Exam-Style Questions Relevant to the New Casualty Actuarial Society Exam 5B

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(*Adapted from The Actuary's Free Study Guide for Exam 6 by Mr. Stolyarov*)

This is an open-source study guide and may be revised pursuant to suggestions.

Sources:
Past Casualty Actuarial Society exams: 2007 Exam 6, 2008 Exam 6, and 2009 Exam 6.

Problem S6-2-1. This problem is similar to Problem 1, Part (a), on the 2009 CAS Exam 6.

You are given the following information as of December 31, 2013:

(1) Paid Claims (including Salvage and Subrogation) by Accident Year (AY)

For AY 2011: 44,224
For AY 2012: 52,143
For AY 2013: 80,087

(2) Selected Ultimate Claims (including Salvage and Subrogation)

For AY 2011: 49,500
For AY 2012: 58,700
For AY 2013: 82,420

(3) Ratio of Received Salvage and Subrogation (S&S) to Paid Claims

For AY 2011: 0.151
For AY 2012: 0.176
For AY 2013: 0.210

(4) Development Factor to Ultimate for S&S Ratio

For AY 2011: 1.000
For AY 2012: 1.014
For AY 2013: 1.114

Using the ratio method, estimate the recoverables for Salvage and Subrogation (S&S) for accident years 2011-2013.

Solution S6-2-1. First, we estimate the ultimate S&S for each accident year. This is done by multiplying Ultimate Claims (2) by the S&S ratio (3) and the development factor to ultimate (4).

(5) Ultimate S&S by Accident Year: (5) = (2)*(3)*(4)

For AY 2011: 49,500*0.151*1.000 = 7474.5
For AY 2012: 58,700*0.176*1.014 = 10475.8368
For AY 2013: 82,420*0.210*1.114 = 19281.3348

Then we estimate paid S&S for each accident year. This is done by multiplying actual paid claims (1) by the S&S ratio (2). No development factors apply because we are only estimating what has already been paid.

(6) Paid S&S by Accident Year: (6) = (1)*(2)

For AY 2011: 44,224*0.151 = 6677.824
For AY 2012: 52,143*0.176 = 9177.168
For AY 2013: 80,087*0.210 = 16818.27

The S&S recoverables are the difference between ultimate S&S and paid S&S. They are *what remains to be recovered.*

(7) S&S Recoverables by Accident Year: (7) = (5) - (6)

For AY 2011: 7474.5 - 6677.824 = 796.676
For AY 2012: 10475.8368 - 9177.168 = 1298.6688
For AY 2013: 19281.3348 - 16818.27 = 2463.0648
Total: 796.676 + 1298.6688 + 2463.0648 = **4558.4096.**

Problem S6-2-2. How is the development for salvage recoveries typically different from the development for subrogation recoveries, and why? (See Friedland, p. 329).

Solution S6-2-2. Salvage is associated with property coverages, where the losses are often quickly reported and settled. Thus, the salvage can also be determined much faster.

Subrogation is associated with liability coverages, where the losses can take years to ascertain, and it may take years to determine who is liable and the ultimate claim payout. Also, subrogation recoveries may take years to materialize after the underlying claim is paid, because the insurer still has to pursue the responsible party.

Friedland (p. 329) notes that some subrogation age-to-age factors may be less than 1. This can happen for older claims where the prospect of recovering from the responsible party diminishes over time.

Problem S6-2-3. This problem is similar to Problem 8 on the 2008 CAS Exam 6.

You are analyzing a contract where the premium is paid in full at the start of the contract term. The upfront premium is $3650.

The expected incurred losses occur in the following percentages per year of the contract:

Year 1: 34%
Year 2: 15%
Year 3: 40%

Years 4-14: 1% per year

For the end of each the years 1, 2, 3, and 4, calculate the unearned premium reserve based on **(a)** the assumption that premium is earned in the same pattern as expected losses, **(b)** the assumption that premium is earned on a pro rata basis, and **(c)** the difference between the answers in (a) and (b).

(d) Based on your answer to part (c), explain the problem with applying the approach in part (b) to this situation.

(e) Name three kinds of insurance-related products for which assuming that premium is earned on a pro rata basis would not be appropriate.

Solution S6-2-3. (a) The unearned premium reserve (here, UPR) is equal to

(Total premium)*(1 - Fraction of premium that is earned).

For Year 1, UPR = 3650*(1 - 0.34) = **2409**.
For Year 2, UPR = 3650*(1 - 0.34 - 0.15) = **1861.5**.
For Year 3, UPR = 3650*(1 - 0.34 - 0.15 - 0.40) = **401.5**.
For Year 4, UPR = 3650*(1 - 0.34 - 0.15 - 0.40 - 0.01) = **365**.

(b) There are 14 years over which the policy is expected to have losses. Thus, the pro rata method assumes that each year, $1/14^{th}$ of the premium is earned, leaving the unearned premium reserve to be (Total premium)*(1 - (1/14)*Number of years elapsed).

For Year 1, UPR = 3650*(1 - 1/14) = **3389.286714**
For Year 2, UPR = 3650*(1 - 2/14) = **3128.571429**
For Year 3, UPR = 3650*(1 - 3/14) = **2867.857143**
For Year 4, UPR = 3650*(1 - 4/14) = **2607.142857**

(c) These answers are simply the difference between the corresponding values in (a) and (b):

For Year 1: 2409 - 3389.286714 = **-980.286714**
For Year 2: 1861.5 - 3128.571429 = **-1267.071429**
For Year 3: 401.5 - 2867.857143 = **-2466.357143**
For Year 4: 365 - 2607.142857 = **-2242.142857**

(d) The answers in part (c) can be thought of as the degree to which the pro rata method of estimating earned premium *underestimates* the true profitability of this product. Most of the losses for this product occur early on - during the first four years. But the pro rata method assumes that the losses occur evenly throughout the 14 years. This might, for instance, lead the company to assume that this product is not profitable and withdraw from offering it, when the product might in fact be a decent revenue source.

(e) The pro rata assumption for earned premium is not appropriate for the following kinds of products:

1. **Warranties**, where losses typically occur later during the contract term;

2. Policies covering **seasonal exposures**, such as hurricane risk. More premium should be earned during the season(s) of peak exposure.

3. **Aggregate excess insurance policies**, which cover losses above a certain attachment point. The attachment point is likely to be reached only later in the policy term, so that is when premium should start to be earned.

Problem S6-6-5. Similar to Problem 1 from the Fall 2008 Exam 6. Based on the CAS *Statement of Principles Regarding Property and Casualty Loss and Loss Adjustment Expense Reserves*, explain how **(a)** settlement patterns and **(b)** frequency/severity considerations could affect the determination of whether or not data in a given set are homogeneous.

Solution S6-6-5. (a) Settlement patterns - how long it takes for reported claims to settle - influence the level of reserve uncertainty. Claims that take longer to settle, such as bodily injury liability claims, have more reserve uncertainty, and the ultimate claim amount can vary considerably from the initial estimate. It may therefore be appropriate to analyze claims with long settlement patterns separately from claims with short settlement patterns, lest the actuary understate development on the former claims.

(b) Frequency/severity considerations: Claims with high frequency and low severity tend to be subject to more accurate reserve estimates than claims with low frequency and high severity. The latter type of claim will often necessitate greater analysis and may therefore need to be examined separately.

Problem S6-8-1. Similar to Problem 2 from the Fall 2008 Exam 6.
You know the following regarding data from policy year 2044:
● Premium was $5,000,000.
● It is expected that 50% of the loss would be emerged at 48 months, and 70% of the loss would be emerged at 60 months.
● Reported loss as of the end of 2047 was $2,120,000.
● The estimate of ultimate loss via the Bornhuetter-Ferguson method is $4,200,000.

(a) What was the expected loss ratio used in the Bornhuetter-Ferguson estimate of ultimate loss?

(b) Use the chain-ladder method to calculate the ultimate loss estimate for policy year 2044.

(c) Use the Bornhuetter-Ferguson method to find the expected 2048 calendar year development for losses from policy year 2044.

Solution S6-8-1.

(a) We use the Reported Bornhuetter-Ferguson method, applying Formula 8.1:

Ultimate Claims = Actual Reported Claims + (Expected Claims)*(% Claims Unreported)

Here, Ultimate Claims = 4,200,000 and Actual Reported Claims = 2,120,000. % Claims Unreported = 50% at the end of 2047, which is the point at which we have reported claim data.

Expected Claims can be expressed as (Premium)*(ELR), where the ELR is the expected loss ratio.

Thus, 4,200,000 = 2,120,000 + 5,000,000*ELR*0.5 →

2,080,000 = 2,500,000*ELR →

ELR = 2,080,000/2,500,000 = **ELR = 0.832 = 83.2%**.

(b) The chain ladder method takes the latest known reported loss figure and asks, "What percentage of the ultimate reported loss is this figure expected to be?" Here, the latest known reported loss figure is $2,120,000, and this is expected to be 50% of the ultimate loss, so the ultimate loss is 2,120,000/0.5 = **$4,240,000.**

(c) The development portion of the Bornhuetter-Ferguson method formula is the

(Expected Claims)*(% Claims Unreported) part. Here, we only want to focus on claims expected to emerge in calendar year 2048. Based on our given expected loss percentages, this is 70% - 50% = 20% of all claims. Based on part (a), we can already calculate expected claims to be 5,000,000*ELR = 5,000,000*0.832 = $4,160,000. Of this, 20% is **$832,000** - our estimate of development during CY 2048.

Problem S6-8-2. Similar to Problem 5 from the Fall 2008 Exam 6.

(a) If an insurer makes a one-time change in its policy limits applicable to all policies written after Day X, which method of data aggregation would be preferable: policy year or accident year? Why?

(b) When an insurer's business is growing rapidly within a particular year, which method of data aggregation would be preferable: accident year or accident quarter? Why?

