# Exam-Style Questions Relevant to the New Casualty Actuarial Society Exam 7 <br> G. Stolyarov II, ARe, AIS <br> Spring 2011 

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(Adapted from The Actuary's Free Study Guide for Exam 6 by Mr. Stolyarov)
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## Sources:

## Past Casualty Actuarial Society exams: 2007 Exam 6, 2008 Exam 6, and 2009 Exam 6.

## Problem S6-8-4. Similar to Problem 12 from the Fall 2008 Exam 6.

For a workers' compensation high-deductible policy, the full-coverage premium is $\$ 120,140$, and the full-coverage expected loss ratio is 0.560 . The excess ratio (the ratio of losses expected to be above the deductible) is 0.222 . The aggregate ratio (the proportion of losses below the deductible for which the insurer may still have to incur expenses, for instance, via administering the policy or collecting the deductible from the insured) is 0.05 . For the time period in question, it is known that reported excess losses were $\$ 16,000$, and the excess loss development factor is 1.11 .
(a) Using the loss ratio method to set reserves, what is the estimated ultimate loss for this policy?
(b) Would the loss ratio method be more preferable for a small start-up workers' compensation insurer or a large, established insurer with plenty of its own credible data? Explain your choice.

## Solution S6-8-4.

(a) The loss ratio method for workers' compensation high-deductible policies does not rely on observed losses. Thus, reported excess losses and their associated development factor are not going to be used. There are two components to the loss ratio method: (1) the expected excess loss and (2) the losses associated with the aggregate ratio.
The expected excess loss component is $($ Premium $) *($ Full-Coverage ELR $) *($ Excess Ratio $)=$ $120140 * 0.560 * 0.222=14935.8048$.
The losses associated with the aggregate ratio are (Premium)*(Full-Coverage ELR)*(1-Excess Ratio) ${ }^{*}($ Aggregate Ratio $)=120140 * 0.560 *(1-0.222) * 0.05=2617.12976$.
The total estimated ultimate loss is $14935.8048+2617.12976=17552.93456=\mathbf{\$ 1 7 , 5 5 2 . 9 3}$.
(b) The loss ratio method to set reserves would be preferable for the small insurer which does not have a lot of its own data to estimate ultimate losses. The approach uses industry data and therefore adds credibility. However, for a large, established insurer, the loss ratio method has the drawback of not considering that insurer's extensive loss experience data. Also, the insurer's own practices and book of business may differ from those of the overall industry, and thus a reliance on the large insurer's own data may be more appropriate.

Problem S6-14-1. Similar to Problem 5 from the 2007 CAS Exam 6. Suppose that an insurance policy is retrospectively rated and the following information is known:

Loss conversion factor $=1.22$
Tax multiplier $=1.04$
Basic premium $=\$ 13,165$
Loss at first retro adjustment: $\$ 15,530$
Capped loss at first retro adjustment: $\$ 15,530$
Loss at second retro adjustment: $\$ 25,010$
Capped loss at second retro adjustment: $\$ 20,000$
What is the ratio of premium development to loss development between the first and second retro adjustments?

Solution S6-14-1. We can already figure out how much loss has developed between the two adjustments: 25010-15530 = \$9480.

To determine premium development, we need to calculate the retrospectively rated premium at each adjustment, using Formula 14.1: $\mathrm{P}=(\mathrm{B}+\mathrm{CL} * \mathrm{LCF}) * \mathrm{TM}$.

At the first retro adjustment, the premium is $(13165+15530 * 1.22)^{*} 1.04=33396.064$.
At the second retro adjustment, the premium is $(13165+20000 * 1.22) * 1.04=39067.6$. Premium development is thus 39067.6-33396.064 $=5671.536$.

The desired ratio is $5671.536 / 9480=\mathbf{0 . 5 9 8 2 6 3 2 9 1 1}$.
Problem S6-18-1. Similar to Question 36 from the 2007 CAS Exam 6. Explain why, for a retrospectively rated policy, retrospective premiums will increase at a decreasing rate as reported losses increase. Also explain why this would apply, in particular, to cases where catastrophic injuries make a person eligible for lifetime benefits.

