirth, p. 247):

1. Checking that the data satisfy all the model's assumptions;
2. Calculating reserve forecast distributions and the total reserve;
3. Calculating distributions of future payment streams and the correlations between such payment streams;

4. Pricing for future underwriting years, including for excess layers and aggregate deductibles;
5. Updating models based on new data and tracking forecasts with relative ease.

Problem S6-67-3. On page 248, Barnett and Zehnwirth describe previous findings that influenced their paper.

(a) Fill in the blank: According to Brosius, using link ratio techniques corresponds to fitting a regression line without _____.

(b) How do Mack's findings relate to the statement in part (a)?

(c) What is heteroscedastic normality?

Solution S6-67-3.

(a) According to Brosius, using link ratio techniques corresponds to fitting a regression line without **an intercept term.**

(b) Mack presented diagnostics suggesting that actual data warrant using an intercept term with a regression line, indicating a possible flaw in link ratio techniques.

(c) Heteroscedastic normality is situation of data following a normal distribution *without* a constant variance for all the errors. (For some data, the variance may be greater or less than for other data.)

Problem S6-67-4. (a) On page 249, Barnett and Zehnwirth state that the link ratio trend or average method is based on the formula $y(i) = b*x(i) + \varepsilon(i)$, where $Var(\varepsilon(i)) = \sigma^2*x(i)^{\delta}$.

Explain the meaning of the variables x(i), y(i), b, $\varepsilon(i)$, σ^2, and δ. (See Barnett and Zehnwirth, pp. 249-250.)

(b) What happens when $\delta = 1$, and what commonly used development technique does this situation reflect? (See Barnett and Zehnwirth, pp. 250-251.)

(c) What happens when $\delta = 2$, and what commonly used development technique does this situation reflect? (See Barnett and Zehnwirth, p. 251.)

(d) What happens when $\delta = 0$, and what commonly used development technique does this situation reflect? (See Barnett and Zehnwirth, p. 251.)

Solution S6-67-4. (a) x(i) for some accident period i is the cumulative developed value at some development period (j-1).

y(i) for the same accident period i is the cumulative developed value at some development period j, i.e., at the development period after the one pertaining to x(i).

b is the slope of the "best" line that includes the origin and the data points (x(i), y(i)). Essentially, the formula y(i) = b*x(i) + ε(i) expresses a value at one development period as a function of the value at the previous development period.

ε(i) is the error term in the regression formula.

σ^2 is the base variance, or underlying variance, that is the same for the entire development period.

The parameter δ is a "weighting parameter" through which the variance of the error term depends to a certain extent on the value of x(i).

(b) When δ = 1, it is the case that the average value of y(i) is b*(x(i)), so b can be estimated (using least-squares estimation) as Σ(x(i)* y(i)/x(i))/Σ(x(i)) = Σ(y(i))/Σ(x(i)). Thus, the estimate of b is the volume-weighted-average chain-ladder ratio between the value at one development period and the value at the previous development period. That is, when δ = 1, we arrive at the **chain-ladder method** of reserving.

(c) When δ = 2, it is the case that the weighted least-squares estimator of b is (1/n) Σ(y(i)/x(i)), or the simple arithmetic average of the link ratios between each pair (x(i), y(i)). This is also the **chain ladder method,** except the averages used are simple arithmetic means instead of volume-weighted averages.

(d) When δ = 0, the least-squares estimator of b is an average weighted by volume-squared, corresponding to the technique of **ordinary least-squares regression through the origin.**

Problem S6-67-5.

(a) What are two advantages described by Barnett and Zehnwirth for estimating link ratios using regressions? (See Barnett and Zehnwirth, p. 251.)

(b) Refer to the formulas in Problem S6-67-4. What is the formula for the standardized errors? (See Barnett and Zehnwirth, p. 251.)

(c) What is the assumption needed to be made regarding the standardized errors in order to render the weighted least-squares estimator of b "efficient"? (See Barnett and Zehnwirth, p. 251.)

Solution S6-67-5.