(c) If there is a significant legal decision that changes typical amounts of damages resulting from particular incidents, which method of data aggregation would be preferable: report year or accident year? Why?

(d) What could happen to claim counts so as to make earned exposures a more reliable measure by comparison?

Solution S6-8-2.

(a) If an insurer makes a one-time change in its policy limits applicable to all policies written after Day X, the **policy year** method would be preferable, because it could separately analyze policies written before the limit change and policies written after the limit change. Accident-year aggregation could mix data on losses occurring in the same period, but pertaining both to policies written before the limit change and policies written after it.

(b) When an insurer's business is growing rapidly within a particular year, the **accident quarter** method of aggregation would be preferable, because a growing book of business would be expected to experience growing amounts of losses as well. This means that losses would be more heavily concentrated toward the end of the year, and separating data into accident quarters could segment the periods of greater losses from the periods of smaller losses.

(c) If there is a significant legal decision that changes typical amounts of damages resulting from particular incidents, the **report year** method of aggregation would be preferable, because claims reported after the decision would be subject to different likely severities than claims reported before the decision, irrespective of when the underlying losses occurred.

(d) Either the definition of what constitutes a claim or the insurer's claim-handling practices might change in such a way as to make "claim counts" non-comparable across time. In such cases, earned exposures are a more reliable measure.

Problem S6-8-3. Similar to Problem 6 from the Fall 2008 Exam 6.

During Year X, an insurer's claim-handling practices changed and each claim is now given a significantly lower initial case reserve than previously. However, the claims are also settled faster.

(a) Which of these methods would lead to definite overstatement of losses - the unadjusted reported loss development method or the unadjusted paid loss development method? Why?

(b) What changes unrelated to claims settlement could be responsible for a lowering of the initial case reserve assigned to each claim?

Solution S6-8-3.

(a) Both methods depend on development factors calculated from historical information and assuming that historical patterns of development will continue into the future. The unadjusted reported loss development method, however, will have to work with initial case reserve estimates that are lower than previously. Thus, an application of a historical loss development factor (LDF) to a lower case reserve will result in a lower estimate. However, the effect of faster claims settlement may or may not compensate for this - depending on the degree. If claims are settled significantly faster, and the historical LDF assumes a longer settlement pattern, then the effect of this would be a relative overstatement of losses. With the paid loss development method, however, the focus is only on the settlement pattern, and in this case a decrease in settlement times would produce an overstatement of ultimate losses if historical assumptions are used. So the **unadjusted paid loss development method** would lead to definite overstatement of losses.

(b) The following changes unrelated to claims settlement could be responsible for a lowering of the initial case reserve assigned to each claim:

1. Increase in deductibles on all policies - reducing the insurer's potential liability per claim.
2. Decrease in limits on all policies - reducing the insurer's potential liability per claim.
3. More rigorous underwriting standards - meaning that the insurer expects lower-risk insureds to be accepted into the program.
4. Movement of business to a different geographical area which tends to be populated by lower-risk insureds.

Problem S6-8-5. Similar to Problem 16 from the Fall 2008 Exam 6.

(a) There are two claims of the exact same type. Claim A has an incurred loss amount of $60,000, while Claim B has an incurred loss amount of $6,000. An actuary is estimating unallocated loss adjustment expenses (ULAE) for these claims. He must choose between the dollar-based approach and the count-based approach. Which of these approaches would be likely to give the same estimate for ULAE for both claims?

(b) For each of the two approaches, give a diagnostic that might suggest the desirability of one approach over the other.

Solution S6-8-5.

(a) The **count-based approach** would be likely to give the same estimate for ULAE for both claims. This is because this approach assumes that ULAE does not correlate with the loss amount and is essentially the same for similar types of claims. The dollar-based approach assumes that ULAE is directly proportional to the loss amount.

(b) If a cost analysis of each claim identifies that the ULAE per claim is close to the same, irrespective of claim size, then the count-based approach can be reliable.

If the ratio of ULAE to paid loss amount is stable across all claims, then the dollar-based approach can be reliable.

The following information applies to Problems S6-10-2 and S6-10-3.

You are aware of the following information for claims pertaining to accident year (AY) 2033. As of December 31, 2034, reported losses were $130. It is expected that ultimate AY 2033 losses will be $200. Based on many years of data, cumulative development factors have also been selected in the following manner:

12-months-to-ultimate factor: 1.850
24-months-to-ultimate factor: 1.628
36-months-to-ultimate factor: 1.374

As of December 31, 2035, AY 2033 reported losses are $166.

Problem S6-10-2. Similar to Problem 16(a) from the Fall 2009 Exam 6. Calculate the difference between (i) actual AY 2033 reported losses in calendar year (CY) 2035 and (ii) expected AY 2033 reported losses in CY 2035, based on the data and assumptions given.

Solution S6-10-2.

(a) The actual AY 2033 reported losses in CY 2035 are 166 - 130 = $36.
We find the expected reported losses as follows.
The expected losses yet-to-be-reported (all the way to ultimate) are 200 - 130 = $70.
We need to determine *what fraction* of this yet-to-be-reported amount is expected to be reported in CY 2035.

The 24-months-to-ultimate factor is 1.628, meaning that, as of the end of 2034, 1/1.628 of the loss is expected to have emerged. The loss yet to emerge is thus (1 - 1/1.628) of the total expected amount. The 36-months-to-ultimate factor is 1.374, meaning that, as of the end of 2035, 1/1.374 of the loss is expected to have emerged.

During 2035, the proportion of the total expected loss that will emerge is thus (1/1.374 - 1/1.628).

Thus, the fraction of yet-to-be-reported losses assigned to CY 2035 is (1/1.374 - 1/1.628)/(1 - 1/1.628) = 0.2943657924.

The expected reported losses for CY 2035 are therefore 70*0.2943657924 = 20.60560547.

The desired (actual - expected) difference is thus 36 - 20.60560547 = 15.39439453 = **$15.39.**

Problem S6-10-3. Similar to Problems 16(b) and 16(c) from the Fall 2009 Exam 6.

(a) Using linear interpolation of the development pattern provided, what are the expected losses emerged between January 1, 2035, and September 30, 2035?

(b) Will the answer in part (a) overestimate or underestimate the projection? Explain your answer.

Solution S6-10-3.

(a) The expected losses yet-to-be-reported (all the way to ultimate) are 200 - 130 = $70.
The 24-months-to-ultimate factor is 1.628.
The 36-months-to-ultimate factor is 1.374.
We want to find, using linear interpolation, the 33-months-to-ultimate factor.
Thus value is 9/12 of the way between 1.628 and 1.374:
1.628 - (9/12)(1.628 - 1.374) = 1.4375.

Thus, the fraction of yet-to-be-reported losses assigned to the first 9 months of CY 2035 is (1/1.4375 - 1/1.628)/(1 - 1/1.628) = 0.2110218776, meaning that the expected losses are 0.2110218776*70 = 14.77153143 = **$14.77.**

(b) Even a visual comparison of the answer in part (a) to the answer in Solution S6-10-2 suggests that the linear interpolation approach would **underestimate** the projection. A real-world reason for this is that development tends to occur at a decreasing rate, with more development occurring earlier. Linear interpolation, however, presumes that development occurs at a uniform rate. The interpolated development factor thus overstates the true factor, leading to an understated estimate for the amount of development occurring up to the time in question.

Problem S6-12-5. Similar to Problem 7 from the 2007 CAS Exam 6.
You are analyzing the following paid loss development triangle, where *cumulative* paid losses for each accident year (AY) are evaluated as of 12, 24, 36, and 48 months, via the following notation, where applicable: (12-month estimate, 24-month estimate, 36-month estimate, 48-month estimate)

AY 2019: (430, 450, 487, 560)
AY 2020: (243, 342, 543)
AY 2021: (1100, 1250)
AY 2022: (320)

These data are valued as of December 31, 2022. What are the losses paid in calendar year (CY) 2022?

Solution S6-12-5. The outermost diagonal of the loss development triangle indicates cumulative paid losses for each AY's experience in CY 2022. To find the *incremental* paid losses in CY 2022, we must subtract (where possible) from the CY 2022 cumulative amounts the prior year's (CY 2021's) cumulative amounts, expressed on the second-outermost diagonal. Our incremental losses paid in CY 2022 are thus (560 - 487) + (543 - 342) + (1250 - 1100) + 320 = **744.**

Problem S6-14-3. Similar to Problem 8 from the 2007 CAS Exam 6. The *disposal rate* for claims measures the proportion of claims from a given report year that are settled within the specified time interval from the report year.

Consider a triangle displaying disposal rates in the following format for each report year:
(Rate at 0-24 months from the report year, rate at 25-48 months, rate from 48 months to ultimate)

Report Year 2028: (0.431, 0.352, 0.217)
Report Year 2029: (0.540, 0.260)
Report Year 2030: (0.410)

Estimate the disposal rate for the group of claims at 25-48 months from report year 2030. Use *only* the information from the most recent available calendar year.

Solution S6-14-3. The information from the most recent available calendar year is the information pertaining to 2029 data. This shows that a 0-24 disposal rate of 0.540 corresponds to a 25-48 month disposal rate of 0.260. In 2029, after 24 months, the proportion of *unsettled* claims was 1 - 0.540 = 0.460. So the proportion of this amount that was settled was 0.260/0.460 = 0.5652173913.

In 2030, after 24 months, the proportion of *unsettled* claims was 1 - 0.410 = 0.590. If 56.52173913% of 0.590 gets settled 25-48 months from 2030, the desired disposal rate is 0.590*0.5652173913 = **0.3334782609**.