Solution S6-18-1. The determination of retrospective premium is subject to per-occurrence caps on losses and also, typically, to a maximum premium amount. Thus, not the entire amount of every subsequent loss would be added retrospective premium, and the proportion of each subsequent loss's contribution is probably going to decrease. With regard to cases where catastrophic injuries make a person eligible for lifetime benefits, the initial injury may not seem like one that will disable an individual for life. Later, as additional information is discovered, the loss would develop, and the cap and maximum premium would be quite likely to apply.

Problem S6-24-1. Similar to Question 11 from the 2007 CAS Exam 6. For each of the following pairs of variables, explain why examining them via an integrated approach in a Dynamic Financial Analysis (DFA) model is superior to using a silo approach.
(a) Underwriting profit and investment profit
(b) Loss ratio and mix of business in terms of new versus renewal business

Solution S6-24-1. In general terms, an integrated approach examines the interrelationships among variables, while a silo approach considers variables in isolation. A good resource on questions of this type is the CAS "Overview of Enterprise Risk Management".

The following are sample answers, and other valid explanations are possible.
(a) Underwriting profit and investment profit both contribute to the overall profitability of the insurer. The insurer may be able to afford to sustain slight losses in a particular year if its investments are performing well and can make up the difference. On the other hand, if the stock market is in decline and bonds are not yielding much income, the insurer may need to raise its rates and/or implement stricter underwriting standards in order to remain profitable overall.
(b) The insurer's loss experience may vary between its established customers and its new customers. If the loss ratio increases, this may be a sign of loosening underwriting standards. On the other hand, if the loss ratio decreases, this may be because the insurer has been able to attract more favorable loss exposures to its book of business.

Problem S6-24-4. Similar to Question 21 from the 2007 CAS Exam 6. What do the StanardBühlmann and Bornhuetter-Ferguson methods have in common? How are they different, in essential terms?

Solution S6-24-4. Both methods rely on the formula
Ultimate Losses $=$ Reported Losses $+(\%$ Losses Unreported $) *($ Expected Losses $)$.
In the Bornhuetter-Ferguson Method, the expected loss ratio is estimated judgmentally. Losses are compared to earned premium that is not brought to the present rate levels.

In the Stanard- Bühlmann Method, adjusted premium is used instead of earned premium; adjusted premium is earned premium adjusted to current rate levels. Also, the expected loss ratio is estimated on the basis of reported claim experience from the overall time period being examined.
(See Friedland, pp. 174-175.)
Problem S6-24-5. Similar to Question 25 from the 2007 CAS Exam 6. Identify four possible problems with industry aggregates of reinsurance data as currently exist in the United States.

Solution S6-24-5. The following are possible problems with industry aggregates of reinsurance data:

1. Different reinsurance policies have different limits and attachment points, and the aggregate data are not separated by limits/attachment points.
2. Various reinsurers may choose to include or omit data about asbestos, pollution, or other environmental hazards in the numbers they submit.
3. Reinsurance contracts are unique, so the data are generally not comparable across reinsurers; there is no homogeneity on the basis of which the law of large numbers could work (Patrik, pp. 436-437).
4. Low claim frequency and extreme report lags may contribute to large fluctuations in the data (Patrik, p. 437).

Problem S6-37-4. Similar to Question 45 from the 2007 CAS Exam 6. There are two excess-of-loss reinsurance treaties. In Treaty A, the primary insurer's retention is $\$ 500,000$. In Treaty B, the primary insurer's retention is $\$ 5,000,000$. For which treaty would you expect the excess loss development factors to be higher? Give two reasons justifying your answer.

Solution S6-37-4. One would expect the excess loss development factors to be higher for Treaty B, because it has the higher retention. Two reasons why this happens is (1) larger losses that would exceed the higher retention are more likely to be reported later, since the primary insurer may not expect certain initial claims to develop to such an extent, and (2) the smaller claims that do not exceed the retention are likely to be reported sooner. More of the smaller claims would contribute to the excess loss development for a treaty with a smaller retention.