(a) Estimating link ratios using regressions has the following advantages (Barnett and Zehnwirth, p. 251):

1. It is possible to obtain both the standard errors of the forecasts and the standard errors of the parameters in the average method selection.
2. It is possible to test the assumptions made by this method.

(b) The formula for the standardized errors is $\varepsilon(i)/\sqrt{(Var(\varepsilon(i))} = \varepsilon(i)/\sqrt{(\sigma^2 * x(i)^{\delta})} = \mathbf{\varepsilon(i)/(\sigma * x(i)^{\delta/2})}$ for all applicable values of i. Dividing by the square root of the variance (i.e., the standard deviation) of the error term is necessary for the standardized error to reflect a standard deviation of 1. (See the answer to part (c)).

(c) In order to render the weighted least-squares estimator of b "efficient", it is needed to assume that the standardized errors are **normally distributed with mean 0 and standard deviation 1** (Barnett and Zehnwirth, p. 251).

Problem S6-67-6. Barnett and Zehnwirth describe a basic assumption of the link ratio method that can be expressed in two equivalent ways:

(a) $E(y(i) \mid x(i)) = bx(i)$
(b) $E((y(i) - x(i)) \mid x(i)) = (b-1)x(i)$

Give a verbal interpretation of each of the above two expressions of this assumption. (See Barnett and Zehnwirth, p. 252.)

(c) What is a way to test this assumption, using a plot of fitted values on the horizontal axis and weighted standardized residuals on the vertical axis? What is typically empirically observed as a result of such tests? (See Barnett and Zehnwirth, pp. 252-253.)

(d) What do the empirical observations discussed in part (c) imply about the differences between actual results and the predictions of the link ratio method?

Solution S6-67-6.

(a) Given that we know the cumulative development at period (j-1), all we need to do to obtain the mean cumulative development at period j is to multiply the cumulative development at period (j-1) by the ratio b.

(b) Given that we know the cumulative development at period (j-1), in order to find the mean *incremental* development at during j, all we need to do is to multiply the cumulative development at period (j-1) by (b - 1), i.e., the ratio b, minus 1 so that we do not count the development through period (j-1).

(c) To test this assumption, plot fitted values on the horizontal axis and weighted standardized residuals on the vertical axis. If the assumption is correct, there should be a random distribution of weighted standard residuals versus fitted values, with no way to fit an intelligible line or curve through them. However, empirically, it has been observed that there is a downward-sloping line that can be fitted to this plot.

(d) The link ratio method *overpredicts large values and underpredicts small values*. (E.g., for large values, the fitted values are too high, and, for small values, the fitted values are too low.)

Problem S6-67-7. (a) How did Murphy modify the equation from Problem S6-67-4? Give the modified equation and explain the meaning of any additional term(s). (See Barnett and Zehnwirth, p. 253.)

(b) What does Murphy's modification imply regarding the ability of prior cumulative development x(i) to predict subsequent incremental development y(i)-x(i)? Does this correspond to observations from empirical data in situations where b = 1? (See Barnett and Zehnwirth, p. 255.)

Solution S6-67-7. (a) Murphy modified the equation from Problem S6-67-4 as follows:
$y(i) = a + b*x(i) + \varepsilon(i)$, where $Var(\varepsilon(i)) = \sigma^2*x(i)^\delta$.

The new term added is a, an intercept that enables the regression line used in estimating the link ratio not to go through the origin.

(b) Murphy's modification implies that x(i) can **no longer** predict y(i)-x(i). This is because the intercept a is estimated via a weighted average of the incremental development in period j pertaining to the values of y(i). Thus, the value of a is independent of any x(i).

This implication **does correspond** to empirical data in situations where b = 1, which show virtually zero correlation between prior cumulative development and subsequent incremental development.

Problem S6-67-8. (a) How can one further modify the equation from Solution S6-67-7 to reflect a situation where there *is* a trend in incremental development down the accident periods? Describe the meaning of any additional term(s) used. (See Barnett and Zehnwirth, p. 256.)

(b) How can the Cape Cod (Stanard-Bühlmann) method be described in terms of the formula from part (a)? (See Barnett and Zehnwirth, p. 257.)