Problem S6-16-1. Similar to Problem 13 from the Fall 2009 CAS Exam 6.
For Accident Year (AY) 2120 through Accident Year 2123, you are aware of the following information, where the ratios are displayed in the format (ratio at 12 months, ratio at 24 months, ratio at 36 months, ratio at ultimate):

Ratio of Paid ALAE to Paid Claims Only
AY 2120: (0.012, 0.016, 0.020, 0.025)
AY 2121: (0.015, 0.015, 0.018)
AY 2122: (0.014, 0.018)
AY 2123: (0.013)

AY 2120 Estimated Ultimate Claims: $31,150
AY 2121 Estimated Ultimate Claims: $33,310
AY 2122 Estimated Ultimate Claims: $30,120
AY 2123 Estimated Ultimate Claims: $38,125

(a) Use the multiplicative-paid-ALAE-to-paid-claims-only method to estimate ultimate ALAE for AY 2123. When calculating age-to-age development factors, use a simple average.

(b) Briefly discuss two possible disadvantages of the multiplicative-paid-ALAE-to-paid-claims-only method.

Solution S6-16-1. (a) First, we calculate the 12-24-month, 24-36-month, and 36-month-to-ultimate age-to-age factors for each accident year where this is possible:

Age-to-Age Factors for Ratio of Paid ALAE to Paid Claims Only
AY 2120: (0.016/0.012, 0.020/0.016, 0.025/0.020)
AY 2121: (0.015/0.015, 0.018/0.015)
AY 2122: (0.018/0.014)

Age-to-Age Factors for Ratio of Paid ALAE to Paid Claims Only
AY 2120: (1.333, 1.250, 1.250)
AY 2121: (1.000, 1.200)
AY 2122: (1.28571286)

We take the simple averages of the age-to-age factors to find our estimates:
12-24 month factor estimate: (1.333333 + 1 + 1.28571286)/3 = 1.206349206.
24-36 month factor estimate: (1.250 + 1.200)/2 = 1.225.
36-month-to-ultimate factor estimate: 1.250 - this is the only value we have.

The estimated 12-month-to-ultimate factor is thus 1.206349206*1.225*1.250 = 1.8472222222. We multiply by this factor the AY 2123 ratio at 12 months of paid ALAE to paid claims only: 0.013*1.8472222222 = 0.02401388889. This is the estimated ultimate ratio of paid ALAE to paid claims only for AY 2123. The estimated ultimate ALAE is the AY 2123 estimated claims, multiplied by the ratio derived above: 0.02401388889*38125 = 915.5295139 = **$915.53**.

(b) The following are two disadvantages of the multiplicative-paid-ALAE-to-paid-claims-only method:

1. The accuracy of the ultimate ALAE estimate is dependent on the accuracy of the ultimate claim estimate, which could be subject to considerable error.

2. If a lot of ALAE are devoted to claims that end up being closed without no payment (CNP), the determination of ultimate ALAE as a percentage of ultimate paid claims would overlook the effect that the CNP claims have on ALAE.

Problem S6-18-4. Similar to Question 39 from the 2007 CAS Exam 6. Define *pure premium* by reference to its constituent terms and name three events that are external to an insurance company but which could affect pure premium. For each event, specify the component of pure premium which would be affected.

Solution S6-18-4. Pure Premium = (Frequency)*(Severity).

The following is a sample response. Many other valid answers are possible.

1. An increased tendency for juries to award higher damages for particular types of cases might increase pure premium by raising claim *severity*.

2. An increased number of uninsured motorists on the road might increase the *frequency* of claims on uninsured motorists coverage.

3. A law that requires the insurer to cover a previously excluded exposure might increase the *frequency* of claims on the policies in question.

Problem S6-24-4. Similar to Question 21 from the 2007 CAS Exam 6. What do the Stanard-Bühlmann and Bornhuetter-Ferguson methods have in common? How are they different, in essential terms?

Solution S6-24-4. Both methods rely on the formula

Ultimate Losses = Reported Losses + (% Losses Unreported)*(Expected Losses).

In the Bornhuetter-Ferguson Method, the expected loss ratio is estimated judgmentally. Losses are compared to earned premium that is not brought to the present rate levels.

In the Stanard-Bühlmann Method, *adjusted premium* is used instead of earned premium; adjusted premium is earned premium adjusted to current rate levels. Also, the expected loss ratio is estimated on the basis of reported claim experience from the overall time period being examined.

(See Friedland, pp. 174-175.)

Problem S6-30-5. Similar to Question 50 from the 2007 CAS Exam 6. You have the following triangle of cumulative closed claim counts per accident year (AY), with age of development being expressed at (12 months, 24 months, 36 months, 48 months):

AY 2034 (13000 earned exposures): (100, 150, 175, 200)
AY 2035 (13500 earned exposures): (102, 148, 155)
AY 2036 (14000 earned exposures): (99, 130)
AY 2037 (13000 earned exposures): (80)

(a) What operational change might have occurred within the insurance company to explain the data above?

(b) How would the operational change in part (a) affect the accuracy of the calculations of ultimate losses on the basis of the corresponding paid loss triangle?

Solution S6-30-5.

(a) We can construct a triangle of claim counts per earned exposure to spot any differences:

AY 2034: (0.00769, 0.01154, 0.01346, 0.01538)
AY 2035: (0.00756, 0.01096, 0.01148)
AY 2036: (0.00707, 0.00929)
AY 2037: (0.00615)

It appears that, over the years 2035-2037, the insurer's claims department has closed increasingly fewer claims at each age of the experience, as compared to prior years. The decline is particularly evident going from AY 2036 to AY 2037. Perhaps the claims department has become less efficient or has chosen to scrutinize claims more closely.

(b) Since claims in more recent time periods are being closed at a slower rate, applying ultimate loss estimates based on the corresponding paid loss triangle, where the diagonals based on the most recent experience will give lower factors, will result in losses from earlier periods being multiplied by smaller development factors, leading to an **underestimate.**

Problem S6-31-3. Similar to Question 33 from the 2007 CAS Exam 6. You are given the following triangles for accident years (AY) 2034 through 2036, where data is expressed in the format (Value at 12 months, Value at 24 months, Value at 36 months), where applicable.

Average Case Reserve per Open Claim
AY 2034: (230, 320, 400)
AY 2035: (260, 370)
AY 2036: (320)

Number of Open Claims
AY 2034: (110, 80, 20)
AY 2035: (140, 70)
AY 2036: (150)

Cumulative Paid Losses
AY 2034: (13000, 18900, 28000)
AY 2035: (14000, 17000)
AY 2036: (18210)

The annual severity trend is +5%. Develop the Berquist-Sherman triangle of adjusted incurred losses for this scenario.

Solution S6-31-3. The Berquist-Sherman triangle of adjusted incurred losses is developed by adjusting the case reserve estimates and de-trending the values at the latest known (outermost) diagonal by the severity trend so as to arrive at the rest of the case reserve triangle:

Adjusted Average Case Reserve per Open Claim
AY 2034: ($320/1.05^2$, 370/1.05, 400)
AY 2035: (320/1.05, 370)
AY 2036: (320)

Adjusted Average Case Reserve per Open Claim
AY 2034: (290.2494331, 352.3809524, 400)
AY 2035: (304.7619048, 370)
AY 2036: (320)

Then the adjusted incurred loss for each time period is equal to
Paid Losses + (Average Case Reserve per Open Claim)*(Number of Open Claims).

Adjusted Incurred Losses
AY 2034: (13000 + 290.2494331*110, 18900 + 352.3809524*80, 28000 + 400*20)
AY 2035: (14000 + 304.7619048*140, 17000 + 370*70)
AY 2036: (18210 + 320*150)

Our answer is
Adjusted Incurred Losses
AY 2034: (44927.44, 47090.48, 36000)
AY 2035: (56666.67, 42900)
AY 2036: (66210)

Problem S6-31-4. Similar to Question 38 from the 2007 CAS Exam 6.

(a) How would the Berquist-Sherman approach be superior to the chain ladder approach in the event of case reserve strengthening by the insurer?

(b) How would the Berquist-Sherman approach be superior to the chain ladder approach in the event of a changing claim settlement rate?

(c) If insureds are purchasing lower policy limits than before, why would it be preferable to switch from accident-year data aggregation to policy-year data aggregation?

Solution S6-31-4.

(a) In the event of case reserve strengthening by the insurer, the chain ladder method, with development factors based in part on prior experience under lower case reserves, would overstate ultimate loss results. The Berquist-Sherman approach can mitigate this by adjusting previous, lower case reserves to the level of reserve adequacy that currently exists. This is done by de-trending the most recent case reserves instead of using historical values prior to the reserve strengthening.

(b) A changing claim settlement rate could result in the chain ladder method either overstating (if the settlement rate increases) or understating (if the settlement rate decreases) ultimate losses. The Berquist-Sherman approach applies the current claim settlement rate to historical closed claims, thereby mitigating any overstatement or understatement.

(c) If insureds are purchasing lower policy limits than before, analysis using the chain ladder method and accident-year aggregation will understate the ultimate losses - in essentially the inverse fashion of what would happen under strengthening case reserves. Accident-year loss data combine losses from policies written in previous years with higher limits and policies written in later years with lower limits, whereas policy-year data are segregated by the year in which policies were written, meaning that there will not be a mix of losses from policies from years with higher limits and years with lower limits. This allows for trending of each policy year's data by any policy limit change that has been observed.

Problem S6-37-1. Similar to Question 42 from the 2007 CAS Exam 6. If there is a clearly identifiable trend in an insurer's loss ratio experience from one year to another, what aspects of **(a)** the Bornhuetter-Ferguson development method and **(b)** the Least-Squares development method would render such methods sub-optimal for developing ultimate loss and unpaid claim estimates?

Solution S6-37-1.

(a) The unreported component of losses under the Bornhuetter-Ferguson development method depends entirely on an expected loss ratio. Unless that expected loss ratio has already been adjusted to reflect the most recent loss ratio trends, there will be an over- or underestimation.

(b) The Least-Squares development method is designed for situations where any changes in loss ratio experience are random. If there is a clear directional trend that the insurer can identify, then this assumption would not hold, and the Least-Squares method would ignore this systematic change in the book of business.