Problem S6-44-2. Similar to Question 3 from the 2009 CAS Exam 6. You are analyzing the following information about cumulative paid losses for Insurer $\Psi$ by accident year (AY):

Cumulative Paid Loss, expressed in the format (Amount at development year 0, Amount at development year 1, Amount at development year 2).
AY 2022: $(343,444,500)$
AY 2023: $(360,500,555)$
AY 2024: $(320,466)$
AY 2025: (350)
(a) Use Brosius's least-squares method to find the expected losses at development year 1 for AY 2025.
(b) Is the least-squares method proper to use in a situation such as the one in part (a)? Explain why or why not.

## Solution S6-44-2.

(a) First we find the various averages necessary for the least-squares method.

Let $x$ be experience at development year 0 , and let y be experience at development year 1 .
$\mathrm{x}^{-}=(343+360+320) / 3=341$.
$y^{-}=(444+500+466) / 3=470$
$(x y)^{-}=(343 * 444+360 * 500+320 * 466) / 3=160470.666667$
$\left(x^{2}\right)^{-}=\left(343^{2}+360^{2}+320^{2}\right) / 3=116549.666667$
$\left(x^{-}\right)^{2}=341^{2}=116281$
Now we find $b=\left((x y)^{-}-x^{-*} y^{-}\right) /\left(\left(x^{2}\right)^{-}-\left(x^{-}\right)^{2}\right)=$ $(160470.666667-341 * 470) /(116549.666667-116281)=b=0.7468982642$.

Now we find $\mathrm{a}=\mathrm{y}^{-}-\mathrm{b}^{*} \mathrm{x}^{-}=470-0.7468982642 * 341=215.3076919$.
For AY 2025, $\mathrm{x}=350$, so $\mathrm{y}=\mathrm{a}+\mathrm{bx}=215.3076919+0.7468982642 * 350=\mathbf{4 7 6 . 7 7 2 2 0 8 4 4}$.
(b) It is proper to use the least-squares method in this situation, because $\mathrm{b}>0$.

## Problem S6-44-5. Similar to Question 14 from the 2008 CAS Exam 6.

You have the following information as of December 31, 2050, for a book of retrospectively rated policies for which business first began to be written in 2047.

## Expected Future Loss Emergence

For Policy Year 2047: 8000
For Policy Year 2048: 56000
For Policy Year 2049: 123000
For Policy Year 2050: 352000

## Cumulative Premium Development to Loss Development (CPDLD) Ratio

For Policy Year 2047: 0.25
For Policy Year 2048: 0.66
For Policy Year 2049: 1.04
For Policy Year 2050: 1.44

## Premium Booked from Prior Adjustments

For Policy Year 2047: 222000
For Policy Year 2048: 180000
For Policy Year 2049: 150000
For Policy Year 2050: 0

## Premium Booked

For Policy Year 2047: 224000
For Policy Year 2048: 201000
For Policy Year 2049: 180000
For Policy Year 2050: 190000
Calculate the premium asset as of December 31, 2050, using the methodology of Teng and Perkins.
Solution S6-44-5. The CPDLD ratio - a ratio of premium development to loss development - can be used to estimate future premium emergence from expected future loss ratio.

Expected Future Premium Emergence
For Policy Year 2047: $8000 * 0.25=2000$
For Policy Year 2048: $56000 * 0.66=36960$
For Policy Year 2049: $123000 * 1.04=127920$
For Policy Year 2050: 352000* $1.44=506880$
The expected ultimate premium is the sum of the prior booked premium and the expected future premium emergence.

Expected Ultimate Premium
For Policy Year 2047: $222000+2000=224000$
For Policy Year 2048: $180000+36960=216960$
For Policy Year 2049: $150000+127920=277920$
For Policy Year 2050: $0+506880=506880$
The premium asset is the expected ultimate premium minus the premium booked.