Solution S6-67-8. (a) The modified formula is as follows:
$y(i) = a_0 + a_1*i + b*x(i) + \varepsilon(i)$, where $Var(\varepsilon(i)) = \sigma^2*x(i)^\delta$.

The old intercept a becomes a_0 and serves the same purpose. The new addition is the parameter a_1, which relates to the accident period i. This is the accident-period trend parameter; it can capture changes in cumulative (and corresponding incremental) development in later accident periods, as compared to earlier ones.

(b) The Cape Cod method can be seen as a special case of the formula in part (a), where it is assumed that b = 1 and $a_1 = 0$. (That is, there is no accident-period trend, and there is no effect of prior cumulative development on subsequent incremental development.) The Cape Cod formula becomes $y(i) = a_0 + x(i) + \varepsilon(i)$, where $Var(\varepsilon(i)) = \sigma^2*x(i)^\delta$.

Problem S6-67-9. In general terms, what does the Cape Cod method assume regarding the relationship among incremental values in the same development period? Contrast this to the

approach of the chain ladder method and discuss the implications regarding coefficients of variation in incremental values for each method. (See Barnett and Zehnwirth, pp. 261-262.)

Solution S6-67-9. The Cape Cod method assumes that incremental values in the same development period are randomly drawn from the same distribution. The chain ladder method makes no such assumption; instead, the incremental values are related to previous cumulative values via the same link ratios for each accident period. Because the Cape Cod method inherently assumes a relationship among incremental values in the same development period, we can expect to observe lower coefficients of variation for such values, as compared to the coefficients of variation in a chain-ladder scenario.

Problem S6-67-10. (a) In what way can the models described by the equation in Solution S6-67-8 be considered illustrative of the bridge that the ELRF forms to the statistical models in the PTF? (See Barnett and Zehnwirth, p. 264.)

(b) What assumption do the ELRF models make regarding the weighted standardized errors, which is rarely true for real-world reserving data? What is generally observed empirically instead, and what approach does this observation suggest? (See Barnett and Zehnwirth, p. 264.)

Solution S6-67-10. (a) As a bridge to the PTF, the ELRF models are capable of more accurately incorporating real-world data and phenomena than the traditional link ratio techniques. What they can do is identify payment-period trend changes diagnostically, but not identify them directly or forecast on their basis; thus, they still fall short of the statistical models in the PTF (Barnett and Zehnwirth, p. 264).

(b) The ELRF models make the assumptions that the weighted standardized errors **are normally distributed**, which is rarely true in the real world. Instead, it is generally observed that weighted standardized errors are **skewed to the right**; that is, more of them are positive than are negative. This observation suggests that **a logarithmic scale** (analyzing the logarithms of the weighted standard errors) would be better suited to analyzing them (Barnett and Zehnwirth, p. 264).

Problem S6-67-11. Explain how the use of logarithms can introduce linearity into observed dollar trends. (See Barnett and Zehnwirth, p. 266.)

Solution S6-67-11. Observed real-world dollar trends are typically percentage trends (e.g., growth at X% per year). But logarithms of incremental data that follow percentage trends themselves follow linear trends.

Problem S6-67-12.(a) If the development year can be expressed by the variable j and the accident year can be expressed by the variable i, find the expression for the *payment year* (t). (See Barnett and Zehnwirth, p. 266.)

(b) Suppose that all data being used are adjusted for wage and price inflation, but the payment-year trend is still positive. What phenomenon does this observation indicate? (See Barnett and Zehnwirth, p. 267.)

Solution S6-67-12. (a) The expression for payment year is **t = i + j**. That is, loss amounts that develop j years after a loss occurs in year i will be paid in year (i + j).

(b) If all data being used are adjusted for wage and price inflation, but the payment-year trend is still positive, this is indicative of **social inflation** (e.g., higher average legal verdicts and increases in medical costs above the increases in the costs of medical inputs).

Problem S6-67-13. Let $y(j) = \alpha + {}_{k=1}^{j}\Sigma(\gamma_k) + \varepsilon_j$, where k = 1, ..., j are the development years, ε_j is the error term (normally distributed with a mean of zero), α is the expected development in the first development year, each γ_k is the mean trend between development years (k - 1) and k.