Problem S6-37-2. Similar to Question 43 from the 2007 CAS Exam 6. You are given the following cumulative paid claim data by accident year (AY) for Insurer Λ as of December 31, 2049, expressed in the format (Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months), where applicable.

Cumulative Paid Claims
AY 2046: (2330, 3345, 4010, 4430)
AY 2047: (2402, 3504, 4123)
AY 2048: (2403, 3450)
AY 2049: (2420)

There are two reserving methods used to determine ultimate claim amounts for each accident year. The following are the development factors to ultimate for each method:

12 months to ultimate - Method 1: 1.89
12 months to ultimate - Method 2: 1.93
24 months to ultimate - Method 1: 1.30
24 months to ultimate - Method 2: 1.35
36 months to ultimate - Method 1: 1.10
36 months to ultimate - Method 2: 1.06
48 months to ultimate - Method 1: 1.00
48 months to ultimate - Method 2: 1.00

In calendar year (CY) 2050, the following losses are actually paid out:

For AY 2046: 0
For AY 2047: 332
For AY 2048: 704
For AY 2049: 1022
For AY 2050: 2450
Total: 4508

(a) Use a retrospective test of reserve adequacy to select either Method 1 or Method 2 as the more appropriate reserving method of the two.

(b) How could the bias of the selected method be corrected via an adjustment? Explain any assumptions in your answer.

Solution S6-37-2.

(a) A retrospective test of reserve adequacy would compare the losses actually paid out in CY 2050 to the projections by each of the methods. The losses for AY 2046-2049 paid out in CY 2050 are 4508 - 2450 = 2058.

We now determine the losses projected by Method 1:

For AY 2046: 0, since losses are already at ultimate.

For AY 2047: 4123*(1.10 - 1) = 412.3
For AY 2048: 3450*(1.30/1.10 - 1) = 627.2727272727
For AY 2049: 2420*(1.89/1.30 - 1) = 1098.307692
Total: 2137.915384
Error: 2137.915384/2058 - 1 = 0.0388315784 = Overestimate of circa 3.88%.

We now determine the losses projected by Method 2:

For AY 2046: 0, since losses are already at ultimate.
For AY 2047: 4123*(1.06 - 1) = 247.38
For AY 2048: 3450*(1.35/1.06 - 1) = 943.8679245
For AY 2049: 2420*(1.93/1.35 - 1) = 1039.703704
Total: 2230.951628
Error: 2230.951628/2058 - 1 = 0.084038692 = Overestimate of circa 8.40%.

Method 1 is preferable because it has a lower overall error.

(b) To adjust for the overestimate in Method 1, one could multiply the result by 1/(1 + Error Amount) - in this case, 1/1.0388315784 = 0.962619948. This would bring the overall reserve for CY 2050 to the level of actual losses in CY 2050 and would presumably correct any bias in estimates for subsequent years. This adjustment requires the assumption that Method 1 would continue having the same bias over time, and that the insurer experiences consistent losses and has a stable book of business.

Problem S6-37-3. Similar to Question 44 from the 2007 CAS Exam 6. In accident years (AY) 2023 through 2026, the number of cumulative reported and closed claims for Insurer Σ did not vary by accident year for any particular age of maturity. Cumulative incurred losses and case loss reserves were as follows, expressed in the format
(Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months), where applicable. Assume all losses are at ultimate at 48 months.

Cumulative Incurred Losses - Data as of December 31, 2026
For AY 2023: (3033, 4044, 4505, 4606)
For AY 2024: (3185, 4246, 4730)
For AY 2025: (3344, 4459)
For AY 2026: (3511)

Case Loss Reserves - Data as of December 31, 2026
For AY 2023: (1000, 500, 200, 0)
For AY 2024: (1050, 525, 210)
For AY 2025: (1050, 525)
For AY 2026: (1103)

(a) Find the IBNR as of December 31, 2026, using the chain ladder method.

(b) What aspect of this scenario renders the IBNR estimate in part (a) inaccurate? Justify your answer by reference to the given data.

Solution S6-37-3.

(a) We first calculate age-to-age development factors for incurred losses, using the format (12-24-month factor, 24-36-month factor, 36-48-month factor), where applicable.

Age-to-Age Factors for Incurred Losses
(4044/3033, 4505/4044, 4606/4505)
(4246/3185, 4730/4246)
(4459/3344)

Age-to-Age Factors for Incurred Losses
(1.333, 1.114, 1.022)
(1.333, 1.114)
(1.333)

Our selection for age-to-age factors is made simple in this scenario. We can also select factors to ultimate:
12-month-to-ultimate factor: 1.333*1.114*1.022 = 1.517631164
24-month-to-ultimate factor: 1.114*1.022 = 1.138508
36-month-to-ultimate factor: 1.022

Now we can estimate IBNR by multiplying each still-not-ultimate value on the outermost diagonal of the incurred loss triangle by (the appropriate factor to ultimate - 1) and adding these products:
4730*(1.022 - 1) + 4459*(1.138508 - 1) + 3511*(1.517631164 - 1) = 2539.070189 = **IBNR = 2539.**
(Slight variations on this are possible if rounding was used at different steps of the calculation.)

(b) We consider the incurred loss trend at each age of maturity and compare it to the case reserve trend:

Cumulative Incurred Loss Trend
AY 2023 to AY 2024: (3185/3033, 4246/4044, 4730/4505)
AY 2024 to AY 2025: (3344/3185, 4459/4246)
AY 2025 to AY 2026: (3511/3344)

Cumulative Incurred Loss Trend
AY 2023 to AY 2024: (1.05, 1.05, 1.05)
AY 2024 to AY 2025: (1.05, 1.05)
AY 2025 to AY 2026: (1.05)

Case Reserve Trend
AY 2023 to AY 2024: (1050/1000, 525/500, 210/200)
AY 2024 to AY 2025: (1050/1050, 525/525)
AY 2025 to AY 2026: (1103/1050)

Case Reserve Trend
AY 2023 to AY 2024: (1.05, 1.05, 1.05)
AY 2024 to AY 2025: (1.00, 1.00)
AY 2025 to AY 2026: (1.05)

While incurred losses increased by 5% from AY 2024 to AY 2025, case reserves did not increase at all. The net result of this is reduced case outstanding strength. The chain ladder method assumes constant case outstanding strength. With reduced case outstanding strength and the same loss development factors calculated via the chain ladder method, there will be an underestimation of IBNR.

Problem S6-37-5. Similar to Question 46 from the 2007 CAS Exam 6. You have the following information about a particular insurance policy from a well-established book of business:

Premium: 200,000
Expected loss ratio: 80%
Observed loss up to December 31, 2020: 130,000
Age-to-ultimate development factor applicable at December 31, 2020: 1.60

(a) According to the Bornhuetter-Ferguson method, what is the estimated ultimate loss amount for this policy?

(b) In the answer from part (a), what is the percentage credibility assigned to the loss development projection?

(c) What is one possible shortcoming of the Bornhuetter-Ferguson method in this case, and what can be used to mitigate this shortcoming?

Solution S6-37-5.

(a) The Bornhuetter-Ferguson method uses the formula
Ultimate Claims = Actual Reported Claims + (Expected Claims)*(% Claims Unreported).
Here, we know that Actual Reported Claims = 130,000.
We calculate Expected Claims = Premium*(Expected Loss Ratio) = 200000*0.8 = 160,000.
We calculate % Claims Unreported = 1 - 1/1.60 = 0.375 = 37.5%
Thus, Ultimate Claims = 130000 + 160000*0.375 = **190,000**.

(b) For the Bornhuetter-Ferguson method, the percentage credibility assigned to the loss development projection is the percentage of claims assumed to be reported at the time as of which the data are being analyzed. This is 1/(Development Factor to Ultimate), which here is 1/1.60 = 0.625 = **62.5%**.

(c) The Bornhuetter-Ferguson method relies on a predetermined expected loss ratio that may not take into account recent changes in loss experience. To assign more credibility to the development projection, one could use the Benktander method, which is an iterative application of the Bornhuetter-Ferguson method, using the result from the first application of the Bornhuetter-Ferguson method as the "Expected Claims" component. One could also use the Stanard-Bühlmann (Cape Cod) method, which contains a systematic way of calculating the expected loss ratio.

Problem S6-38-1. Similar to Question 47 from the 2007 CAS Exam 6. You know the following about paid defense and cost containment (DCC) expenses as of December 31, 2047, and ultimate losses for an insurer by accident year (AY):

AY 2044: Ultimate loss: 5550; Paid DCC: 200
AY 2045: Ultimate loss: 5200; Paid DCC: 150
AY 2046: Ultimate loss: 5100; Paid DCC: 110
AY 2047: Ultimate loss: 6000; Paid DCC: 20

Ratio of Cumulative Paid DCC to Cumulative Paid Loss, expressed in the format
(Ratio at 12 months, Ratio at 24 months, Ratio at 36 months, Ratio at Ultimate).

AY 2044: (0.24%, 2.22%, 3.20%, 4.25%)
AY 2045: (0.28%, 2.24%, 3.25%)
AY 2046: (0.25%, 2.30%)
AY 2047: (0.22%)

For AY 2044 through AY 2047, calculate the total DCC reserve. Show all intermediate steps contributing to the result.

Solution S6-38-1. First, we want to calculate age-to-age factors for ratio of cumulative DCC to cumulative paid loss.

Age-to-Age Factors for Ratio of Cumulative Paid DCC to Cumulative Paid Loss, expressed in the format
(Factor for 12-24 months, Factor for 24-36 months, Factor for 36 months to Ultimate).