## Premium Asset

For Policy Year 2047: 224000-224000 $=0$
For Policy Year 2048: 216960-201000 $=15960$
For Policy Year 2049: 277920-180000 $=97920$
For Policy Year 2050: 506880-190000 $=316880$
Our total premium asset is thus $0+15960+97920+316880=\mathbf{4 3 0 7 6 0}$.
Problem S6-45-2. Similar to Question 6 from the 2009 CAS Exam 6. You have the following information on a book of retrospectively rated workers' compensation policies:

Standard premium: $\$ 260,000$
Expected loss ratio to standard premium: 70\%
Tax multiplier: 1.03
Loss cost factor: 1.25
Basic premium factor: 0.30

## Percentages at Third Retrospective Adjustment

Loss eliminated by maximum and minimum: $16 \%$ Loss eliminated by accident limit: $10 \%$
Total loss emerged: 55\%

## Percentages at Ultimate

Loss eliminated by maximum and minimum: $24 \%$ Loss eliminated by accident limit: $13 \%$
Total loss emerged: 100\%
What is the estimated future premium after the third retrospective adjustment?
Solution S6-45-2. The estimated future premium after the third retrospective adjustment is the difference between the estimated future premium at ultimate and the estimated premium up to the third retrospective adjustment.

The basic premium is the same irrespective of loss experience. Since it does not vary among retrospective adjustments, it can be disregarded for the purposes of this calculation. We want to find the components of premium that vary on the basis of losses. We need to convert standard premium to losses, take into account the losses eliminated, and then multiply the result by the loss cost factor and the tax multiplier to have the estimated premium take into account other elements besides losses (e.g., taxes and other expenses).

Estimated non-basic premium up to the third retrospective adjustment:
(Standard Premium)*(Expected Loss Ratio)*(\% Loss Emerged)(1-\% Loss Eliminated)*(Loss Cost Factor $)^{*}($ Tax Multiplier $)=260000^{*} 0.7 * 0.55 *(1-0.16-0.1) * 1.25^{*} 1.03=95370.275$
Estimated ultimate non-basic premium:
(Standard Premium)*(Expected Loss Ratio)*(\% Loss Emerged)*(1-\% Loss Eliminated)*(Loss Cost Factor)*(Tax Multiplier) $=260000^{*} 0.7^{*} 1^{*}(1-0.24-0.13) * 1.25^{*} 1.03=147624.75$.

Thus, the estimated future premium after the third retrospective adjustment is $147624.75-95370.275=$ $52254.475=\mathbf{5 2}, \mathbf{2 5 4 . 4 8}$.

Problem S6-45-3. Similar to Question 10 from the 2008 CAS Exam 6. You are given the following information for an insurer's book of business for a particular accident year:

Earned premium: $\$ 34,350$
Reported losses as of 24 months: $\$ 14,515$
Expected loss ratio: 80\%
Coefficient of variation of loss ratio: 0.66
Coefficient of variation of percent of loss reported: 0.88
Expected percent of loss reported at 24 months: 55\%
(a) What is the linear approximation to the Bayesian credibility estimate, as of 24 months, for the ultimate loss for this accident year?
(b) Calculate estimates of ultimate loss at age 24 months using each of the following methods (i) the chain ladder method, (ii) the Bornhuetter-Ferguson method, (iii) the Benktander method.
(c) Explain, for this particular situation, how the Benktander method incorporates the idea of credibility.

## Solution S6-45-3.

(a) Let Y denote losses and $\mathrm{X} / \mathrm{Y}$ denote the reporting pattern. Then $\mathrm{E}(\mathrm{Y})=$ (Expected Loss Ratio $) *($ Earned Premium $)=0.8 * 34350=\mathrm{E}(\mathrm{Y})=27480$.

The coefficient of variation (CV) is (Standard Deviation)/(Mean). $\operatorname{So} \operatorname{SD}(\mathrm{Y}) / \mathrm{E}(\mathrm{Y})=0.66$, and thus $\mathrm{SD}(\mathrm{Y})=0.66 * \mathrm{E}(\mathrm{Y})=0.66 * 27480=18136.8$, and $\operatorname{Var}(\mathrm{Y})=\mathrm{SD}(\mathrm{Y})^{2}=18136.8^{2}=328943514.2$.
$\mathrm{E}(\mathrm{X} / \mathrm{Y})$ is given as 0.55 , and $\mathrm{SD}(\mathrm{X} / \mathrm{Y})=\mathrm{CV}(\mathrm{X} / \mathrm{Y}) * \mathrm{E}(\mathrm{X} / \mathrm{Y})=0.55 * 0.88=0.484$. Thus, $\operatorname{Var}(\mathrm{X} / \mathrm{Y})=$ $0.484^{2}=\operatorname{Var}(\mathrm{X} / \mathrm{Y})=0.234256$.