Fill in the blanks (See Barnett and Zehnwirth, p. 271-272):

(a) y(0) has a mean of _____ and a variance of σ^2.

(b) Let p(j) = exp(y(j)). Then the median of p(j) is _____.

(c) The mean of p(j) is the median of p(j), multiplied by _____.

(d) The standard deviation of p(j) is the mean of p(j), multiplied by _____.

(e) The distribution of p(j) is _____.

Solution S6-67-13.

(a) y(0) has a mean of **α** and a variance of σ^2.

(b) Let p(j) = exp(y(j)). Then the median of p(j) is $\mathbf{\exp(\alpha + {}_{k=1}^{j}\Sigma(\gamma_k))}$.

(c) The mean of p(j) is the median of p(j), multiplied by $\mathbf{\exp(0.5\sigma^2)}$.

(d) The standard deviation of p(j) is the mean of p(j), multiplied by $\mathbf{\sqrt{(\exp(\sigma^2) - 1)}}$.

(e) The distribution of p(j) is **lognormal**.

Problem S6-67-14. (a) Give the general formula for the Probabilistic Trend Family (PTF) of models and explain the meaning of each term. (See Barnett and Zehnwirth, p. 273.)

(b) Based on this formula, with relation to accident year i and development year j, what is the mean trend between (i, j -1) and (i, j)? (See Barnett and Zehnwirth, p. 273.)

(c) Based on this formula, with relation to accident year i and development year j, what is the mean trend between (i-1, j) and (i, j)? (See Barnett and Zehnwirth, p. 273.)

Solution S6-67-14. (a) The general formula for the Probabilistic Trend Family (PTF) of models is $\mathbf{y(i,j) = \alpha_i + {}_{k=1}^{j}\Sigma(\gamma_k) + {}_{t=1}^{i+j}\Sigma(\iota_t) + \varepsilon_{i,j}}$, where

i = accident year
j = development year
t = i+j = payment year
α_i = expected development in the first development year (j = 0)
γ_k = mean trend between development year (k-1) and development year k.
ι_t = mean of the inflation between payment year t and t + 1
$\varepsilon_{i,j}$ = error term (normally distributed with mean 0)
y(i,j) = natural logarithm of incremental paid loss for accident year i, development year j.

(b) The mean trend between (i, j -1) and (i, j) is $\boldsymbol{\gamma_j + \iota_{i+j}}$.

(c) The mean trend between (i-1, j) and (i, j) is $\boldsymbol{\alpha_i - \alpha_{i-1} + \iota_{i+j}}$.

Problem S6-67-15. What is a possible problem resulting from several parameters in the same model pertaining to payment-year and accident-year trends? Briefly, how might such a problem be overcome? (See Barnett and Zehnwirth, p. 295.)

Solution S6-67-15. Multicollinearity is a possible problem resulting from several parameters in the same model pertaining to payment-year and accident-year trends. This is because payment year is a linear combination of accident year and development year, and so there might be some definitional overlap among the parameters. To overcome this problem, one could use a varying-parameter stochastic model, especially one in which the α parameter varies, instead of adding new parameters (Barnett and Zehnwirth, p. 295).

Problem S6-67-16. If you were analyzing a simulated cumulative loss development array created using traditional ratio methods, how would using a PTF model enable you to distinguish this array from an array based on actual observed data? (See Barnett and Zehnwirth, p. 297.)

Solution S6-67-16. Using a PTF model to analyze the array would show that, after removing the trends in the direction of development year, there are still clear patterns in the direction of accident year. Since the same ratios are applied to data for all accident years, the initial discrepancies among accident years at early maturities are never mitigated; there is much more volatility among accident years than would be observed for real data (Barnett and Zehnwirth, p. 297).

Problem S6-67-17.
(a) In order for a forecast distribution to accurately predict the future, at least three assumptions about trends need to be true. Identify the three assumptions as listed by Barnett and Zehnwirth on p. 298.
(b) What is the difference between a fitted distribution and a predictive distribution? (See Barnett and Zehnwirth, p. 299.)