AY 2044: (2.22%/0.24%, 3.20%/2.22%, 4.25%/3.20%)
AY 2045: (2.24%/0.28%, 3.25%/2.24%)
AY 2046: (2.30%/0.25%)

Age-to-Age Factors for Ratio of Cumulative Paid DCC to Cumulative Paid Loss

AY 2044: (9.25, 1.441441441, 1.328125)
AY 2045: (8.00, 1.450892857)
AY 2046: (9.20)

We select the simple arithmetic means of the available age-to-age factors for a given age to maturity:

12-24 months: 8.8166666667
24-36 months: 1.446167149
36 months to ultimate: 1.328125

We can now select factors to ultimate:

12-24 months: 8.8166666667*1.446167149*1.328125 = 16.93409007
24-36 months: 1.446167149*1.328125 = 1.920690745
36 months to ultimate: 1.328125

Now, for each accident year, we can project to ultimate the ratios of cumulative paid DCC to cumulative paid loss:

AY 2044: 4.25% -- Already at ultimate
AY 2045: 3.25%*1.328125 = 4.31640625%
AY 2046: 2.30%*1.920690745 = 4.417588714%
AY 2047: 0.22%*16.93409007 = 3.725499815%

This allows us to find the ultimate DCC for each accident year:

AY 2044: 5550*4.25% = 235.875
AY 2045: 5200*4.31640625% = 224.452125
AY 2046: 5100*4.417588714% = 225.2970244
AY 2047: 6000*3.725499815% = 223.5299889

Now we can find the DCC reserve for each accident year by subtracting paid DCC from ultimate DCC:

AY 2044: 235.875 - 200 = 35.875
AY 2045: 224.452125 - 150 = 74.452125
AY 2046: 225.2970244 - 110 = 115.2970244
AY 2047: 223.5299889 - 20 = 203.5299889

Total: 35.875 + 74.452125 + 115.2970244 + 203.5299889 = 429.1541383 = circa **429.15.**

Problem S6-38-2. Similar to Question 3 from the 2008 CAS Exam 6. The annual projected severity trend is +2%. All data are at ultimate at 48 months. You also know the following information by accident year (AY):

Incremental Loss and ALAE Payments on Closed Claims, expressed in thousands of dollars and in the format (Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months).

AY 2030: (200, 250, 180, 80)
AY 2031: (250, 290, 200)
AY 2032: (300, 300)
AY 2033: (350)

Incremental Number of Claims Closed, expressed in the format
(Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months).

AY 2030: (30, 60, 50, 40) **Ultimate claims:** 180
AY 2031: (36, 72, 60) **Ultimate claims:** 216
AY 2032: (24, 48) **Ultimate claims:** 144
AY 2033: (42) **Ultimate claims:** 252

(a) Use Adler and Kline's claim-closure projection method to find the projected reserve as of December 31, 2033.

(b) Describe two aspects of the calculation you performed in part (a) that would recommend it as a reserve estimation technique.

Solution S6-38-2.

(a) First we note the incremental claim closure pattern, which appears to be the same for every accident year. Let N be the number of claims closed at 12 months. Then the pattern is
(N, 2N, (5/3)N, (4/3)N). Using this formula, we can extrapolate the numbers of closed claims:

AY 2030: (30, 60, 50, 40) **Ultimate claims:** 180
AY 2031: (36, 72, 60, 48) **Ultimate claims:** 216
AY 2032: (24, 48, 40, 32) **Ultimate claims:** 144
AY 2033: (42, 84, 70, 56) **Ultimate claims:** 252

We can also figure out the incremental paid severities (Paid Amounts/Closed Claims):

Incremental Paid Severities on Closed Claims, expressed in thousands of dollars and in the format (Amount at 12 months, Amount at 24 months, Amount at 36 months, Amount at 48 months).

AY 2030: (200/30, 250/60, 180/50, 80/40)
AY 2031: (250/36, 290/72, 200/60)
AY 2032: (300/24, 300/48)
AY 2033: (350/42)

Really, we are just interested in the outermost diagonal:
Incremental Paid Severities on Closed Claims

AY 2030: (200/30, 250/60, 180/50, 2)
AY 2031: (250/36, 290/72, 3.3333)
AY 2032: (300/24, 6.25)
AY 2033: (8.3333)

Now we can apply our annual multiplicative severity trend of 1.02:
Incremental Paid Severities on Closed Claims

AY 2030: (200/30, 250/60, 180/50, 2)
AY 2031: (250/36, 290/72, 3.3333, 2.04)
AY 2032: (300/24, 6.25, 3.40, 2.0808)
AY 2033: (8.3333, 6.375, 3.468, 2.122416)

Now we can calculate the reserve amounts for each year as the sum of 1000*(Number of Closed Claims*Closed Claim Severity) for each age to maturity. There is no reserve for AY 2030, since losses are already at ultimate.

AY 2031: 1000*(2.04*48) = 97920
AY 2032: 1000*(3.40*40 + 2.0808*32) = 202585.6
AY 2033: 1000*(6.375*84 + 3.468*70 + 2.122416*56) = 897115.296
Total: 97920 + 202585.6 + 897115.296 = 1197620.896 ≈ **$1,197,620.90**.

(b) Two advantages of the calculation in part (a) are 1) explicit incorporation of claim severity trends, which could be accounted for by economic or social inflation, and 2) no reliance on incurred losses and independence from the accuracy or lack thereof of case reserve estimates.

Problem S6-38-3. Similar to Question 4 from the 2008 CAS Exam 6. You have the following information as of December 31, 2066, all expressed in the format
(Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months), where applicable.

Cumulative Reported Loss ($000)

AY 2063: (3030, 4506, 4990, 5200)
AY 2064: (3133, 4666, 5000)
AY 2065: (3002, 4556)
AY 2066: (3000)

Cumulative Paid Loss ($000)

AY 2063: (1525, 2344, 2990, 4560)
AY 2064: (1498, 2200, 3000)
AY 2065: (1555, 2660)
AY 2066: (1500)

Average Case Reserve Per Open Claim ($000)

AY 2063: (10.03, 27.15, 66.6667, 64)
AY 2064: (11.68, 35.55, 50)
AY 2065: (11.13, 24)
AY 2066: (12)

Number of Open Claims

AY 2063: (150, 80, 30, 10)
AY 2064: (140, 75, 40)
AY 2065: (130, 79)
AY 2066: (125)

(a) Assume an annual severity trend of -4% and use the Berquist-Sherman method to create an adjusted cumulative reported loss triangle, based on a severity-adjusted case reserve triangle. Round your answers to the nearest whole number.

(b) Using the adjusted cumulative reported loss triangle from part (a) and loss development factors calculated as weighted averages of all relevant years' experience, estimate the ultimate loss for AY 2066. Use a 48-month-to-ultimate factor of 1.05.

Solution S6-38-3.

(a) We take the average case reserve triangle and project backward from the outermost diagonal using the annual severity trend of -4%. We divide each subsequent vertical entry by 0.96 to get the preceding entry.

Adjusted Average Case Reserve Per Open Claim ($000)
AY 2063: (13.56336806, 26.0416667, 52.083333, 64)
AY 2064: (13.02083333, 25, 50)
AY 2065: (12.5, 24)
AY 2066: (12)

Now, to get the adjusted reported claim triangle, for each entry except the outermost diagonal (where reported claims are unchanged from what is given), we calculate

Cumulative Paid Loss + (Number of Open Claims)*(Adjusted Average Case Reserve Per Open Claim).

Sample calculation, for AY 2063 at 12 months:
1525 + 150*13.56336806 = 3559.505209 = circa 3560.

Adjusted Cumulative Reported Loss ($000)
AY 2063: (3560, 4427, 4553, 5200)
AY 2064: (3321, 4075, 5000)
AY 2065: (3180, 4556)
AY 2066: (3000)

(b) We can calculate weighted-average age-to-age factors as follows:
For 12-24 months: (4427 + 4075 + 4556)/(3560 + 3321 + 3180) = 1.297882914.
For 24-36 months: (4553 + 5000)/(4427 + 4075) = 1.123617972
For 36-48 months: 5200/4553 = 1.142104107
12-month-to-ultimate factor: 1.297882914*1.123617972*1.142104107*1.05 = 1.748836402.
Ultimate loss for AY 2066: 1000*3000*1.748836402 = **$5,246,509.21**.

Problem S6-38-4. Similar to Question 11 from the 2008 CAS Exam 6. You have the following IBNR estimates from three different methods:

Loss development method: $6000
Bornhuetter-Ferguson method: $5000
Percent of premium method: $5300

The insurer's book of business has been showing a deteriorating loss ratio, with no changes in case reserve adequacy or loss emergence patterns.

(a) Rank these methods in order of accuracy in this situation. Justify your answer.

(b) For any of these methods that are inaccurate, which are self-correcting in the long term? Why?

Solution S6-38-4.

(a) The most accurate method here is the development method. Since there are no changes in case reserve adequacy or loss emergence patterns, the loss development pattern has not altered at all, and the loss development factors based on historical losses will still fully reflect the current situation. Less accurate is the percent of premium method, where the IBNR estimate is based on the premiums and losses during the time periods in question, and only part of the experience will be based on the more recent time periods of deteriorating loss ratios. The percent of premium method would thus underestimate the true IBNR. The Bornhuetter-Ferguson method would produce an even greater underestimate, as the IBNR component of ultimate losses is based on an expected loss ratio that is determined *a priori.* This expected loss ratio would be lower than warranted by the more recent experience. The ranking in terms of accuracy would thus be
Loss development method > Percent of premium method > Bornhuetter-Ferguson method - with the ">" sign denoting greater accuracy.

(b) The **percent of premium method** would be self-correcting over time, as the earlier time periods' experience falls outside the time period being analyzed and new experience, based on more recent loss-ratio behavior, would replace it. The Bornhuetter-Ferguson method would require deliberate adjustment of the expected loss ratio to reflect more current conditions.

Problem S6-44-4. Similar to Question 9 from the 2008 CAS Exam 6.
You are analyzing the following information about cumulative paid losses for Insurer Ψ by accident year (AY):

Cumulative Paid Loss, expressed in the format
(Amount at development year 0, Amount at development year 1, Amount at development year 2).
AY 2022: (343, 444, 500)
AY 2023: (360, 500, 555)
AY 2024: (320, 466)
AY 2025: (350)

Find the cumulative paid loss amount for AY 2024 at development year 2 using the following methods:
(a) The development method;
(b) The budgeted loss method;
(c) The least-squares method.