We recall the formula for the credibility percentage $\mathrm{Z}: \mathrm{Z}=(\mathrm{VHM}) /(\mathrm{EVPV}+\mathrm{VHM})$.
We need to find $\mathrm{VHM}=\operatorname{Var}_{\mathrm{Y}}(\mathrm{E}(\mathrm{X} / \mathrm{Y}) * \mathrm{Y})=\operatorname{Var}(0.55 \mathrm{Y})=0.55^{2} * \operatorname{Var}(\mathrm{Y})=0.55^{2} * 328943514.2=\mathrm{VHM}$ $=99505413.06$.

We also need to find $\mathrm{EVPV}=\operatorname{Var}(\mathrm{X} / \mathrm{Y})^{*}\left(\operatorname{Var}(\mathrm{Y})+\mathrm{E}(\mathrm{Y})^{2}\right)=0.234256 *\left(328943514.2+27480^{2}\right)=$ 2539455504.

Thus, $\mathrm{Z}=(\mathrm{VHM}) /(\mathrm{EVPV}+\mathrm{VHM})=99505413.06 /(2539455504+99505413.06)=\mathrm{Z}=0.2815174416$.
This credibility is being assigned to expected ultimate losses based on losses that are already developed, while the complement of credibility is assigned to $\mathrm{E}(\mathrm{Y})$, the expected losses. Here, expected ultimate losses based on losses that are already developed, are (Losses already developed)/(\% Losses reported) $=14515 / 0.55=26390.0909090909$.

Thus, our estimate is $0.2815174416 * 26390.0909090909+(1-0.2815174416) * 27480=27173.40191=$ $\mathbf{\$ 2 7 , 1 7 3 . 4 0}$.
(b) (i) Using the chain ladder method, the expected loss is (Loss already reported) $/(\%$ Loss reported) $=$ $14515 / 0.55=26390.0909090909=\mathbf{\$ 2 6 , 3 9 0 . 0 9}$.
(ii) Using the Bornhuetter-Ferguson method, one assumes that unreported losses will be (Expected losses)*( $1-\%$ Losses already reported), and adds already reported losses to this value: $27480 *(1-0.55)+14515=\mathbf{\$ 2 6 , 8 8 1}$.
(iii) The Benktander method is an iteration of the Bornhuetter-Ferguson method, with the BornhuetterFerguson estimate substituted in place of expected losses:
$26881 *(1-0.55)+14515=\$ 26,611.45$.
(c) The Benktander method can be seen as a credibility-weighted estimate in the sense that the percentage of credibility assigned to the chain-ladder estimate is equal to the percent of ultimate losses reported. The value 14515 in part (b)(iii) can be seen as the chain ladder estimate (26390.0909090909) multiplied by the percentage of credibility ( $55 \%$ ). The complement of credibility is assigned to the Bornhuetter-Ferguson estimate (26881).

Problem S6-48-1. Similar to Question 36 from the 2008 CAS Exam 6. You have the following information as of December 31, 2050:

Earned Premium for Calendar/Accident Year 2047: 35055
Earned Premium for Calendar/Accident Year 2048: 38899
Earned Premium for Calendar/Accident Year 2049: 37600
Earned Premium for Calendar/Accident Year 2050: 41400

Adjusted Premium for Calendar/Accident Year 2047: 34400
Adjusted Premium for Calendar/Accident Year 2048: 36011
Adjusted Premium for Calendar/Accident Year 2049: 37000
Adjusted Premium for Calendar/Accident Year 2050: 40000
Aggregate Reported Loss for Calendar/Accident Year 2047: 22222
Aggregate Reported Loss for Calendar/Accident Year 2048: 16244
Aggregate Reported Loss for Calendar/Accident Year 2049: 12522
Aggregate Reported Loss for Calendar/Accident Year 2050: 4040
Aggregate Loss Report Lag for Calendar/Accident Year 2047: 0.95
Aggregate Loss Report Lag for Calendar/Accident Year 2048: 0.75
Aggregate Loss Report Lag for Calendar/Accident Year 2049: 0.60
Aggregate Loss Report Lag for Calendar/Accident Year 2050: 0.20
Note that "report lag" here refers to the proportion of losses assumed to be already reported, not the proportion assumed to have yet to be reported.