Solution S6-67-17.

(a) In order for a forecast distribution to accurately predict the future, at least the following assumptions about trends need to be true (Barnett and Zehnwirth, p. 298):

1. Assumptions regarding mean trends
2. Assumptions regarding standard deviations of trends
3. Assumptions regarding distributions about the trends

(b) A predictive distribution incorporates into itself parameter risk (the risk of estimating a parameter erroneously), whereas a fitted distribution does not (Barnett and Zehnwirth, p. 299).

Problem S6-67-18. What three important observations with regard to risk-based capital do Barnett and Zehnwirth offer on p. 301?

Solution S6-67-18. Barnett and Zehnwirth offer the following three observations regarding on page 301:

1. Loss reserve uncertainty should be based on a future-oriented probabilistic model and does not have a necessary relationship to the company's past reserves.
2. The company may have unique experience, not reflective of the industry's experience as a whole.
3. For each line of the company's business, the reserve uncertainty should be modeled by a probabilistic model unique to that line and using company-specific experience, as a model derived from the experience of another line will typically not be accurate when applied to the line in question.

Problem S6-67-19. What are the three steps for booking a reserve described by Barnett and Zehnwirth on p. 302?

Solution S6-67-19. The following are the three steps for booking a reserve (Barnett and Zehnwirth, p. 302):

1. Extract information about trends for the loss development array - their stability, distribution about them - especially for incremental paid losses. This is done by identifying the best model in the PTF.
2. Formulate assumptions about the future: will future trends be stable or not?
3. Select a security margin for the combined lines of business. Determine the percentile at which the reserve will be established, based on the reserve distribution, the security margin, and the company's available risk capital.

Problem S6-67-20. What two additional benefits does the statistical modeling framework offer? (See Barnett and Zehnwirth, pp. 302-303.)

Solution S6-67-20. The following are two additional benefits of the statistical modeling framework:

1. **Incorporation of credibility:** It is possible to bring a trend parameter estimate that is lacking credibility closer to the level of the complement of credibility (e.g., the industry estimate).
2. **Segmentation and layers:** The model can be applied specifically to some of the segments of the company's book of business or some of the insured layers.

About Mr. Stolyarov

Gennady Stolyarov II (G. Stolyarov II) is an actuary, science-fiction novelist, independent philosophical essayist, poet, amateur mathematician, composer, and Editor-in-Chief of The Rational Argumentator, a magazine championing the principles of reason, rights, and progress.

In December 2013, Mr. Stolyarov published *Death is Wrong*, an ambitious children's book on life extension illustrated by his wife Wendy. *Death is Wrong* can be found on Amazon in paperback and Kindle formats.

Mr. Stolyarov has contributed articles to the Institute for Ethics and Emerging Technologies (IEET), The Wave Chronicle, Le Quebecois Libre, Brighter Brains Institute, Immortal Life, Enter Stage Right, Rebirth of Reason, The Liberal Institute, and the Ludwig von Mises Institute. Mr. Stolyarov also published his articles on Associated Content (subsequently the Yahoo! Contributor Network) from 2007 until its closure in 2014, in an effort to assist the spread of rational ideas. He held the highest Clout Level (10) possible on the Yahoo! Contributor Network and was one of its Page View Millionaires, with over 3.1 million views.

Mr. Stolyarov holds the professional insurance designations of Associate of the Society of Actuaries (ASA), Associate of the Casualty Actuarial Society (ACAS), Member of the American Academy of Actuaries (MAAA), Chartered Property Casualty Underwriter (CPCU), Associate in Reinsurance (ARe), Associate in Regulation and Compliance (ARC), Associate in Personal Insurance (API), Associate in Insurance Services (AIS), Accredited Insurance Examiner (AIE), and Associate in Insurance Accounting and Finance (AIAF).

Mr. Stolyarov has written a science fiction novel, *Eden against the Colossus*, a philosophical treatise, *A Rational Cosmology*, a play, *Implied Consent*, and a free self-help treatise, *The Best Self-Help is Free*. You can watch his YouTube Videos. Mr. Stolyarov can be contacted at gennadystolyarovii@gmail.com.