Solution S6-44-4.

(a) We use the development method with a weighted-average loss development factor from year 1 to year 2: (500 + 555)/(444 + 500) = 1.117584746. Our answer is thus 466*1.117584746 = **520.7944915**.

(b) The budgeted loss method simply takes the expected value of the known losses at development year 2 and sets that as the loss for AY 2024: (500 + 555)/2 = **527.5**.

(c) First we find the various averages necessary for the least-squares method.
Let x be experience at development year 1, and let y be experience at development year 2.

x^{-} = (444 + 500)/2 = 472.
y^{-} = (500 + 555)/2 = 527.5.
$(xy)^{-}$ = (444*500 + 500*555)/2 = 249750
$(x^2)^{-} = (444^2 + 500^2)/2$ = 223568
$(x^{-})^2 = 472^2$ = 222784
Now we find b = $((xy)^{-} - x^{-}*y^{-})/((x^2)^{-} - (x^{-})^2)$ = (249750 - 472*527.5)/(223568 - 222784) = b = 0.9821428571.
Now we find a = y^{-}- b* x^{-} = 527.5 - 0.9821428571*472 = 63.92857143.
For AY 2024, x = 466, so y = a+bx = 63.92857143 + 0.9821428571*466 = **521.6071429**.

Problem S6-45-1. Similar to Question 4 from the 2009 CAS Exam 6. You are given the following information for Insurer Ξ:

Cumulative Closed Claim Counts, expressed in the format
(Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months):
AY 2056: (124, 234, 304, 350)
AY 2057: (150, 225, 320)
AY 2058: (130, 240)
AY 2059: (144)

Selected Ultimate Claim Counts
AY 2056: 380
AY 2057: 400
AY 2058: 390
AY 2059: 410

Selected Cumulative Disposal Rates
At 12 Months: 0.35
At 24 Months: 0.60
At 36 Months: 0.78
At 48 Months: 0.87

Cumulative Paid Loss ($000)
AY 2056: (1000, 2150, 3340, 4400)
AY 2057: (1200, 2000, 3600)
AY 2058: (1200, 1950)
AY 2059: (1240)

Find the following using the disposal-rate frequency-severity technique and a 4% annual severity trend factor:

(a) The expected incremental claim counts for the time periods 36-48 months and 48 months to ultimate for accident year 2057;
(b) The 36-months-to-ultimate tail severity at 2059 levels;
(c) The ultimate losses for accident year 2057.

Solution S6-45-1.

(a) We use the selected disposal rates to estimate the incremental claim count for 36-48 months: (Not-Yet-Opened Claims for 2057)*(Disposal rate at 48 months - Disposal rate at 36 months)/(1 - Disposal rate at 36 months) = (400-320)*(0.87-0.78)/(1-0.78) = 32.727272727, which we round to 33. Since 80 claims remain to be opened in total, the claims opened during the time period 48 months to ultimate will be 80 - 33 = 47.

Incremental claim count for 36-48 months: **33**
Incremental claim count for 48 months to ultimate: **47**

(b) We consider the incremental severity between 36 and 48 months for AY 2056 as (Incremental Paid Loss)/(Incremental Claim Counts) = (4400 - 3340)*1000/(350-304) = 23043.47826. Since we have no information about paid losses from 48 months to ultimate, this is our best estimate regarding severity from 36 months to ultimate. Now we need to trend this answer to 2059 by using the factor 1.04^3: $23043.47826*1.04^3$ = 25920.77913 = **$25,920.78.**

(c) There are 80 claims remaining to be paid in 2057. The losses that have already been paid are $3,600,000. For the remaining claims, we multiply the claim count by the estimated 2057 36-months-to-ultimate severity, which is the 2056 severity trended by one year: 23043.47826*1.04 = 23965.21739. Our answer is thus 3600000 + 80*23965.21739 = **$5,517,217.39**.

Problem S6-47-5. Similar to Question 8 from the 2009 CAS Exam 6. You have the following information about cumulative paid losses for an insurer, expressed in the format
(Number at 12 months, Number at 24 months, Number at 36 months, Number at 48 months).

Cumulative Paid Losses
AY 2044: (40000, 60000, 80000, 90000)
AY 2045: (45000, 64000, 86000)
AY 2046: (51000, 69000)
AY 2047: (80000)

You also have the following information about on-level premiums and exposures:

AY 2044: On-level premium: 60000; Exposures: 300; Average premium: 200
AY 2045: On-level premium: 66000; Exposures: 327; Average premium: 201.83
AY 2046: On-level premium: 70000; Exposures: 350; Average premium: 200
AY 2047: On-level premium: 80000; Exposures: 404; Average premium: 198.02

Use two diagnostics to show why it would not be proper to use the paid development method to estimate ultimate losses for AY 2047.

Solution S6-47-5. First we consider the ratio of cumulative paid loss to on-level premium at 12 months of development:

Ratios of Cumulative Paid Loss to On-Level Premium
AY 2044: 40000/60000 = 0.666667
AY 2045: 45000/66000 = 0.681818
AY 2046: 51000/70000 = 0.728571
AY 2047: 80000/80000 = 1

We note the immense increase in the ratio of cumulative paid loss to on-level premium in AY 2047. It is possible that faster claim settlement in part contributed to this, in which case the paid development method would overestimate the claim development factors and thus the ultimate loss for AY 2047. We can also compare the trend in exposures to the trend in paid losses at 12 months.

Exposure Trend - AY 2044 to AY 2045: 327/300 - 1 = +9%.
Exposure Trend - AY 2045 to AY 2046: 350/327 - 1 = +7.03%
Exposure Trend - AY 2046 to AY 2047: 404/350 - 1 = +15.43%

Paid Loss Trend - AY 2044 to AY 2045: 45000/40000 - 1 = +12.5%
Paid Loss Trend - AY 2045 to AY 2046: 51000/45000 - 1 = +13.33%
Paid Loss Trend - AY 2046 to AY 2047: 80000/51000 - 1 = +56.87%

The paid loss trend is higher than the exposure trend in all cases, but the difference is especially pronounced from AY 2046 to AY 2047. Since average premiums are close to constant, one can infer that a faster settlement rate has been manifested in later accident years, especially in AY 2047. Claim development factors using the chain ladder method will overestimate the ultimate loss in this situation.

Problem S6-48-1. Similar to Question 36 from the 2008 CAS Exam 6. You have the following information as of December 31, 2050:
Earned Premium for Calendar/Accident Year 2047: 35055
Earned Premium for Calendar/Accident Year 2048: 38899
Earned Premium for Calendar/Accident Year 2049: 37600
Earned Premium for Calendar/Accident Year 2050: 41400

Adjusted Premium for Calendar/Accident Year 2047: 34400
Adjusted Premium for Calendar/Accident Year 2048: 36011
Adjusted Premium for Calendar/Accident Year 2049: 37000
Adjusted Premium for Calendar/Accident Year 2050: 40000

Aggregate Reported Loss for Calendar/Accident Year 2047: 22222
Aggregate Reported Loss for Calendar/Accident Year 2048: 16244
Aggregate Reported Loss for Calendar/Accident Year 2049: 12522
Aggregate Reported Loss for Calendar/Accident Year 2050: 4040

Aggregate Loss Report Lag for Calendar/Accident Year 2047: 0.95
Aggregate Loss Report Lag for Calendar/Accident Year 2048: 0.75
Aggregate Loss Report Lag for Calendar/Accident Year 2049: 0.60
Aggregate Loss Report Lag for Calendar/Accident Year 2050: 0.20

Note that "report lag" here refers to the proportion of losses assumed to be *already reported,* not the proportion assumed to have yet to be reported.

Use the Cape Cod method to calculate IBNR as of December 31, 2050.

Solution S6-48-1. We can calculate the expected loss ratio (ELR) as follows:

ELR = (Sum of aggregate reported losses for each year)/(Sum of used-up premiums for each year), where the used-up premium for each year is (Adjusted Premium)*(Aggregate Loss Report Lag).

Thus, ELR = (22222 + 16244 + 12522 + 4040)/(0.95*34400 + 0.75*36011 + 0.60*37000 + 0.20*40000) = 0.6121823486.

By the Cape Cod Method, IBNR = ELR*(Sum of adjusted premiums for each year) - (Sum of aggregate reported losses for each year) =
0.6121823486*(34400 + 36011 + 37000 + 40000) - (22222 + 16244 + 12522 + 4040) = 35214.4122 = **$35,214.41**.

Problem S6-49-4. Similar to Question 27 from the 2008 CAS Exam 6. You have the following information for an insurer by accident year (AY).

AY 2042
Reported loss as of December 31, 2044: 20300
Selected IBNR as of December 31, 2044: 4000
Reported loss as of December 31, 2045: 23500

AY 2043
Reported loss as of December 31, 2044: 12444
Selected IBNR as of December 31, 2044: 10340
Reported loss as of December 31, 2045: 21230

AY 2044
Reported loss as of December 31, 2044: 6005
Selected IBNR as of December 31, 2044: 18000
Reported loss as of December 31, 2045: 13000

You also have the following selected reported loss development factors to ultimate:
From 12 months: 2.602
From 24 months: 1.983
From 36 months: 1.252
From 48 months: 1.055

(a) Based on the selected IBNR and the selected reported loss development factors, what is the expected loss emergence during calendar year 2045 for AY 2042 through AY 2044?

(b) Based on the data and calculations, what conclusions can be drawn regarding the accuracy of the expected loss emergence estimate, compared to actual loss emergence?

Solution S6-49-4. (a) We calculate the expected loss emergence for each accident year as (Selected IBNR)*((LDF as of Dec. 31. 2044)/(LDF as of Dec. 31. 2045) - 1)/((LDF as of Dec. 31. 2044) - 1).