Use the Cape Cod method to calculate IBNR as of December 31, 2050.

Solution S6-48-1. We can calculate the expected loss ratio (ELR) as follows:
ELR = (Sum of aggregate reported losses for each year)/(Sum of used-up premiums for each year), where the used-up premium for each year is (Adjusted Premium)*(Aggregate Loss Report Lag). Thus, ELR $=(22222+16244+12522+4040) /(0.95 * 34400+0.75 * 36011+0.60 * 37000+$ $0.20 * 40000)=0.6121823486$.

By the Cape Cod Method, IBNR $=E L R *$ (Sum of adjusted premiums for each year) - (Sum of aggregate reported losses for each year) $=$
$0.6121823486 *(34400+36011+37000+40000)-(22222+16244+12522+4040)=35214.4122=$ $\mathbf{\$ 3 5 , 2 1 4 . 4 1}$.

Problem S6-48-2. Similar to Question 37 from the 2008 CAS Exam 6. Provide two advantages and disadvantages of using a reinsurer's own experience, as opposed to using reinsurance industry data, in applying the Stanard-Bühlmann (Cape Cod) reserving technique.

Solution S6-48-2. This is a sample answer, and other valid answers are possible.
Advantages:

1. The reinsurer may have a history of rate changes that is not reflected in industry data.
2. The reinsurer may have a unique book of business that consists of a mix of treaty types not adequately represented in industry data.

Disadvantages:

1. The reinsurer's experience may be sparse and therefore not credible.
2. The reinsurer's data may be less stable than industry data, and a lot of the volatility in reinsurer data may be "noise" that would not provide a meaningful reserve estimate.

Problem S6-50-4. Similar to Question 31 from the 2009 CAS Exam 6.
You have the following information as of December 31, 2022:
Earned Premium for Calendar/Accident Year 2020: 11500
Earned Premium for Calendar/Accident Year 2021: 12024
Earned Premium for Calendar/Accident Year 2022: 14444
Adjusted Premium for Calendar/Accident Year 2020: 11000
Adjusted Premium for Calendar/Accident Year 2021: 12000
Adjusted Premium for Calendar/Accident Year 2022: 13000
Aggregate Reported Loss for Calendar/Accident Year 2020: 9700
Aggregate Reported Loss for Calendar/Accident Year 2021: 6400 Aggregate Reported Loss for Calendar/Accident Year 2022: 4100

Aggregate Loss Report Lag for Calendar/Accident Year 2020: 0.85
Aggregate Loss Report Lag for Calendar/Accident Year 2021: 0.66
Aggregate Loss Report Lag for Calendar/Accident Year 2022: 0.42
Note that "report lag" here refers to the proportion of losses assumed to be already reported, not the proportion assumed to have yet to be reported. Use the Cape Cod method to calculate IBNR as of December 31, 2022.

Solution S6-50-4. We can calculate the expected loss ratio (ELR) as follows:
$E L R=($ Sum of aggregate reported losses for each year)/(Sum of used-up premiums for each year), where the used-up premium for each year is (Adjusted Premium)*(Aggregate Loss Report Lag).

Thus, ELR $=(9700+6400+4100) /(11000 * 0.85+12000 * 0.66+13000 * 0.42)=0.8886933568$.
By the Cape Cod Method, IBNR = ELR*(Sum of adjusted premiums for each year) - (Sum of aggregate reported losses for each year $)=0.8886933568 *(11000+12000+13000)-(9700+6400+$ 4100 ) $=11792.96084=\mathbf{\$ 1 1 , 7 9 2 . 9 6}$.