Expected loss emergence for AY 2042: 4000*(1.252/1.055 - 1)/(1.252 - 1) = **2963.965007**.
Expected loss emergence for AY 2043: 10340*(1.983/1.252 - 1)/(1.983 - 1) = **6141.579373**.
Expected loss emergence for AY 2044: 18000*(2.602/1.983 - 1)/(2.602 - 1) = **3504.340484**.

(b) We calculate the actual loss emergence.

For AY 2042: 23500 - 20300 = 3200.
For AY 2043: 21230 - 12444 = 8786.
For AY 2044: 13000 - 6005 = 6995.

For each accident year, actual loss emergence considerably exceeds expected loss emergence. Thus, we can conclude that the selected reserving method is prone to underreserving.

Problem S6-50-4. Similar to Question 31 from the 2009 CAS Exam 6.
You have the following information as of December 31, 2022:
Earned Premium for Calendar/Accident Year 2020: 11500
Earned Premium for Calendar/Accident Year 2021: 12024
Earned Premium for Calendar/Accident Year 2022: 14444

Adjusted Premium for Calendar/Accident Year 2020: 11000
Adjusted Premium for Calendar/Accident Year 2021: 12000
Adjusted Premium for Calendar/Accident Year 2022: 13000

Aggregate Reported Loss for Calendar/Accident Year 2020: 9700
Aggregate Reported Loss for Calendar/Accident Year 2021: 6400
Aggregate Reported Loss for Calendar/Accident Year 2022: 4100

Aggregate Loss Report Lag for Calendar/Accident Year 2020: 0.85
Aggregate Loss Report Lag for Calendar/Accident Year 2021: 0.66
Aggregate Loss Report Lag for Calendar/Accident Year 2022: 0.42

Note that "report lag" here refers to the proportion of losses assumed to be *already reported,* not the proportion assumed to have yet to be reported.

Use the Cape Cod method to calculate IBNR as of December 31, 2022.

Solution S6-50-4. We can calculate the expected loss ratio (ELR) as follows:

ELR = (Sum of aggregate reported losses for each year)/(Sum of used-up premiums for each year), where the used-up premium for each year is (Adjusted Premium)*(Aggregate Loss Report Lag).

Thus, ELR = (9700 + 6400 + 4100)/(11000*0.85 + 12000*0.66 + 13000*0.42) = 0.8886933568.

By the Cape Cod Method, IBNR = ELR*(Sum of adjusted premiums for each year) - (Sum of aggregate reported losses for each year) = 0.8886933568*(11000 + 12000 + 13000) - (9700 + 6400 + 4100) = 11792.96084 = **$11,792.96.**

Problem S6-52-1. Similar to Question 10 from the 2009 CAS Exam 6. It is estimated that the expected loss rate for an insurance company is $200 per exposure unit. The company has no exposure prior to 2030. You also have the following information by accident year (AY):

AY 2030
Exposure Units: 2340
Incurred Loss: 400530
Incurred Loss Development Factor to Ultimate: 1.15

AY 2031
Exposure Units: 3000
Incurred Loss: 360470
Incurred Loss Development Factor to Ultimate: 1.45

AY 2032
Exposure Units: 3560
Incurred Loss: 350900
Incurred Loss Development Factor to Ultimate: 1.90

(a) What is the Bornhuetter-Ferguson estimate of IBNR at December 31, 2032, for all accident years?

(b) What is the Cape Cod estimate of IBNR at December 31, 2032, for all accident years?

(c) Based on the given information and your information, which method from parts (a) and (b) would produce the more accurate estimate of IBNR? Why?

Solution S6-52-1.

(a) The Bornhuetter-Ferguson estimate of IBNR does not depend on losses reported to date. For each accident year it is equal to (Expected Loss)*(1 - 1/(LDF to Ultimate). In this case, the expected loss is (Number of Exposures)*(Loss Rate Per Exposure). Thus, the Bornhuetter-Ferguson estimate of IBNR is Σ((Number of Exposures)*(Loss Rate Per Exposure)*(1 - 1/(LDF to Ultimate)) = 2340*200*(1-1/1.15) + 3000*200*(1-1/1.45) + 3560*200*(1-1/1.90) = 584513.5327 = $**584,513.53**.

(b) The Cape Cod method first derives an empirical expected loss rate per exposure by first calculating "used-up" exposures equal to (Number of Exposures)*(1/(LDF to Ultimate)) and then obtaining the loss rate as (Sum of Reported Losses)/(Sum of Used-Up Exposures). The Cape Cod expected loss rate is thus (400530 + 360470 + 350900)/(2340/1.15 + 3000/1.45 + 3560/1.90) = 186.0163256.

The total Cape Cod IBNR is the Bornhuetter-Ferguson IBNR, multiplied by the ratio of the Cape Cod expected loss rate to the given expected loss rate: 584513.5327*186.0163256/200 = 543645.2981 = $**543,645.30**.

(c) As a diagnostic, we can calculate the expected loss per exposure for each accident year via the chain ladder method as (Reported Losses)*(LDF)/(Number of Exposures):

AY 2030: 400530*1.15/2340 = 196.8416667
AY 2031: 360470*1.45/3000 = 174.2271667
AY 2032: 350900*1.90/3560 = 187.2780899

We note that, based on losses reported to date, the a priori loss rate of $200 per exposure is too high. Changes in the recent loss reporting pattern or the nature of losses have reduced this rate, and the Cape Cod method is more responsive to such changes, as compared to the Bornhuetter-Ferguson method, which relies on predetermined expected losses for its IBNR calculation. Thus, the **Cape Cod method** is preferable.

Problem S6-52-2. Similar to Question 9 from the 2009 CAS Exam 6. The annual loss ratio trend is +3.0%. You are also given on-level earned premiums for each accident year (AY):
AY 2055: On-level premium is 55353.
AY 2056: On-level premium is 62444.
AY 2057: On-level premium is 65725.

Cumulative incurred losses are as follows, expressed in the format
(Amount at 12 months, Amount at 24 months, Amount at 36 months), where applicable:

<u>Cumulative Incurred Losses</u>
AY 2055: (23400, 34440, 40222)
AY 2056: (25650, 37000)
AY 2057: (28000)

The following development factors to ultimate were selected:
12 months to ultimate: 1.333
24 months to ultimate: 1.155
36 months to ultimate: 1.052

(a) Use the expected claims technique to find the IBNR for AY 2057 as of December 31, 2057.

(b) Identify three situations where it might be desirable to use the expected claims technique.

Solution S6-52-2.

(a) We want to calculate the expected ultimate loss ratio for every accident year. We develop and trend the most recent known incurred losses (outermost diagonal of the triangle) and divide them by the on-level earned premium for each accident year.

AY 2055: Expected loss ratio is $40222*1.052*1.03^2$/ 55353 = 0.8109847493.
AY 2056: Expected loss ratio is 37000*1.155*1.03/62444 = 0.7049043943.
AY 2057: Expected loss ratio is 28000*1.333/65725 = 0.5678813237.

To get our total expected loss ratio, we can take the arithmetic mean of the three accident-year expected loss ratios: (0.8109847493 + 0.7049043943 + 0.5678813237)/3 = 0.6945901558.

Expected ultimate losses for AY 2057 are the AY 2057 on-level earned premium, multiplied by the expected loss ratio: 65725*0.6945901558 = 45651.93799. To get IBNR, we subtract already reported losses from expected losses: 45651.93799 - 28000 = 17651.93799 = **17651.94**.

(b) The expected claims method might be useful in the following situations:

1. A new book of business for which prior experience is not available;
2. A long-tailed book of business at the early stages of development, where there is too much volatility in reported losses to use methods that rely on them;
3. A book of business subject to recent macroeconomic or regulatory changes which render past data largely irrelevant.

Problem S6-52-3. Similar to Question 11 from the 2009 CAS Exam 6. You are given the following information, expressed in the format

(Number at 12 months, Number at 24 months, Number at 36 months) by accident year (AY):

Cumulative Paid Losses
AY 2044: (5505, 6666, 7044)
AY 2045: (5880, 6900)
AY 2046: (5400)

Number of Open Claims
AY 2044: (72, 38, 32)
AY 2045: (70, 34)
AY 2046: (66)

Average Case Reserve
AY 2044: (100, 194, 202)
AY 2045: (99, 180)
AY 2046: (103)

The annual case reserve severity trend is selected to be +2.0%.

The 36-month-to-ultimate incurred loss development factor is selected to be 1.04.

(a) What does the Berquist-Sherman case reserve adjustment do, and what is its purpose?

(b) Use the Berquist-Sherman case reserve adjustment to arrive at ultimate losses for AY 2046.

Solution S6-52-3.

(a) The Berquist-Sherman case reserve adjustment takes the most recent (outermost diagonal) known average case reserves and de-trends them by the annual case reserve severity trend to arrive at average case reserve estimates for the same ages of maturity for experience of prior accident years. This done in order to facilitate the assumption of the same case outstanding adequacy for all calendar years as exists in the current calendar year.

(b) We first de-trend average case outstanding to get the adjusted average case outstanding:

Adjusted Average Case Reserve
AY 2044: ($103/1.02^2$, 180/1.02, 202)
AY 2045: (103/1.02, 180)
AY 2046: (103)

Adjusted Average Case Reserve
AY 2044: (99, 176.47, 202)
AY 2045: (100.98, 180)
AY 2046: (103)

Now we can estimate adjusted reported claims for each time period as (Paid Claims) + (Adjusted Average Case Reserve)*(Number of Open Claims):

Adjusted Reported Claims
AY 2044: (5505 + 99*72, 6666 + 176.47*38, 7044 + 202*32)
AY 2045: (5880 + 100.98*70, 6900 + 180*34)
AY 2046: (5400 + 103*66)

Adjusted Reported Claims
AY 2044: (12633, 13371.86, 13508)
AY 2045: (12948.6, 13020)
AY 2046: (12198)

Using this information, we can calculate weighted-average age-to-age factors for adjusted reported claims:
Factor for 12 months to 24 months: (13371.86 + 13020)/(12633 + 12948.6) = 1.031673547
Factor for 24 months to 36 months: 13508/13371.86 = 1.010181082
Factor for 36 months to ultimate: 1.04 (given)

Factor for 12 months to ultimate: 1.031673547*1.010181082*1.04 = 1.083864183.

Our estimate of ultimate losses for AY 2046 is thus 12198*1.083864183 = 13220.97531 = **13220.98.**
(Note that some minor discrepancies with this answer may arise due to rounding.)

Problem S6-52-4. Similar to Question 12 from the 2009 CAS Exam 6. You know that the following incremental paid losses occurred by accident year:

AY 2034: Valuation Date: December 31, 2034; Incremental paid loss: 1140
AY 2034: Valuation Date: December 31, 2035; Incremental paid loss: 240
AY 2035: Valuation Date: December 31, 2035; Incremental paid loss: 210
AY 2034: Valuation Date: December 31, 2036; Incremental paid loss: 140
AY 2035: Valuation Date: December 31, 2036; Incremental paid loss: 1240
AY 2036: Valuation Date: December 31, 2036; Incremental paid loss: 1000

You also know that the 36-month-to-ultimate paid loss development factor is 1.03.

(a) Develop a triangle of cumulative paid losses on the basis of this information.

(b) Use the chain ladder method and volume-weighted-average development factors to estimate the unpaid claim liability for AY 2036 as of December 31, 2036.

Solution S6-52-4.

(a) We develop our cumulative paid loss triangle by accident year in the format (Number at 12 months, Number at 24 months, Number at 36 months):

Cumulative Paid Losses
AY 2034: (1140, 1140+240, 1140+240+140)
AY 2035: (210, 210+1240)
AY 2036: (1000)

Cumulative Paid Losses
AY 2034: (1140, 1380, 1520)
AY 2035: (210, 1450)
AY 2036: (1000)

(b) We find the volume-weighted age-to-age factors:

Factor for 12 months to 24 months: (1380 + 1450)/(1140 + 210) = 2.096296296
Factor for 24 months to 36 months: 1520/1380 = 1.101449275
Factor for 36 months to ultimate: 1.03 (given)

Factor for 12 months to ultimate: 2.096296296*1.101449275*1.03 = 2.378232958.

The estimated unpaid claims for AY 2036 are thus 1000*2.378232958 - 1000 = 1378.232958 = **1378.23**.

Problem S6-54-1. Similar to Question 14 from the 2009 CAS Exam 6. You are given the following information about an insurer's book of business by calendar year (CY):
CY 2024: Paid ULAE: 4444; Paid Loss & ALAE: 24000; Reported Loss & ALAE: 33000; Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year: 40000

CY 2025: Paid ULAE: 3800; Paid Loss & ALAE: 18000; Reported Loss & ALAE: 29000; Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year: 38000

CY 2026: Paid ULAE: 5190; Paid Loss & ALAE: 30000; Reported Loss & ALAE: 34000; Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year: 42000

You also know the following information as of December 31, 2026, for all accident years combined:
Case Reserves: 90990
IBNR: 40440
Ultimate Loss & ALAE: 120111

(a) Find the ULAE reserve as of December 31, 2026, using the classical method.

(b) Find the ULAE reserve as of December 31, 2026, using Kittel's refinement to the classical method.

(c) Find the ULAE reserve as of December 31, 2026, using the Conger-Nolibos generalized approach with the Bornhuetter-Ferguson method. Assume that 75% of work is expended when opening a claim, and 25% of the work is expended when maintaining the claim. No work is expended when closing the claim.

(d) How does the Conger-Nolibos generalized approach (i) more satisfactorily address growing books of business and (ii) make possible more realistic assumptions regarding the distribution of ULAE over the lifetime with regard to opening, closing, and maintaining claims, as compared to the classical method and Kittel's refinement?

Solution S6-54-1.

(a) In the classical method, we first calculate the ratio of paid ULAE to paid claims. We can select as our total ratio the sum of paid ULAE over all the given years, divided by the sum of paid claims (loss and ALAE) over all the given years: (4444 + 3800 + 5190)/(24000 + 18000 + 30000) = 0.186583333.

The classical method's estimate of the ULAE reserve is (ULAE Ratio)*(0.5*Case Reserve + IBNR) = 0.186583333*(0.5*90990 + 40440) = 16034.03872 = **16034.04**.

(b) Using the Kittel refinement, the ULAE ratio is calculated as (Paid ULAE)/(Claims Basis), where in the case of the Kittel refinement, the Claims Basis is the average of reported claims and paid claims. Here, our claims basis for all of the years is ((24000 + 33000)/2 + (18000 + 29000)/2 + (30000 + 34000)/2) = 84000, and our ULAE ratio is (4444 + 3800 + 5190)/84000 = 0.1599285714.

The Kittel refinement preserves the classical method's estimate of the ULAE reserve: (ULAE Ratio)*(0.5*Case Reserve + IBNR) = 0.1599285714*(0.5*90990 + 40440) = 13743.46178 = **13743.46**.

(c) Using the Conger-Nolibos approach, the ULAE ratio is calculated as (Paid ULAE)/(Claims Basis). The total Claims Basis is calculated as follows (assuming no work is expended closing claims): (% of Work Expended Opening a Claim)*(Estimated Ultimate Loss & ALAE on Claims Reported in Calendar Year) + (% of Work Maintaining a Claim)*(Paid Claims) = 0.75*(40000 + 38000 + 42000) + 0.25*(24000 + 18000 + 30000) = 108000.

The ULAE ratio is thus (4444 + 3800 + 5190)/108000 = 0.124388889.

Applying the Bornhuetter-Ferguson method to the Conger-Nolibos approach, the ULAE Reserve is (ULAE Ratio)*(Ultimate Loss & ALAE - Total Claims Basis) = 0.124388889*(120111 - 108000) = 1506.473835 = **1506.47**.

(d) The Conger-Nolibos generalized approach (i) more satisfactorily addresses growing books of business than approaches that depend on comparing paid ULAE solely to paid claims or to the average of paid and reported claims, because the ULAE for a particular calendar year may not match to the paid claims of that calendar year, and the mismatch may be material if the volume of business written is changing. The Conger-Nolibos approach allows more flexibility in its claims basis and ties the claims basis to a more realistic assessment of how ULAE are distributed throughout the life of the claim.

The Conger-Nolibos generalized approach (ii) also relaxes the assumption that 50% of ULAE are spent opening a claim, and the other 50% are spent opening a claim. The generalized approach allows any mathematically legitimate percentage distribution of costs among opening, maintaining, and closing claims.

Problem S6-54-2. Similar to Question 15 from the 2009 CAS Exam 6.

(a) If it is known that the payment rate for claims slowed down over a particular time period, which of the following methods would it be inappropriate to use, and why?

(i) The Unadjusted Reported Development Method
(ii) The Unadjusted Paid Development Method
(iii) The Both-Case-and-Payment-Rate-Adjusted Reported Development Method
(iv) The Payment-Rate-Adjusted Paid Development Method

(b) You are given the following values for Accident Year 2033 as of December 31, 2033:

Claims Reported: 55555
Claims Paid: 14244
Earned Premium: 130555
Estimated Ultimate Claim Count: 57
Open and IBNR Count: 50
Ultimate Claims, Using Unadjusted Reported Development Method: 60000
Ultimate Claims, Using Unadjusted Paid Development Method: 52222
Ultimate Claims, Using Both-Case-and-Payment-Rate-Adjusted Reported Development Method: 80220
Ultimate Claims, Using Payment-Rate-Adjusted Paid Development Method: 81232

Using each of the methods that were *not* rejected in part (a), calculate (i) the ultimate claim ratio, (ii) the ultimate severity, and (iii) the unpaid severity for Accident Year 2033.

(c) Describe one conclusion that might be drawn from the diagnostic results in part (b).

Solution S6-54-2.

(a) The **(ii) Unadjusted Paid Development Method** should be rejected, as it does not take into account the change in paid development as a result of the slowed payment rate. As a result, the unadjusted paid development will understate ultimate losses for years after the slowdown of the payment rate.

(b)

Using Unadjusted Reported Development Method
Claim Ratio: (Ultimate Claims/Earned Premium) = 60000/130555 = **0.459576424**.
Ultimate Severity: (Ultimate Claims/Ultimate Claim Count) = 60000/57 = **1052.631579**.
Unpaid Severity: (Ultimate Claims - Paid Claims)/(Open & IBNR Count) = (60000-14244)/50 = **915.12**.

Using Both-Case-and-Payment-Rate-Adjusted Reported Development Method
Claim Ratio: (Ultimate Claims/Earned Premium) = 80220/130555 = **0.614453679**.
Ultimate Severity: (Ultimate Claims/Ultimate Claim Count) = 80220/57 = **1407.368421**.
Unpaid Severity: (Ultimate Claims - Paid Claims)/(Open & IBNR Count) = (80220-14244)/50 = **1319.52**.

Using Payment-Rate-Adjusted Paid Development Method
Claim Ratio: (Ultimate Claims/Earned Premium) = 81232/130555 = **0.622205201**.
Ultimate Severity: (Ultimate Claims/Ultimate Claim Count) = 81232/57 = **1425.122807**.
Unpaid Severity: (Ultimate Claims - Paid Claims)/(Open & IBNR Count) = (81232-14244)/50 = **1339.76**.

(c) It can be seen that the Unadjusted Reported Development Method gives a dramatically lower estimate of the claim ratio and the severities than the other two methods. This might be due to other changes that unadjusted methods do not capture, such as case outstanding adequacy, which would affect a method based on reported claim development, even though payment rates would not.