Problem S6-52-1. Similar to Question 10 from the 2009 CAS Exam 6. It is estimated that the expected loss rate for an insurance company is $\$ 200$ per exposure unit. The company has no exposure prior to 2030. You also have the following information by accident year (AY):

## AY 2030

Exposure Units: 2340
Incurred Loss: 400530
Incurred Loss Development Factor to Ultimate: 1.15

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AY 2031
Exposure Units: 3000
Incurred Loss: 360470
Incurred Loss Development Factor to Ultimate: 1.45
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AY 2032
Exposure Units: 3560
Incurred Loss: 350900
Incurred Loss Development Factor to Ultimate: 1.90
(a) What is the Bornhuetter-Ferguson estimate of IBNR at December 31, 2032, for all accident years?
(b) What is the Cape Cod estimate of IBNR at December 31, 2032, for all accident years?
(c) Based on the given information and your information, which method from parts (a) and (b) would produce the more accurate estimate of IBNR? Why?

## Solution S6-52-1.

(a) The Bornhuetter-Ferguson estimate of IBNR does not depend on losses reported to date. For each accident year it is equal to (Expected Loss)*(1-1/(LDF to Ultimate). In this case, the expected loss is (Number of Exposures)*(Loss Rate Per Exposure). Thus, the Bornhuetter-Ferguson estimate of IBNR is $\Sigma(($ Number of Exposures $) *($ Loss Rate Per Exposure $) *(1-1 /($ LDF to Ultimate $))=2340 * 200 *(1-$ $1 / 1.15)+3000 * 200 *(1-1 / 1.45)+3560 * 200 *(1-1 / 1.90)=584513.5327=\$ \mathbf{5 8 4}, 513.53$.
(b) The Cape Cod method first derives an empirical expected loss rate per exposure by first calculating "used-up" exposures equal to (Number of Exposures)*(1/(LDF to Ultimate)) and then obtaining the loss rate as (Sum of Reported Losses)/(Sum of Used-Up Exposures).

The Cape Cod expected loss rate is thus $(400530+360470+350900) /(2340 / 1.15+3000 / 1.45+$ $3560 / 1.90)=186.0163256$.

The total Cape Cod IBNR is the Bornhuetter-Ferguson IBNR, multiplied by the ratio of the Cape Cod expected loss rate to the given expected loss rate: $584513.5327 * 186.0163256 / 200=543645.2981=$ $\$ 543,645.30$.
(c) As a diagnostic, we can calculate the expected loss per exposure for each accident year via the chain ladder method as (Reported Losses)*(LDF)/(Number of Exposures):

AY 2030: $400530^{*} 1.15 / 2340=196.8416667$
AY 2031: $360470^{*} 1.45 / 3000=174.2271667$
AY 2032: $350900^{*} 1.90 / 3560=187.2780899$

We note that, based on losses reported to date, the a priori loss rate of $\$ 200$ per exposure is too high. Changes in the recent loss reporting pattern or the nature of losses have reduced this rate, and the Cape Cod method is more responsive to such changes, as compared to the Bornhuetter-Ferguson method, which relies on predetermined expected losses for its IBNR calculation. Thus, the Cape Cod method is preferable.

Problem S6-54-5. Similar to Question 32 from the 2009 CAS Exam 6. Suppose you are faced with a large set of reinsurance data from various sources.
(a) Identify three ways in which you might partition the data.
(b) Identify four considerations you might examine in deciding how to partition the data.

Solution S6-54-5. The following is a sample answer, and other valid answers are possible.
(a) One might partition the data

1. By reinsurance treaty type;
2. By type of underlying primary coverage;
3. By type of exposure being insured (e.g., reinsurance for policies written on manufacturing businesses might be treated separately from reinsurance for policies written on service businesses, if enough credible data exist).
(b) One might examine the following considerations:
4. How similar or different the reporting patterns are for the various categories of data into which partitions are contemplated;
5. How much data would exist in each category post-partition, and whether that is a credible amount of data;
6. How similar or different the underlying insured risks are;
7. How the treaty terms affect the reinsurer's exposure to loss.
